Taiwan Invitational Mathematics Competitions

1999-2012

with

answer keys

Invitational World Youth Mathematics Intercity Competition 1999

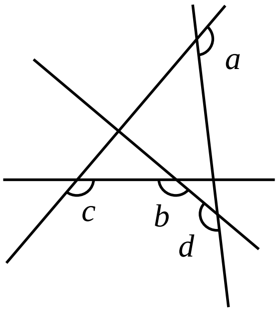
Individual Contest

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Find the remainder when 122333444455555666666777777788888888999999999 is divided by 9.

2. Find the sum of the angles *a*, *b*, *c* and *d* in the following figure.



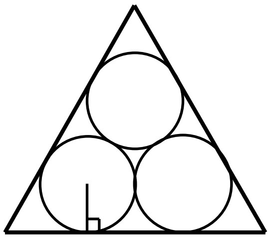
3. How many of the numbers $1^2,\ 2^2,\ \ldots,\ 1999^2$ have odd numbers as their tens-digits?

4. The height of a building is 60 metres. At a certain moment during daytime, it casts a shadow of length 40 metres. If a vertical pole of length 2 metres is erected on the roof of the building, find the length of the shadow of the pole at the same moment.

5. Calculate $1999^2 - 1998^2 + 1997^2 - 1996^2 + \cdots + 3^2 - 2^2 + 1^2$.

6. Among all four-digits numbers with 3 as their thousands-digits, how many have exactly two identical digits?

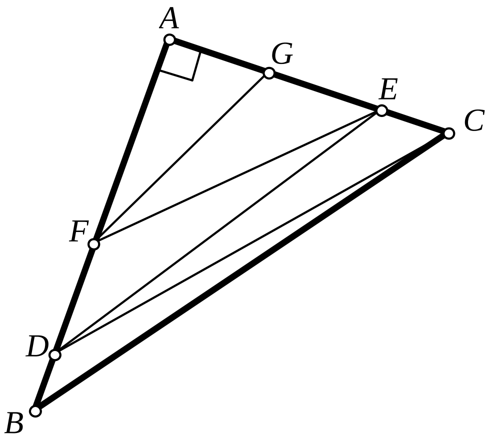
7. The diagram below shows an equilateral triangle of side 1. The three circles touch each other and the sides of the triangle. Find the radii of the circles.



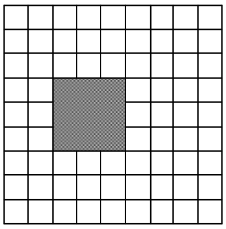
8. Let *a*, *b* and *c* be positive integers. The sum of 160 and the square of *a* is equal the sum of 5 and the square of *b*. The sum of 320 and the square of *a* is equal to the sum of 5 and the square of *b*. Find *a*.

9. Let *x* be a two-digit number. Denote by $f(x)$ the sum of *x* and its digits minus the product of its digits. Find the value of *x* which gives the largest possible value for $f(x)$.

10. The diagram below shows a triangle *ABC*. The perpendicular sides *AB* and *AC* have lengths 15 and 8 respectively. *D* and *F* are points on *AB*. *E* and *G* are points on *AC*. The segments *CD*, *DE*, *EF* and *FG* divide triangle *ABC* into five triangles of equal area. The length of only one of these segments is integral. What is that length?



11. How many squares are formed by the grid lines in the diagram below?

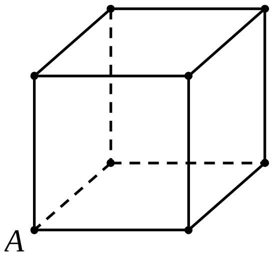


12. There are two committees A and B. Committee A had 13 members while committee B had 6 members. Each member is paid $6000 per day for attending the first 30 days of meetings, and $9000 per day thereafter. Committee B met twice as many days as Committee A, and the expenditure on attendance were the same for the two committees. If the total expenditure on attendance for these two committees was over $3000000, how much was it?

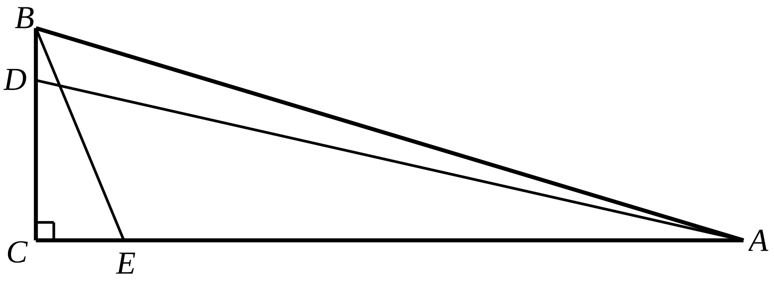
Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. The diagram below shows a cubical wire framework of side 1. An ant starts from a vertex and crawls along the sides of the framework. If it does not repeat any part of its path and finally returns to the starting vertex, what is the longest possible length of the path it has travelled?



2. In the diagram below, BC is perpendicular to AC. D is a point on BC such that $BC = 4BD$. E is a point on AC such that $AC = 8CE$. If $AD = 164$ and $BE = 52$, determine AB.



3. When a particular six-digit number is multiplied by 2, 3, 4, 5 and 6 respectively, each of the products is still a six-digit number with the same digits as the original number but in a different order. Find the original number.

Invitational World Youth Mathematics Intercity Competition 1999

Team Contest

1. (a) Decompose $9^8+7^6+5^4+3^2+1$ into prime factors.
   (b) Find two distinct prime factors of $2^{30}+3^{20}$.

2. The cards in a deck are numbered 1, 3, $\cdots$, $2n-1$. In the k-th step, $1 \le k \le n$, $2k-1$ cards from the top of the deck are transferred to the bottom one at a time. We want the new card on the top to be $2k-1$, which is then set aside. After n steps, the whole deck should be set aside in increasing order. How should the deck be stacked in order for this to happen, if
   (a) n=10;
   (b) n=30?

3. (a) Express 1 as a sum of trhe reciprocals of distinct integers, one of which is 5.
   (b) Express 1 as a sum of trhe reciprocals of distinct integers, one of which is 1999.

4. (a) Show how to dissect a square into 1999 squares which may have different sizes.
   (b) Dissect the first two shapes in the diagram below into the ten or fewer pieces which can be reassembled to form the third shape.

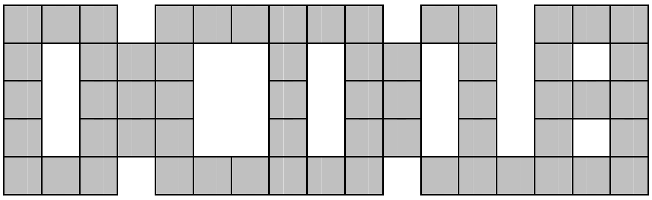


Figure (1)

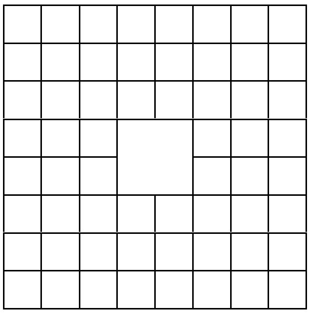
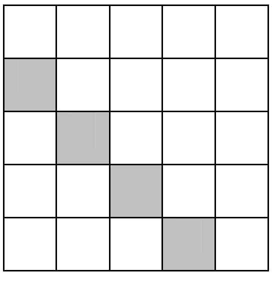


Figure (2)

5. The diagram below shows a blank 5 × 5 table. Each cell is to be filled in with one of the numbers 1, 2, 3, 4 and 5, so there is exactly one number of each kind in each row, each column and each of the two long diagonals. The score of a completed table is the sum of the numbers in the four shaded cells. What is the highest possible score of a completed table? ◦



1999 IWYMIC Answers

Individual

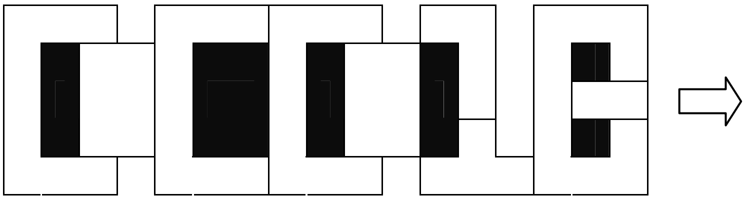
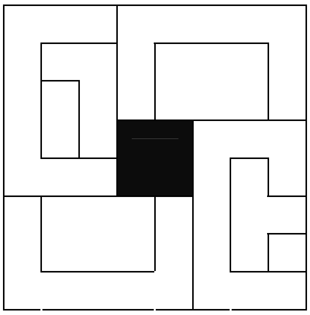
Part I

1.	6	**2.**	540°	**3.**	400	**4.**	$\frac{4}{3}$
5.	1999000	**6.**	432	**7.**	$\frac{\sqrt{3}-1}{4}$	**8.**	13
9.	90	**10.**	10	**11.**	190	**12.**	14040000

Part II

1.	8	**2.**	$16\sqrt{109}$	**3.**	142587

Team

1.	(a)	$43165005 = 3\times5\times13\times41\times5399$
	(b)	13、61
2.	(a)	11、1、5、7、15、3、13、19、9、19
	(b)	13、1、47、33、25、3、57、45、49、55、43、5、19、39、11、17、21、51、29、7、41、15、31、23、27、59、35、53、37、9
3.	(a)	2、5、8、12、20、24
	(b)	1x2、2x3、3x4、4x5、⋯、1998x1999、1999
4.	(b)	 Figure A  Figure B
5.	17, for example	

3	5	4	2	1
5	1	2	3	4
2	4	5	1	3
1	2	3	4	5
4	3	1	5	2

Invitational World Youth Mathematics Intercity Competition 2000

Individual Contest

Section A.
In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Find the unit digit of 17^{2000}.

2. The sum of four of the six fractions $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{12}$, $\frac{1}{15}$ and $\frac{1}{18}$ is equal to $\frac{2}{3}$. Find the product of the other two fractions.

3. Find the smallest odd three-digit multiple of 11 whose hundreds digit is greater than its units digit.

4. Find the sum of all the integers between 150 and 650 such that when each is divided by 10, the remainder is 4.

5. Find the quotient when a four-thousand-digit number consisting of two thousand 1s followed by two thousand 2s is divided by a two-thousand-digit numbers every digit of which is 6.

6. Find two unequal prime numbers p and q such that $p+q=192$ and $2p-q$ is as large as possible.

7. D is a point on the side BC of a triangle ABC such that $AC=CD$ and $\angle CAB = \angle ABC + 45°$. Find $\angle BAD$.

8. Let a, b, c, d and e be single-digit numbers. If the square of the fifteen-digit number $100000035811ab1$ is the twenty-nine-digit number $1000000cde22474824442657353 61$, find the value of $a+b+c-d-e$.

9. P is a point inside a rectangle $ABCD$. If $PA=4$, $PB=6$ and $PD=9$, find PC.

10. In the Celsius scale, water freezes at $0°$ and boils at $100°$. In the Sulesic scale, water freezes at $20°$ and boils at $160°$. Find the temperature in the Sulesic scale when it is $215°$ in the Celsius scale.

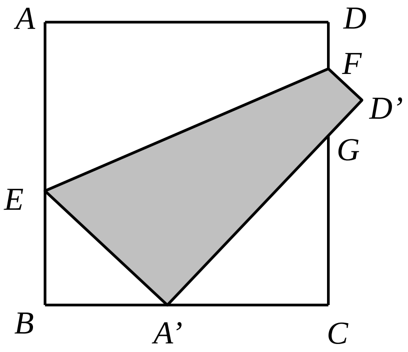
11. The vertices of a square all lie on a circle. Two adjacent vertices of another square lie on the same circle while the other two lie on one of its diameters. Find the ratio of the area of the second square to the area of the first square.

12. Ten positive integers are written in a row. The sum of any three adjacent numbers is 20. The first number is 2 and the ninth number is 8. Find the fifth number.

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. E is a point on the side AB and F is a point on the side CD of a square $ABCD$ such that when the square is folded along EF, the new position A' of A lies on BC. Let D' denote the new position of D and let G be the point of intersection of CF and $A'D'$. Prove that $A'E+FG=A'G$.



2. Twenty distinct positive integers are written on the front and back of ten cards, one on each face of every card. The sum of the two integers on each card is the same for all ten cards, and the sum of the ten integers on the front of the cards is equal to the sum of the ten integers on the back of the cards. The integers on the front of nine of the cards are 2, 5, 17, 21, 24, 31, 35, 36 and 42. Find the integer on the front of the remaining card.

3. Given are two three-digit numbers a and b and a four-digit number c. If the sums of the digits of the numbers $a+b$, $b+c$ and $c+a$ are all equal to 3, find the largest possible sum of the digits of the number $a+b+c$.

Invitational World Youth Mathematics Intercity Competition 2000

Team Contest

1. E is the midpoint of side BC of a square $ABCD$. H is the point on AE such that $BE = EH$. X is the point on AB such that $AH = AX$. Prove that : $AB \times BX = AX^2$.

2. Four non-negative integers have been entered in the following 5×5 table. Fill in the remaining 21 spaces with positive integers so that the sum of all the numbers in each row and in each column is the same.

	82			
				79
		103		
0				

3. For $n \geq 1$, define $a_n = 1000 + n^2$. Find the greatest value of the greatest common divisor of a_n and a_{n+1}.

4. Five teachers predict the order of finish of five classes A, B, C, D and E in an examination.

Guesses	**First**	**Second**	**Third**	**Fourth**	**Fifth**
Teacher 1	A	B	C	D	E
Teacher 2	E	D	A	B	C
Teacher 3	E	B	C	D	A
Teacher 4	C	E	D	A	B
Teacher 5	E	B	C	A	D

After the examination, which produces no ties between classes, it turns out that each of two teachers guesses correctly the ranks of two of the classes but is wrong about the ranks of the other three. The other three teachers are wrong about the rank of every class. Find the order of finish of the classes.

5. Find all triples (a, b, c) of positive integers such that $a \leq b \leq c$ and $\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right) = 2$.

6. Each team is given 50 square cardboard pieces and 50 equilateral triangular cardboard pieces. Using as many of these pieces as faces, construct a set of different convex polyhedra. Two polyhedra with the same numbers of vertices, edges, square faces and triangular faces are not considered different.

2000 IWYMIC Answers

Individual

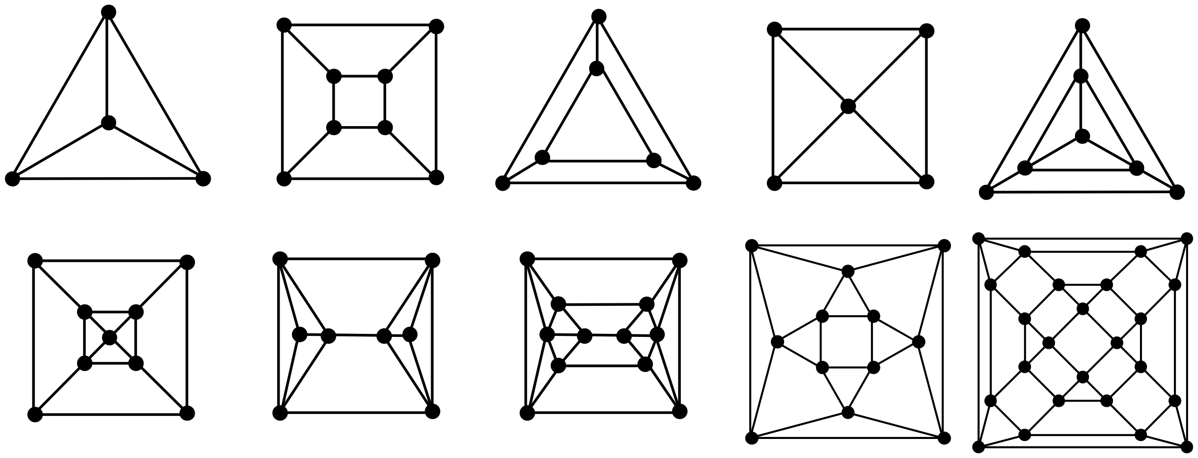
Part I

1.	1	**2.**	$\frac{1}{180}$	**3.**	231	**4.**	19950
5.	$\underbrace{166\cdots66}_{1998\text{ terms}}7$	**6.**	(181, 11)	**7.**	22.5°	**8.**	5
9.	$\sqrt{101}$	**10.**	321	**11.**	2:5	**12.**	10

Part II

2.	37	**3.**	10800

Team

<table>
<tr><td>2.</td><td><table><tr><td>83</td><td>82</td><td>2</td><td>79</td><td>21</td></tr><tr><td>103</td><td>1</td><td>82</td><td>2</td><td>79</td></tr><tr><td>79</td><td>103</td><td>1</td><td>82</td><td>2</td></tr><tr><td>2</td><td>79</td><td>103</td><td>1</td><td>82</td></tr><tr><td>0</td><td>2</td><td>79</td><td>103</td><td>83</td></tr></table></td><td>3.</td><td>4001</td></tr>
<tr><td>4.</td><td>C、D、A、E、B</td><td>5.</td><td>(2, 4, 15)、(2, 5, 9)、(2, 6, 7)、(3, 3, 8)及(3, 4, 5)</td></tr>
<tr><td>6.</td><td colspan="3">Using all the pieces, we construct the following set of ten convex polyhedra. Each is represented in two dimensions by what is known as its Schlegel diagram.
</td></tr>
</table>

Invitational World Youth Mathematics Intercity Competition 2001

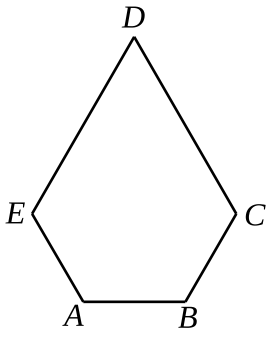
Individual Contest

Section A.

In this section, there are 10 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 6 points.

1. Find all integers *n* such that 1 + 2 + .. + *n* is equal to a 3-digit number with identical digits.

2. In a convex pentagon *ABCDE*, $\angle A = \angle B = 120°$, $EA = AB = BC = 2$, and $CD = DE = 4$. Find the area of the pentagon *ABCDE*.



3. If I place a 6 cm by 6 cm square on a triangle, I can cover up to 60% of the triangle. If I place the triangle on the square, I can cover up to $\frac{2}{3}$ of the square. What is the area of the triangle?

4. Find a set of four consecutive positive integers such that the smallest is a multiple of 5, the second is a multiple of 7, the third is a multiple of 9, and the largest is a multiple of 11.

5. Between 5 and 6 o'clock, a lady looked at her watch. She mistook the hour hand for the minute hand and vice versa. As a result, she thought the time was approximately 55 minutes earlier. Exactly how many minutes earlier was the mistaken time?

6. In triangle *ABC*, the incircle touches the sides *BC*, *CA* and *AB* at *D*, *E* and *F* respectively. If the radius of the incircle is 4 units and if *BD*, *CE* and *AF* are consecutive integers, find the length of the three sides of *ABC*.

7. Determine all primes *p* for which there exists at least one pair of integers *x* and *y* such that $p+1=2x^2$ and $p^2+1=2y^2$.

8. Find all real solutions of
$$\sqrt{3x^2-18x+52}+\sqrt{2x^2-12x+162}=\sqrt{-x^2+6x+280}.$$

9. Simplify $\sqrt{12-\sqrt{24}+\sqrt{39}-\sqrt{104}}-\sqrt{12+\sqrt{24}+\sqrt{39}+\sqrt{104}}$ into a single numerical value.

10. Let *M* = 1010101…01 where the digit 1 appears *k* times. Find the least value of *k* so that 1001001001001 divides *M* ?

Section B.

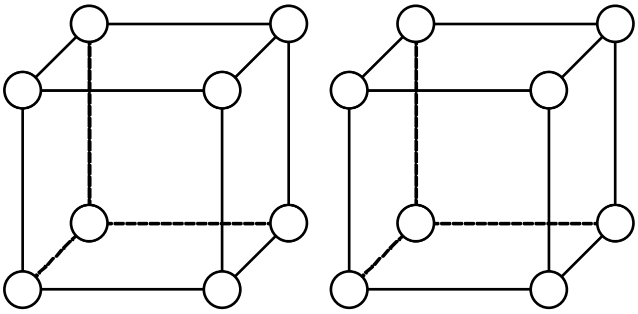
Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. Given that *a* and *b* are unequal positive real numbers, let $A=\frac{a+b}{2}$ and $G=\sqrt{ab}$. Prove that the following inequality holds: $G<\frac{(a-b)^2}{8(A-G)}<A$.
2. Find the range of *p* such that the equation $3^{2x}-3^{x+1}=p$ has two different real positive roots.
3. The four vertices of a square lie on the perimeter of an acute scalene triangle, with one vertex on each of two sides and the other two vertices on the third side. If the square is be as large as possible, should the side of the triangle containing two vertices of the square be the longest, the shortest or neither? Justify your answer.

Invitational World Youth Mathematics Intercity Competition 2001

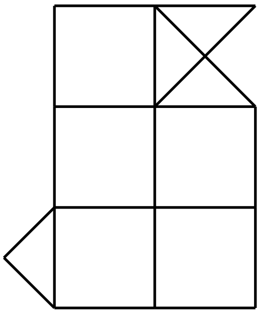
Team Contest

1. Fill in the numbers 1 to 16 on the vertices of two cubes, one number on each vertex with no repetition, such that the sum of the numbers on the four vertices of each face is the same.

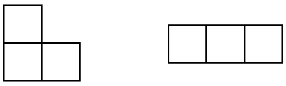


2. Arrange the numbers 1 to 20 in a circle such that the sum of two adjacent numbers is prime.

3. The figure in the diagrm below is a 2 × 3 rectangle, with one-quarter of the top right square cut off and attached to the bottom left square. Cut the figure along some polygonal line into two identical pieces.

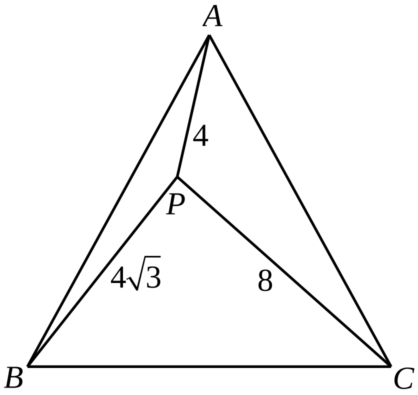


4. A 1×1 cell said to be removable if its removal from an 8×8 square leaves behind a figure which can be tiled by 21 copies of each of the two figures shown in the diagram below. How many removable cells are there in an 8 × 8 square?



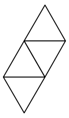
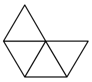
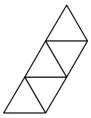
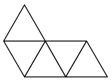
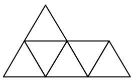
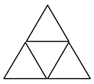
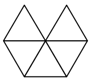
5. The four-digit number 3025 is the square of the sum of the number formed of its first two digits and the number formed of its last two digits, namely, $(30 + 25)^2 = 3025$. Find all other four-digit numbers with this property.

6. P is a point inside an equilateral triangle ABC such that $PA=4$, $PB=4\sqrt{3}$ and $PC=8$. Find the area of triangle ABC.

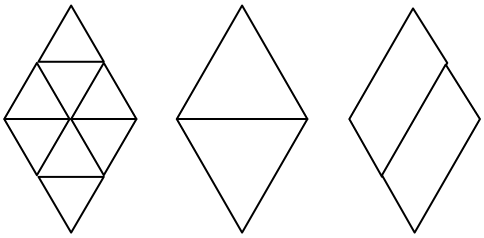


7. The fraction $\frac{1}{4}$ has an interesting property. The numerator is a single-digit number 1 and the denominator is a larger single-digit number 4. If we add the digit 6 after the digit 1 in the numerator n times and add the digit 6 before the digit 4 in the denominator n times also, the fraction $\frac{166\cdots6}{66\cdots64}=\frac{1}{4}$ has the same value. Determine all other fractions with this property, except that the added digit does not have to be 6.

8. There are seven shapes formed of three or four equilateral triangles connected edge-to-edge, as shown in the 2 × 5 chart below.

					
	1	2	3	4	5
	6	7	8	9	10

For each of the numbered spaces in the chart, find a figure which can be formed from copies of the shape at the head of the row, as well as from copies of the shape at the head of the column. The pieces may be rotated or reflected. The problem in Space 1 has been solved in the diagram below as an example.



2001 IWYMIC Answers

Individual

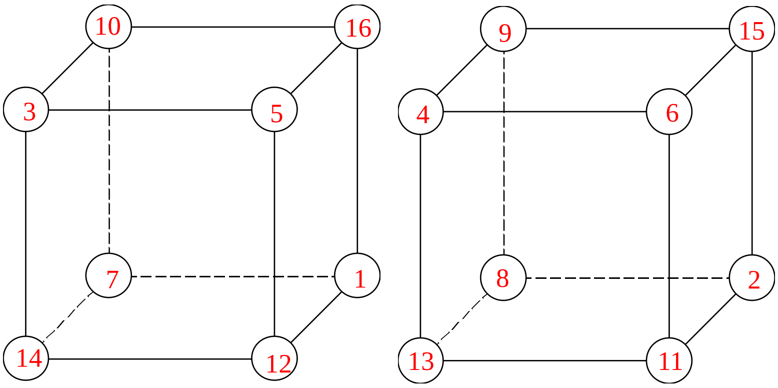
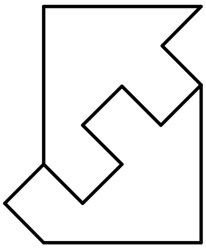
Part I

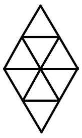
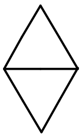
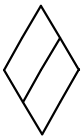
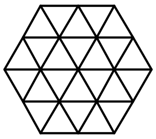
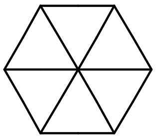
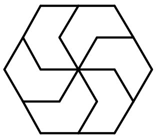
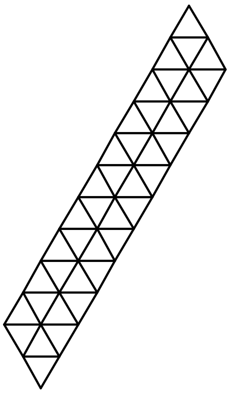
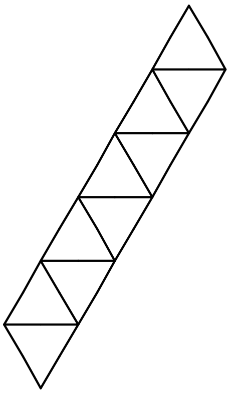
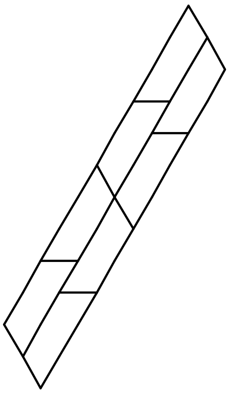
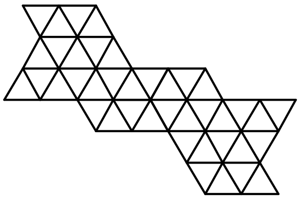
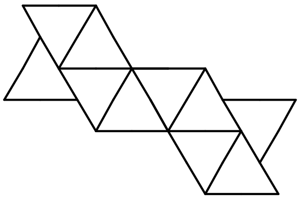
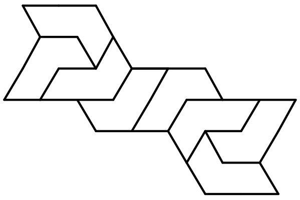
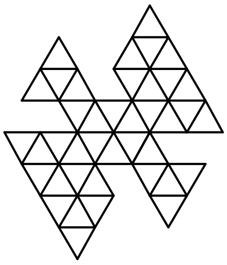
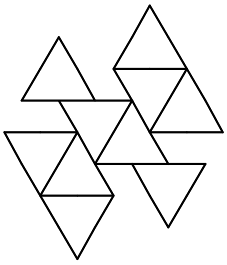
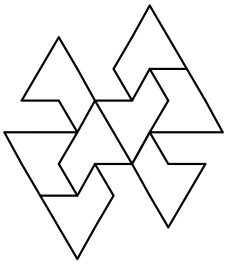
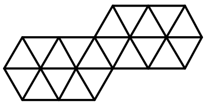
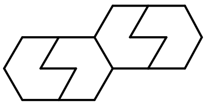
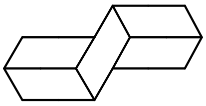
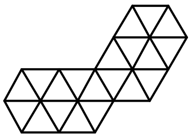
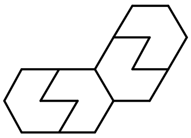
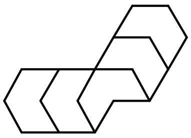
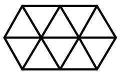
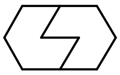
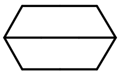
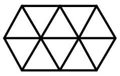
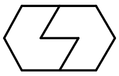
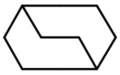
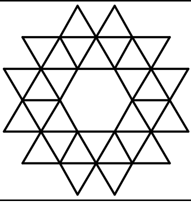
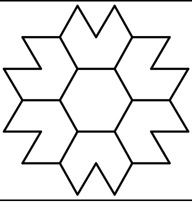
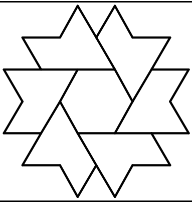
1.	36	**2.**	$7\sqrt{3}$	**3.**	40	**4.**	1735, 1736, 1737, 1738
5.	5:24	**6.**	13, 14, 15	**7.**	7	**8.**	3
9.	-4	**10.**	15				

Part II

2.	$-2\frac{1}{4} < p < -2$	**3.**	One vertex on side a, one vertex on side b and two vertices on side c

Team

1.			
2.	1,2,5,18,19,12,17,14,15,8,3,16,13,10,9,20,11,6,7,4	**3.**	
4.	4	**5.**	2025, 3025, 9801
6.	$28\sqrt{3}$	**7.**	$\frac{2}{5}, \frac{1}{5}, \frac{4}{8}$

8.			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

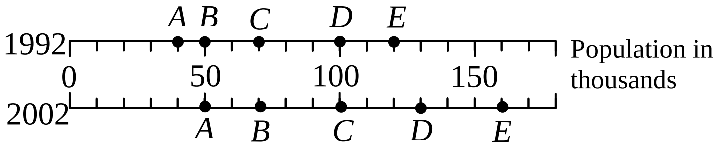
Invitational World Youth Mathematics Intercity Competition 2002

Individual Contest

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. On each horizontal line in the figure below, the five large dots indicate the populations of five branches of City Montessori School in Lucknow: *A, B, C, D* and *E* in the year indicated. Which City Montessori School, Lucknow had the greatest percentage increase in population from 1992 to 2002?

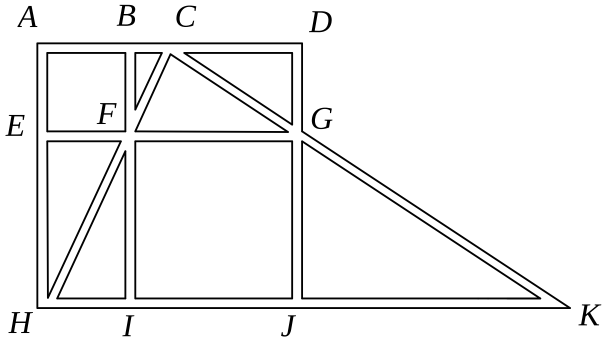


2. If $x = \dfrac{\sqrt{(a+2b)}+\sqrt{(a-2b)}}{\sqrt{(a+2b)}-\sqrt{(a-2b)}}$, what is the numerical value of $bx^2 - ax + b$?

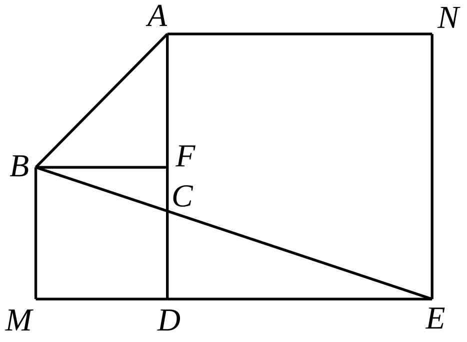
3. To find the value of x^8 given *x*, you need three arithmetic operations: $x^2 = x \cdot x$, $x^4 = x^2 \cdot x^2$ and $x^8 = x^4 \cdot x^4$. To find x^{15}, five operations will do: the first three of them are the same; then $x^{16} = x^8 \cdot x^8$ and $x^{15} = x^{16} \div x$. What is the minimum number of operations (multiplications and divisions) will be needed to find the value of x^{1000}?

4. Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$ where *a*, *b*, *c* and *d* are constants. If $P(1) = 10$, $P(2) = 20$, $P(3) = 30$, what is the value of $P(10) + P(-6)$?

5. The diagram below shows the street map of a city. If three police offcers are to be positioned at street corners so that any point on any street can be seen by at least one offcer, what are the letter codes of these street corners?



6. *ADEN* is a square. *BMDF* is a square such that *F* lies on *AD* and *M* lies on the extension of *ED*. *C* is the point of intersection of *AD* and *BE*. If the area of triangle *CDE* is 6 square units, what is the area of triangle *ABC*?



7. If the 18-digit number $A36\ 405\ 489\ 812\ 706\ 44B$ is divisible by 99, what are all the possible values of (A, B)?

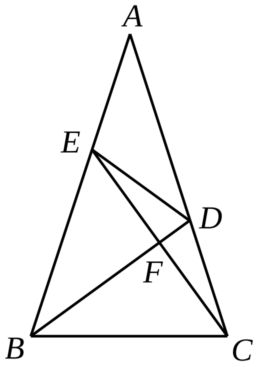
8. Ten people stand in a line. The first goes to the back of the line and the next person sits down so that the person who was third in the line is now first in line. Now the person on the first in line goes to the back of the line and the next person sits down. This process is repeated until only one person remains. What was the original position in line of the only remaining person?

9. In triangle ABC, bisectors AA_1, BB_1 and CC_1 of the interior angles are drawn. If $\angle ABC = 120°$, what is the measure of $\angle A_1B_1C_1 = ?$

10. For how many different real values of k do there exist real numbers x, y and z such that
$$\frac{x+y}{z} = \frac{y+z}{x} = \frac{z+x}{y} = k\ ?$$

11. L is a point on the diagonal AC of a square $ABCD$ such that $AL = 3\ LC$. K is the midpoint of AB. What is the measure of $\angle KLD$?

12. In triangle ABC, $\angle A = 36°$, $\angle ACB = 72°$. D is a point on AC such that BD bisects $\angle ABC$. E is a point on AB such that CE is perpendicular to BD. How many isosceles triangles are in figure?



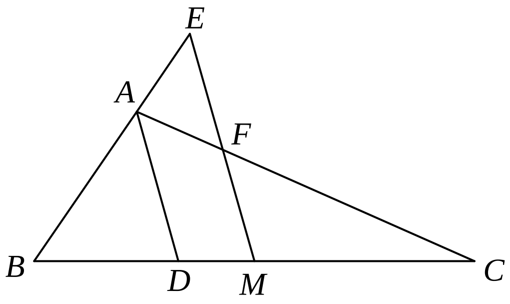
Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. There are two distinct 2-digit numbers which have the same units digit but different tens digits. The quotient when one of them is divided by 9 is equal to the remainder when the other is divided by 9, and vice versa. What is the common units digit?

2. Solve for x, y and z if
$$(x+y)(x+z) = 15$$
$$(y+z)(y+x) = 18$$
$$(z+x)(z+y) = 30$$

3. In triangle ABC, D is the point on BC such that AD bisects $\angle CAB$, and M is the midpoint of BC. E is the point on the extension of BA such that ME is parallel to AD and intersects AC at F. Prove that $\overline{BE} = \overline{CF} = \frac{1}{2}(\overline{AB} + \overline{AC})$.



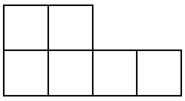
Invitational World Youth Mathematics Intercity Competition 2002

Team Contest

1. Let *m*, *n* and *p* be real numbers. If $a = x^{m+n} \cdot y^{p}$; $b = x^{n+p} \cdot y^{m}$; and $c = x^{p+m} \cdot y^{n}$, what is the numercial value of $a^{m-n} \cdot b^{n-p} \cdot c^{p-m}$?

2. Let $f(x) = \dfrac{bx+1}{2x+a}$ where *a* and *b* are constants such that $ab \neq 2$.
   (a) If $f(x) \cdot f(\frac{1}{x}) = k$ for all *x*, what is the numerical value of *k*?
   (b) Using the result of (a), if $f(x) \cdot f(\frac{1}{x}) = k$, then find the numerical value of *a* and *b*.

3. Prove or disprove that it is possible to form a rectangle using an odd number of copies of the figure shown in the diagram below.



4. Find all integers $x \geq y$, positive and negative, such that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{14}$

5. Four brothers divide 137 gold coins among themselves, no two receiving the same number. Each brother receives a number of gold coins equal to an integral multiple of that received by the next younger brother. How many gold coins does each brother receive? Find all solutions.

6. In $\triangle ABC$, $AB = BC$. A line through *B* cuts *AC* at *D* so that inradius of triangle *ABD* is equal to the exradius of triangle *CBD* opposite *B*. Prove that this common radius is equal to one quarter of the altitude from *C* to *AB*.

7. Two circles of radii *a* and *b* respectively touch each other externally. A third circle of radius *c* touches these two circles as well as one of their common tangents. Prove that $\dfrac{1}{\sqrt{c}} = \dfrac{1}{\sqrt{a}} + \dfrac{1}{\sqrt{b}}$.

8. Robbie the robot is locked in a solar panel and must get out through the hatch located at the centre of the panel, marked by ∗. Locked in with him are other dummy robots under his control. Each robot is mobile, but it can only move along a row or a column directly toward another robot, and can only stop when it bumps into the target robot, stopping in the empty space in front. In each scenario, four moves are allowed, where a continuous sequence of motions by the same robot counts as one move. Robbie is denoted by R.EXAMPLE

			A			
B			∗			
R	C			D		

As an example, C-D-A-B and R-D-A is a two-move solution to the above scenario.

Scenario 1

A						B
			C			R
D						E

Scenario 2

			A			B
C			*		R	
			D			E

Scenario 3

A		B				C
			*			
R	D					E

Scenario 4

A		B				C
			*			
	R	D				E

Scenario 5

	A			B		C
D			*			
		R				E

Scenario 6

A				B		C
			*			
		R	D			E

Scenario 7

	A				B	
			*			C
	D		R			E

Scenario 8

A						B
C			*			
R	D					E

2002 IWYMIC Answers

Individual

Part I

1.	C	**2.**	0	**3.**	12	**4.**	8104
5.	B, G, H	**6.**	6	**7.**	1	**8.**	5
9.	90°	**10.**	2	**11.**	90°	**12.**	7

Part II

1.	5	**2.**	(1, 2, 4) and (-1, -2, -4)

Team

1.	1	**2.**	(a)	$\frac{1}{4}$
			(b)	$a=8$ 、$b=\frac{1}{4}$
3.	Yes	**4.**	9 solutions: (15, 210), (16, 112), (18, 63), (21, 42), (28, 28), (13, -182), (12, -84), (10, -35), (7, -14).	
5.	120 、12 、4 、1 or 88 、44 、4 、1 or 112 、16 、8 、1 or 96 、32 、8 、1			
8.	1	E-D，B-A-E，C-B，R-C		
	2	E-D，D-A-C，B-E-C，R-B		
	3	E-C，B-A-D-E，C-A-D-B，R-A-C		
	4	A-B-D，C-B，D-E，R-D-C-A		
	5	C-B-E，D-C，B-A，R-B-D		
	6	E-D-B，B-A，C-B，R-C-E		
	7	D-A，A-B，C-D，R-E-A-C		
	8	A-B，D-E-A-C，B-E-D，R-E-A-B		

5th Invitational World Youth Mathematics Inter-City Competition

第五屆青少年數學國際城市邀請賽

Individual Contest Time limit: 120 minutes 2004/8/3, Macau

Team: ________________Contestant No.______________ Score: __________________

Name:______________________________

Section I:
In this section, there are 12 questions, fill in the correct answers in the spaces provided at the end of each question. Each correct answer is worth 5 points.

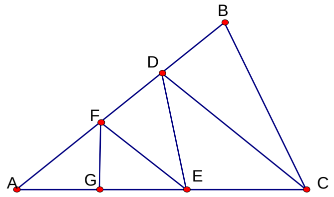
1. Let O_1, O_2 be the centers of circles C_1, C_2 in a plane respectively, and the circles meet at two distinct points A, B. Line O_1A meets the circle C_1 at point P_1, and line O_2A meets the circle C_2 at point P_2. Determine the maximum number of points lying in a circle among these 6 points A, B, O_1, O_2, P_1 and P_2.

   Answer:___________________________.

2. Suppose that a, b, c are real numbers satisfying $a^2 + b^2 + c^2 = 1$ and $a^3 + b^3 + c^3 = 1$. Find all possible value(s) of $a + b + c$.

   Answer:___________________________.

3. In triangle *ABC* as shown in the figure below, *AB*=30, *AC*=32. *D* is a point on *AB*, *E* is a point on *AC*, *F* is a point on *AD* and *G* is a point on *AE*, such that triangles *BCD*, *CDE*, *DEF*, *EFG* and *AFG* have the same area. Find the length of *FD*.



   Answer:___________________________.

4. The plate number of each truck is a 7-digit number. None of 7 digits starts with zero. Each of the following digits: 0, 1, 2, 3, 5, 6, 7 and 9 can be used only once in a plate, but 6 and 9 cannot both occur in the same plate. The plates are released in ascending order (from smallest number to largest number), and no two plates have the same numbers. So the first two numbers to the last one are listed as follows: 1023567, 1023576, ... , 9753210. What is the plate number of the 7,000 th truck?

   Answer:___________________________.

5. Determine the number of ordered pairs (x, y) of positive integers satisfying the equation $x^2 + y^2 - 16y = 2004$.

Answer:__________________________pair(s).

6. There are plenty of 2×5 、 1×3 small rectangles, it is possible to form new rectangles without overlapping any of these small rectangles. Determine all the ordered pairs (m, n) of positive integers where $2 \le m \le n$, so that no $m\times n$ rectangle will be formed.

Answer:__________________________.

7. Fill nine integers from 1 to 9 into the cells of the following 3×3 table, one number in each cell, so that in the following 6 squares (see figure below) formed by the entries labeled with * in the table, the sum of the 4 entries in each square are all equal.

*		*
*		*

	*	
*		*
	*	

*	*	
*	*	

	*	*
	*	*

*	*	
*	*	

	*	*
	*	*

Answer:

8. A father distributes 83 diamonds to his 5 sons according to the following rules:

(i) no diamond is to be cut;

(ii) no two sons are to receive the same number of diamonds;

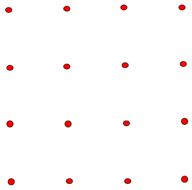
(iii) none of the differences between the numbers of diamonds received by any two sons is to be the same;

(iv) Any 3 sons receive more than half of total diamonds.

Give an example how the father distribute the diamonds to his 5 sons.

Answer:__________________________.

9. There are 16 points in a 4×4 grid as shown in the figure. Determine the largest integer n so that for any n points chosen from these 16 points, none 3 of them can form an isosceles triangle.



Answer:___________________________.

10. Given positive integers x and y, both greater than 1, but not necessarily different. The product xy is written on Albert's hat, and the sum $x+y$ is written on Bill's hat. They can not see the numbers on their own hat. Then they take turns to make the statement as follows:

Bill: " I don't know the number on my hat."
Albert: " I don't know the number on my hat."
Bill: "I don't know the number on my hat."
Albert: "Now, I know the number on my hat."

Given both of them are smart guys and won't lie, determine the numbers written on their hats.

Answer: Albert's number =___________, Bill's number =_____________.

11. Find all real number(s) x satisfying the equation $\{(x+1)^3\} = x^3$, where $\{y\}$ denotes the fractional part of y, for example {3.1416….}= 0.1416…..

Answer:___________________________.

12. Determine the minimum value of the expression

$$x^2 + y^2 + 5z^2 - xy - 3yz - xz + 3x - 4y + 7z,$$

where x, y and z are real numbers.

Answer:___________________________.

Section II: Answer the following 3 questions, and show your detailed solution in the space provided after each question. Write down the question number in each paper. Each question is worth 20 points.

1. A sequence $\left(x_1, x_2, \cdots, x_m\right)$ of m terms is called an OE-sequence if the following two conditions are satisfied:
   a. for any positive integer $1 \le i \le m-1$, we have $x_i \le x_{i+1}$;
   b. all the odd numbered terms x_1, x_3, x_5, ...are odd integer, and all the even numbered terms x_2, x_4, x_6,... are even integer.

   For instance, there are only 7 OE-sequences in which the largest term is at most 4, namely, (1), (3), (1,2), (1,4), (3, 4), (1, 2, 3) and (1, 2, 3, 4).
   How many OE-sequences are there in which the largest terms are at most 20? Explain your answer.

2. Suppose the lengths of the three sides of ΔABC are 9, 12 and 15 respectively. Divide each side into n (≥ 2) segments of equal length, with $n-1$ division points, and let S be the sum of the square of the distances from each of 3 vertices of ΔABC to the $n-1$ division points lying on its opposite side.
   If S is an integer, find all possible positive integer n, with detailed answers.

3. Let ABC be an acute triangle with $AB=c$, $BC=a$, $CA=b$. If D is a point on the side BC, E and F are the foot of perpendicular from D to the sides AB and AC respectively. Lines BF and CE meet at point P. If AP is perpendicular to BC, find the length of BD in terms of a, b, c, and prove that your answer is correct.

5th Invitational World Youth Mathematics Inter-City Competition

第五屆青少年數學國際城市邀請賽

Team Contest 4th August, 2004, Macau

Team: ________________________ Score: _________________

1. In right-angled triangle ΔABC, $\angle A = 30^\circ$, $BC = 1$, $\angle C = 90^\circ$. Consider all the equilateral triangles with all the vertices on the sides of the triangle ΔABC (i.e., the inscribed equilateral triangle of ΔABC). Determine the maximum area among all these equilateral triangles? Justify your answer.

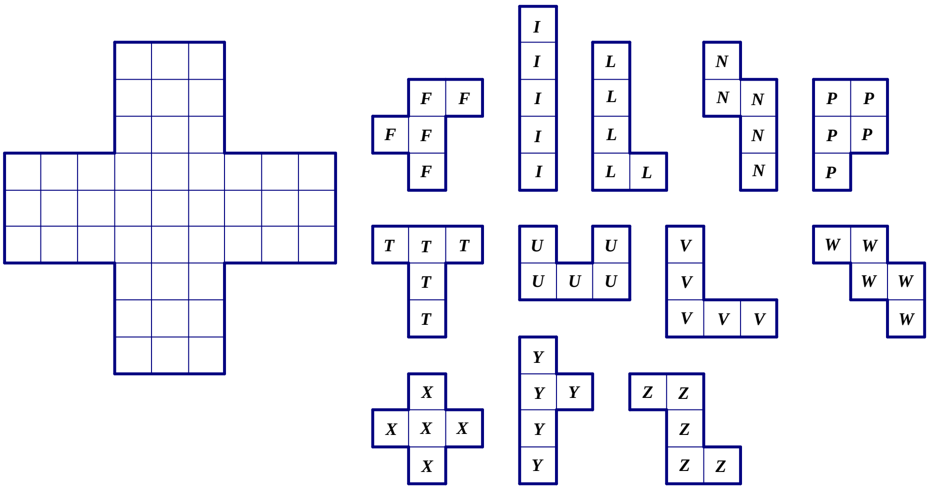
5th Invitational World Youth Mathematics Inter-City Competition

第五屆青少年數學國際城市邀請賽

Team Contest 4th August, 2004, Macau

Team: ________________________ Score: _________________

2. Below are the 12 pieces of pentominoes and a game board. Select four different pentominoes and place on the board so that all the other eight pieces can't placed in this game board. The Pentominoes may be rotated and/or reflected and must follow the grid lines and no overlapping is allowed.



5th Invitational World Youth Mathematics Inter-City Competition

第五屆青少年數學國際城市邀請賽

Team Contest 4th August, 2004, Macau

Team: ________________________ Score: _________________

3. Locate five buildings with heights 1, 2, 3, 4, 5 into every row and every column of the grid (figure A), once each. The numbers on the four sides in figure A below are the number of buildings that one can see from that side, looking row by row or column by column. One can see a building only when all the buildings in front of it are shorter. An example is given as shown in the figure B below, in which the number 5 is replaced by 4, under the similar conditions.

Answer:

			10			
	1				2	
12			5			11
	4				3	
			13			

Figure A

			8				
		3	2	1	2		
	3	1	3	4	2	2	
9	2	3	4	2	1	3	8
	3	2	1	3	4	1	
	1	4	2	1	3	2	
		1	2	3	2		
			8				

Figure B

5^{th} Invitational World Youth Mathematics Inter-City Competition

第五屆青少年數學國際城市邀請賽

Team Contest　　　　　　　　4th August, 2004,　　Macau

Team: ________________________ Score: ________________

4. Let $|x|$ be the absolute value of real number x. Determine the minimum value of the expression $|25^n - 7^m - 3^m|$ where m and n can be any positive integers.

5th Invitational World Youth Mathematics Inter-City Competition

第五屆青少年數學國際城市邀請賽

Team Contest 4th August, 2004, Macau

Team: ________________________ Score: _________________

5. There are m elevators in a building. Each of them will stop exactly in n floors and these floors does not necessarily to be consecutively. Not all the elevators start from the first floor. For any two floors, there is at least one elevator will stop on both floors. If m=11, n=3, determine the maximum number of floors in this building, and list out all the floors stop by each of these m elevators.

5th Invitational World Youth Mathematics Inter-City Competition

第五屆青少年數學國際城市邀請賽

Team Contest　　4th August, 2004,　Macau

Team: ________________________　　Score: ________________

6. In a soccer tournament, every team plays with other team once. In each game under the old scoring system, a winning team gains two points, and in the new score system, this team gains three points instead, while the losing team still get no points as before. A draw is worth one point for both teams without any changes. Is it possible for a team to be the winner of the tournament under the new system, and yet it finishes as the last placer under the old system? If this is possible, at least how many teams participate in this tournament, and list out the results of each game among those teams?

5th Invitational World Youth Mathematics Inter-City Competition

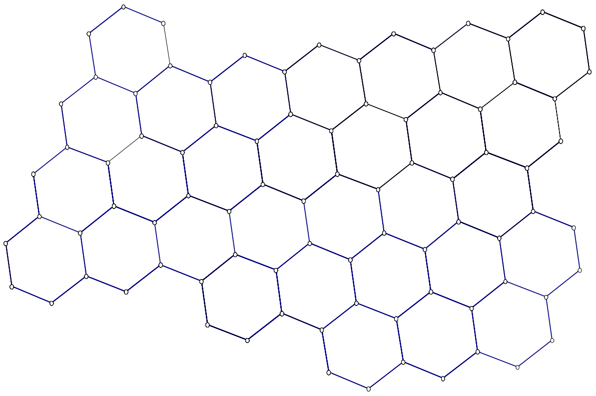
第五屆青少年數學國際城市邀請賽

Team Contest 4th August, 2004, Macau

Team: ________________________ Score: _________________

7. Determine the smallest integer *n* satisfying the following condition: one can divide the following figure into *n* ($n \geq 2$) congruent regions along the grid lines.

Answer:



th Invitational World Youth Mathematics Inter-City Competition

第五屆青少年數學國際城市邀請賽

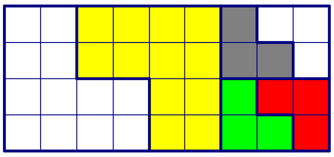
Team Contest 4th August, 2004, Macau

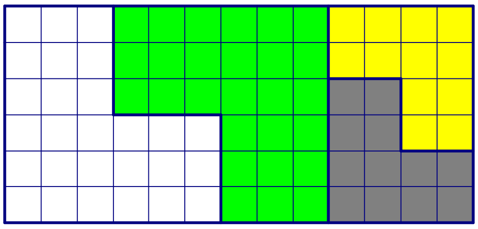
Team: ________________________ Score: _________________

8. A polyomino is a figure formed of several unit squares joined along complete edges. Now one can only construct rectangle with at most 10 pieces of polyominoes where overlapping or gaps are not allowed, and satisfying the following conditions:
   a. the linear dimension of each piece, with at least one square, must be an integral multiple of the smallest piece, under rotation or reflection (if necessary);
   b. each piece is not rectangular;
   c. there are at least two pieces of different sizes.

The diagram on the left is a 9×4 rectangle constructed with six pieces of polyominoes while the diagram on the right is a 13×6 rectangle constructed with four pieces of polyominoes, but it does not satisfy the condition (a) stated above (namely the scale is not integral multiple).

Construct 10 rectangles with no two of them are similar and follow the rules stated above.





2004 IWYMIC Answers

Individual

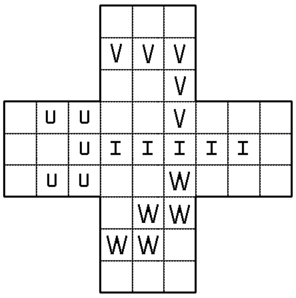
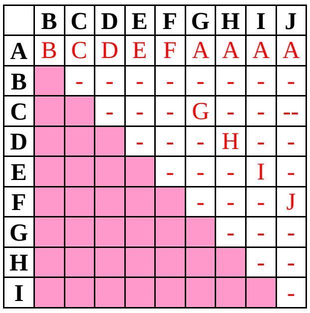
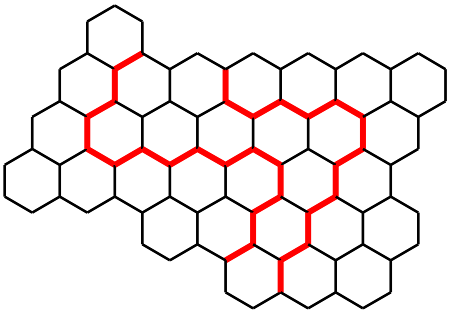
Part I

1.	4	**2.**	1	**3.**	8	**4.**	7206351
5.	0	**6.**	(2, 2), (2, 4), (2, 7) and (4, 4)	**7.**	9 2 7 4 5 6 3 8 1	**8.**	11, 13, 18, 19, 22
9.	6	**10.**	16, 8				
11.	$0, \frac{\sqrt{21}-3}{6}, \frac{\sqrt{33}-3}{6}, \frac{\sqrt{5}-1}{2}, \frac{\sqrt{57}-3}{6}, \frac{\sqrt{69}-3}{6}$					**12.**	$-\frac{75}{8}$

Part II

1.	17710	**2.**	3, 5, 15, 25 and 75	**3.**	$\frac{ac}{b+c}$

Team

1.	$\frac{\sqrt{3}}{4}$		
2.		**3.**	1 4 3 5 2 3 1 2 4 5 2 3 5 1 4 5 2 4 3 1 4 5 1 2 3
4.	15	**5.**	8. (1, 2, 4), (2, 3, 5), (3, 4, 6), (4, 5, 7), (5, 6, 1), (6, 7, 2), (7, 1, 3), (1, 2, 8), (3, 4, 8), (5, 6, 8), (1, 7, 8)
6.	10. 	**7.**	

2005 Kaohsiung Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2005/8/3 Kaohsiung

Section I:
In this section, there are 12 questions, fill in the correct answers in the spaces provided at the end of each question. Each correct answer is worth 5 points.

1. The sum of a four-digit number and its four digits is 2005. What is this four-digit number ?

Answer:_______________

2. In triangle *ABC*, *AB*=10 and *AC*=18. *M* is the midpoint of of *BC*, and the line through *M* parallel to the bisector of $\angle CAB$ cuts *AC* at *D*. Find the length of *AD*.

Answer:_______________

3. Let *x*, *y* and *z* be positive numbers such that $\begin{cases} x+y+xy=8, \\ y+z+yz=15, \\ z+x+zx=35. \end{cases}$ Find the value of

$x+y+z+xy$.

Answer:_________

4. The total number of mushroom gathered by 11 boys and *n* girls is n^2+9n-2, with each gathering exactly the same number. Determine the positive integer *n*.

Answer:____________

5. The positive integer x is such that both x and $x + 99$ are squares of integers. Find the total value of all such integers x.

Answer:__________

6. The lengths of all sides of a right triangle are positive integers, and the length of one of the legs is at most 20. The ratio of the circumradius to the inradius of this triangle is 5:2. Determine the maximum value of the perimenter of this triangle.

Answer: ____________

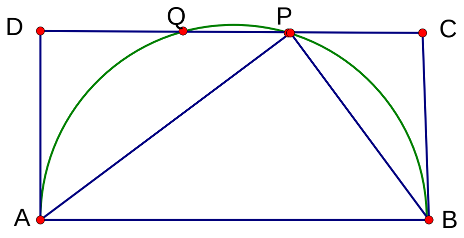
7. Let α be the larger root of $(2004x)^2 - 2003 \cdot 2005x - 1 = 0$ and β be the smaller root of $x^2 + 2003x - 2004 = 0$. Determine the value of $\alpha - \beta$.

Answer: ______________

8.Let a be a positive number such that $a^2 + \frac{1}{a^2} = 5$ ，Determine the value of $a^3 + \frac{1}{a^3}$.

Answer:_____________

9.In the figure, $ABCD$ is a rectangle with AB=5 such that the semicircle on AB as diameter cuts CD at two points. If the distance from one of them to A is 4, find the area of $ABCD$.



Answer:____________

2005 Kaohsiung Invitational World Youth Mathematics Intercity Competition

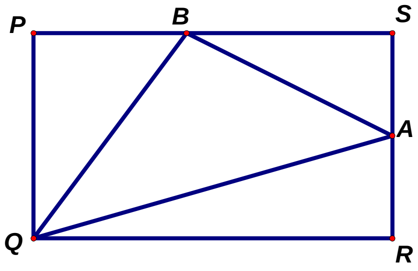
10.Let $a=9\left[n\left(\frac{10}{9}\right)^{n}-1-\left(\frac{10}{9}\right)-\left(\frac{10}{9}\right)^{2}-\cdots-\left(\frac{10}{9}\right)^{n-1}\right]$ where *n* is a positive integer. If *a* is an integer, determine the maximum value of *a*.

Answer: ____________

11. In a two-digit number, the tens digit is greater than the units digit, and the units digit is nonzero. The product of these two digits is divisible by their sum. What is this two-digit number?

Answer: ____________

12. In Figure, *PQRS* is a rectangle of area 10. *A* is a point on *RS* and *B* is a point on *PS* such that the area of triangle *QAB* is 4. Determine the smallest possible value of *PB*+*AR*.



Answer:____________

2005 Kaohsiung Invitational World Youth Mathematics Intercity Competition

Section II:

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Write down the question number in each paper. Each question is worth 20 points.

1. Let a, b and c be real numbers such that $a+bc=b+ca=c+ab=501$. If M is the maximum value of $a+b+c$ and m is the minimum value of $a+b+c$. Determine the value of $M+2m$.

2. The distance from a point inside a quadrilateral to the four vertices are 1, 2, 3 and 4. Determine the maximum value of the area of such a quadrilateral.

3. We have an open-ended table with two rows. Initially, the numbers 1, 2, …, 2005 are written in the first 2005 squares of the first row. In each move, we write down the sum of the first two numbers of the first row as a new number which is then added to the end of this row, and drop the two numbers used in the addition to the corresponding squares in the second row. We continue until there is only one number left in the first row, and drop it to the corresponding square in the second row. Determine the sum of all numbers in the second row. (For example, if 1, 2, 3, 4 and 5 are written in the first row, at the end, we have 1, 2, 3, 4, 5, 3, 7, 8 and 15 in the second row. Hence its sum is 48.)

2005 Kaohsiung Invitational World Youth Mathematics Intercity Competition

Team Contest

2005/8/3 Kaohsiung

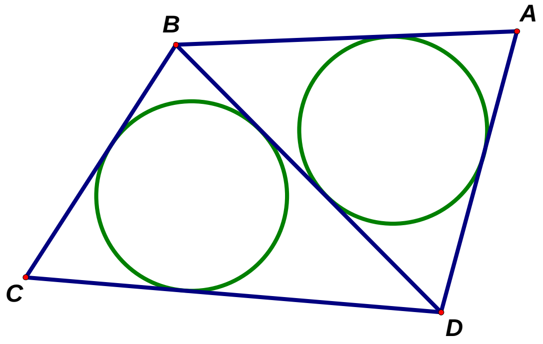
Team: ________________________ ***Score:*** _________________

1. The positive integers a, b and c are such that $a + b + c = 20 = ab + bc - ca - b^2$. Determine all possible values of abc.

2. The sum of 49 positive integers is 624. Prove that three of them are equal to one another.

3. The list 2, 3, 5, 6, 7, 10, … consists of all positive integers which are neither squares nor cubes in increasing order. What is the 2005th number in this list?

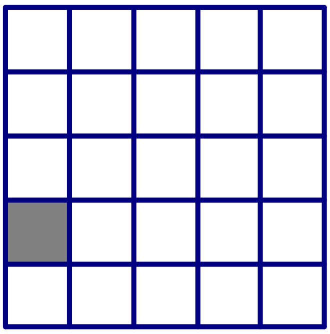
4. *ABCD* is a convex quadrilateral such that the incricles of triangles *BAD* and *BCD* are tangent to each other. Prove that *ABCD* has an incircle.



5. Find a dissection of a triangle into 20 congruent triangles.

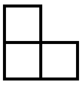
6. You are gambling with the Devil with 3 dollars in your pocket. The Devil will play five games with you. In each game, you give the Devil an integral number of dollars, from 0 up to what you have at the time. If you win, you get back from the Devil double the amount of what you pay. If you lose, the Devil just keeps what you pay. The Devil guarantees that you will only lose once, but the Devil decides which game you will lose, after receiving the amount you pay. What is the highest amount of money you can guarantee to get after the five games?

7. A frog is sitting on a square adjacent to a corner square of a 5×5 board. It hops from square to adjacent square, horizontally or vertically but not diagonally. Prove that it cannot visit each square exactly once.



8. Determine all integers n such that $n^4-4n^3+15n^2-30n+27$ is a prime number.

9. A V-shaped tile consists of a 2×2 square with one corner square missing. Show that no matter which square is omitted from a 7×7 board, the remaining part of the board can be covered by 16 tiles.



V-shaped

10. Let $a_0, a_1, a_2, \dots, a_n$ be positive integers and $a_0 > a_1 > a_2 > \dots > a_n > 1$ such that

$$\left(1-\frac{1}{a_1}\right)+\left(1-\frac{1}{a_2}\right)+\dots+\left(1-\frac{1}{a_n}\right) = 2\left(1-\frac{1}{a_0}\right).$$

Find all possible solutions for $(a_0, a_1, a_2, \dots, a_n)$.

2005 IWYMIC Answers

Individual

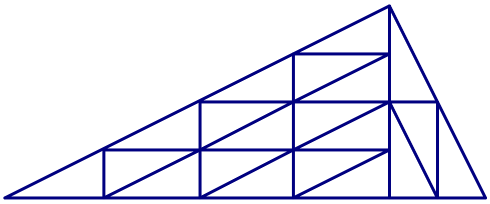
Part I

1.	1979	**2.**	4	**3.**	15	**4.**	9
5.	2627	**6.**	72	**7.**	2005	**8.**	$4\sqrt{7}$
9.	12	**10.**	81	**11.**	63	**12.**	$2\sqrt{2}$

Part II

1.	$499-3\sqrt{2005}$	**2.**	$\frac{25}{2}$	**3.**	24046868

Team

1.	112 and 154	**3.**	2059
5.		**6.**	16
8.	2	**10.**	(24, 4, 3,2) and (60, 5, 3, 2)

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes　　　　2006/7/12　Wenzhou, China

Team:________________ Name:________________ Score:______________

Section I:
In this section, there are 12 questions, fill in the correct answers in the spaces provided at the end of each question. Each correct answer is worth 5 points.

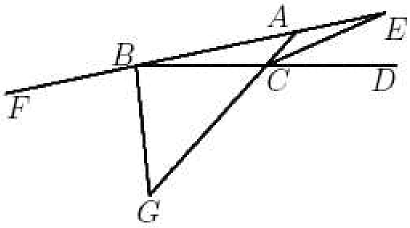
1. Colleen used a calculator to compute $\frac{a+b}{c}$, where *a*, *b* and *c* are positive integers. She pressed *a*, +, *b*, /, *c* and = in that order, and got the answer 11. When he pressed *b*, +, *a*, /, *c* and = in that order, she was surprised to get a different answer 14. Then she realized that the calculator performed the division before the addition. So she pressed (, *a*, +, *b*,), /, *c* and = in that order. She finally got the correct answer. What is it?

Answer:_______________

2. The segment *AB* has length 5. On a plane containing *AB*, how many straight lines are at a distance 2 from *A* and at a distance 3 from *B*?

Answer:_______________

3. In triangle *ABC*, *D* is a point on the extension of *BC*, and *F* is a point on the extension of *AB*. The bisector of $\angle ACD$ meets the extension of *BA* at *E*, and the bisector of $\angle FBC$ meets the extension of *AC* at *G*, as shown in the diagram below. If *CE* = *BC* = *BG*, what is the measure of $\angle ABC$?



Answer:_______________

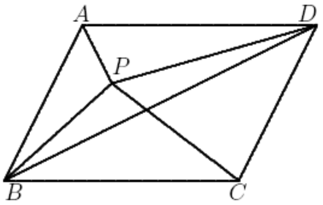
4. The teacher said, “I have two numbers *a* and *b* which satisfy $a+b-ab=1$. I will tell you that *a* is not an integer. What can you say about *b*?” Alex said, “Then *b* is not an integer either.” Brian said, “No, I think *b* must be some positive integer.” Colin said, “No, I think *b* must be some negative integer.” Who was right?

Answer:_______________

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Individual Contest

5. *ABCD* is a parallelogram and *P* is a point inside triangle *BAD*. If the area of triangle *PAB* is 2 and the area of triangle *PCB* is 5, what is the area of triangle *PBD*?



Answer:_______________

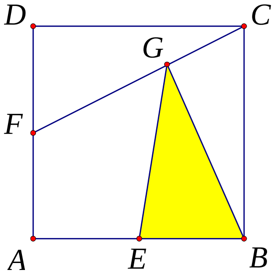
6. The non-zero numbers *a, b, c, d, x, y* and *z* are such that $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$. What is the value of $\frac{xyz(a+b)(b+c)(c+a)}{abc(x+y)(y+z)(z+x)}$?

Answer:_______________

7. On level ground, car travels at 63 kilometres per hour. Going uphill, it slows down to 56 kilometres per hour. Going downhill, it speeds up to 72 kilometres per hour. A trip from *A* to *B* by this car takes 4 hours, when the return trip from *B* to *A* takes 4 hours and 40 minutes. What is the distance between *A* and *B*?

Answer:_______________

8. The square *ABCD* has side length 2. *E* and *F* are the respective midpoints of *AB* and *AD*, and *G* is a point on *CF* such that $3\,CG = 2\,GF$. Determine the area of triangle *BEG*.



Answer:_______________

9. Determine *x*+*y* where *x* and *y* are real numbers such that $(2x+1)^2+y^2+(y-2x)^2=\frac{1}{3}$.

Answer:_______________

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Individual Contest

10. A shredding company has many employees numbered 1, 2, 3, and so on along the disassembly line. The foreman receives a single-page document to be shredded. He rips it into 5 pieces and hands them to employee number 1. When employee n receives pieces of paper, he takes n of them and rips each piece into 5 pieces and passes all the pieces to employee $n+1$. What is the value of k such that employee k receives less than 2006 pieces of paper but hands over at least 2006 pieces?

Answer:_______________

11. A convex polyhedron Q is obtained from a convex polyhedron P with 36 edges as follows. For each vertex V of P, use a plane to slice off a pyramid with V as its vertex. These planes do not intersect inside P. Determine the number of edges of Q.

Answer:_______________

12. Let m and n be positive integers such that $\sqrt{m-174}+\sqrt{m+34}=n$. Determine the maximum value of n.

Answer:_______________

Section II:

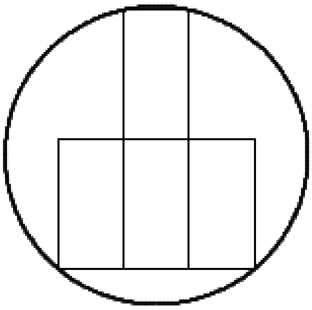
Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. There are four elevators in a building. Each makes three stops, which do not have to be on consecutive floors or include the main floor. For any two floors, there is at least one elevator which stops on both of them. What is the maximum number of floors in this building?

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Individual Contest

2. Four 2×4 rectangles are arranged as shown in the diagram below and may not be rearranged. What is the radius of the smallest circle which can cover all of them?



3. Partition the positive integers from 1 to 30 inclusive into *k* pairwise disjoint groups such that the sum of two distinct elements in a group is never the square of an integer. What is the minimum value of *k*?

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest

2006/7/12 Wenzhou, China

Team: ______________________ ***Score:*** ________________

1. The teacher said, “I want to fit as large a circle as possible inside a triangle whose side lengths are 2, 2 and 2*x* for some positive real number *x*. What should the value of *x* be?” Alex said, “I think *x* should be 1.” Brian said, “I think *x* should be $x=\sqrt{2}$.” Colin said, “Both of you are wrong.” Who was right?

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest 2006/7/12 Wenzhou, China

Team: ________________________ ***Score:*** _________________

2. A triangle can be cut into two isosceles triangles. One of the angles of the original triangle is 36°. Determine all possible values of the largest angle of the original triangle.

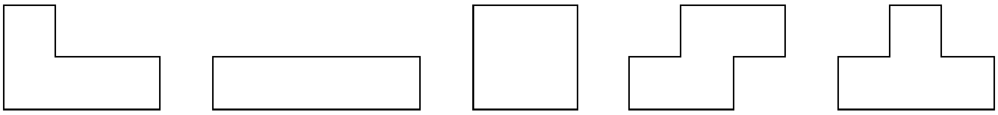
2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest

2006/7/12 Wenzhou, China

Team: ______________________ ***Score:*** ________________

3. There are five Tetris pieces, each consisting of four unit squares joined edge to edge. Use the piece shaped like the letter L (the first one in the diagram below) and each of the other four pieces to form a shape with an axis of reflectional symmetry.



2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest

2006/7/12 Wenzhou, China

Team: ________________________ ***Score:*** _________________

4. A domino consists of two unit squares joined edge to edge, each with a number on it. Fifteen dominoes, numbered 11, 12, 13, 14, 15, 22, 23, 24, 25, 33, 34, 35, 44, 45 and 55, are assembled into the 5 by 6 rectangle shown in the diagram below. However, the boundary of the individual dominoes have been erased. Reconstruct them.

1	1	3	5	2	3
1	4	3	1	5	2
2	4	5	5	3	2
3	3	1	1	2	4
2	5	4	5	4	4

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest 2006/7/12 Wenzhou, China

Team: ________________________ ***Score:*** _________________

5. A lucky number is a positive integer which is 19 times the sum of its digits (in base ten). Determine all the lucky numbers.

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest

2006/7/12 Wenzhou, China

Team: ________________________ ***Score:*** _________________

6. Alice and Betty play the following game on an $n\times n$ board. Starting with Alice, they alternately put either 0 or 1 into any of the blank squares. When all the squares have been filled, Betty wins if the sum of all the numbers in each row is even. Otherwise, Alice wins.
   (a) Which player has a winning strategy when $n = 2006$?
   (b) Answer the question in (a) for an arbitrary positive integer n**.**

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest 2006/7/12 Wenzhou, China

Team: ______________________ ***Score:*** ________________

7. Prove that $1596^n + 1000^n - 270^n - 320^n$ is divisible by 2006 for all positive odd integer *n*.

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest 2006/7/12 Wenzhou, China

Team: ______________________ ***Score:*** ________________

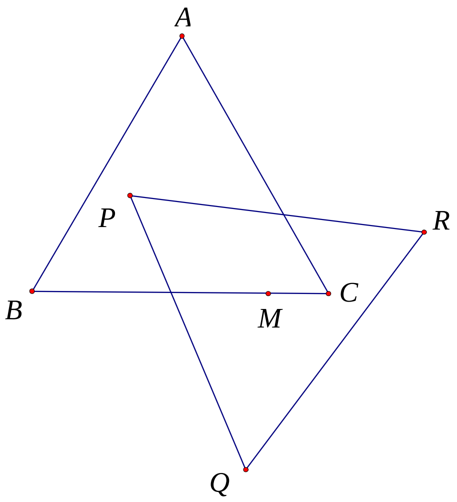
8. From the list of positive integers in increasing order, delete all multiples of 4 and all numbers 1 more than a multiple of 4. Let S_n be the sum of the first *n* terms in the sequence which remains. Compute $\left\lfloor\sqrt{S_1}\right\rfloor+\left\lfloor\sqrt{S_2}\right\rfloor+\cdots\cdots+\left\lfloor\sqrt{S_{2006}}\right\rfloor$.

2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest 2006/7/12 Wenzhou, China

Team: ________________________ ***Score:*** _________________

9. *ABC* and *PQR* are both equilateral triangles of area 1. The centre *M* of *PQR* lies on the perimeter of *ABC*. Determine the minimal area of the intersection of the two triangles.



2006 Wenzhou Invitational World Youth Mathematics Intercity Competition

Team Contest 2006/7/12 Wenzhou, China

Team: ________________________ ***Score:*** _________________

10. For a certain positive integer m, there exists a positive integer n such that mn is the square of an integer and $m-n$ is prime. Determine all such positive integers m in the range $1000 \leqq m < 2006$.

2006 IWYMIC Answers

Individual

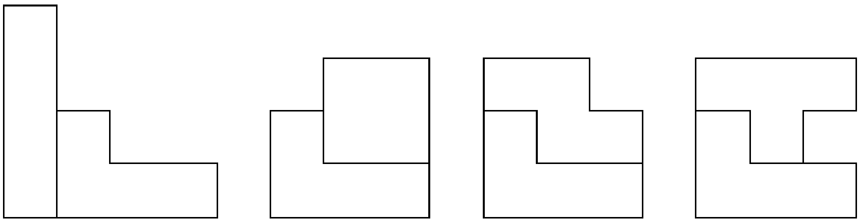
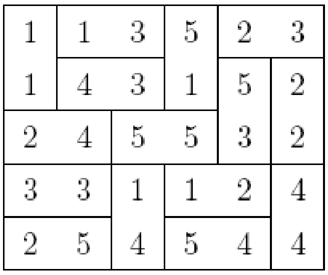
Part I

1.	5	**2.**	3	**3.**	$12°$	**4.**	B
5.	3	**6.**	1	**7.**	273	**8.**	$\frac{4}{5}$
9.	$-\frac{2}{3}$	**10.**	32	**11.**	108	**12.**	104

Part II

1.	5	**2.**	$\frac{\sqrt{85}}{2}$	**3.**	3

Team

<table>
<tr><td>1.</td><td>C</td><td>2.</td><td>$72^\circ, 90^\circ, 108^\circ, 126^\circ, 132^\circ$</td></tr>
<tr><td>3.</td><td colspan="3"></td></tr>
<tr><td>4.</td><td>
1 1 3 5 2 3
1 4 3 1 5 2
2 4 5 5 3 2
3 3 1 1 2 4
2 5 4 5 4 4</td><td>5.</td><td>114, 133, 152, 171, 190, 209, 228, 247, 266, 285 and 399</td></tr>
<tr><td rowspan="2">6.</td><td>(a) Betty</td><td rowspan="2">8.</td><td rowspan="2">2013021</td></tr>
<tr><td>(b) If <i>n</i> is even, Betty can use the same winning strategy. If <i>n</i> is odd, Alice has a winning strategy.</td></tr>
<tr><td>9.</td><td>$\frac{1}{9}$</td><td>10.</td><td>1156, 1296, 1369, 1600 and 1764</td></tr>
</table>

2007 Changchun Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2007/7/23 Changchun, China

Team:________________ Name:________________ Score:______________

Section I:

In this section, there are 12 questions, fill in the correct answers in the spaces provided at the end of each question. Each correct answer is worth 5 points.

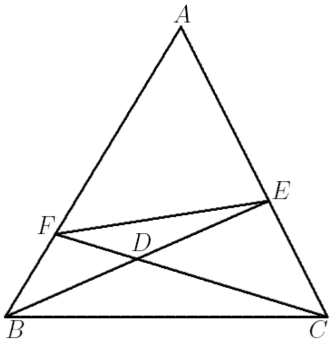
1. Let A_n be the average of the multiples of n between 1 and 101. Which is the largest among A_2, A_3, A_4, A_5 and A_6?

Answer：________________

2. It is a dark and stormy night. Four people must evacuate from an island to the mainland. The only link is a narrow bridge which allows passage of two people at a time. Moreover, the bridge must be illuminated, and the four people have only one lantern among them. After each passage to the mainland, if there are still people on the island, someone must bring the lantern back. Crossing the bridge individually, the four people take 2, 4, 8 and 16 minutes respectively. Crossing the bridge in pairs, the slower speed is used. What is the minimum time for the whole evacuation?

Answer：________________

3. In triangle *ABC*, *E* is a point on *AC* and *F* is a point on *AB*. *BE* and *CF* intersect at *D*. If the areas of triangles *BDF*, *BCD* and *CDE* are 3, 7 and 7 respectively, what is the area of the quadrilateral *AEDF*?



Answer：________________

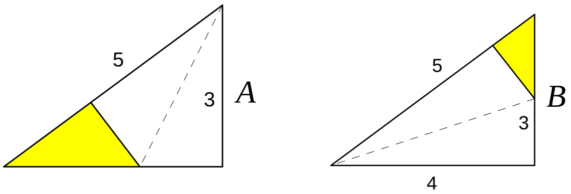
4. A regiment had 48 soldiers but only half of them had uniforms. During inspection, they form a 6×8 rectangle, and it was just enough to conceal in its interior everyone without a uniform. Later, some new soldiers joined the regiment, but again only half of them had uniforms. During the next inspection, they used a different rectangular formation, again just enough to conceal in its interior everyone without a uniform. How many new soldiers joined the regiment?

Answer：________________

5. The sum of 2008 consecutive positive integers is a perfect square. What is the minimum value of the largest of these integers?

Answer：________________

6. The diagram shows two identical triangular pieces of paper *A* and *B*. The side lengths of each triangle are 3, 4 and 5. Each triangle is folded along a line through a vertex, so that the two sides meeting at this vertex coincide. The regions not covered by the folded parts have respective areas S_A and S_B. If $S_A+S_B=39$, find the area of the original triangular piece of paper *A*.



Answer：________________

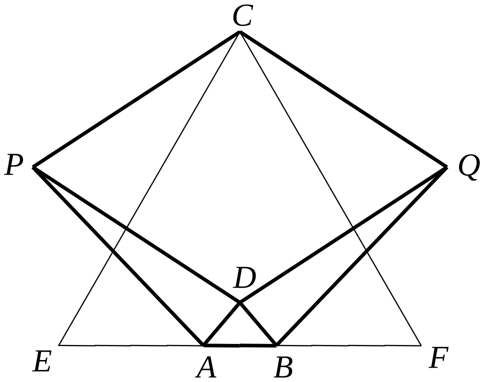
7. Find the largest positive integer *n* such that 3^{1024}-1 is divisible by 2^n.

Answer：________________

8. A farmer use four straight fences, with respective lengths 1, 4, 7 and 8 units to form a quadrilateral. What is the maximum area of the quadrilateral the farmer can enclose?

Answer：________________

9. In the diagram, $CE=CF=EF$, $EA=BF=2AB$, and $PA=QB=PC=QC=PD=QD=1$, Determine *BD*.



Answer：________________

10. Each of the numbers 2, 3, 4, 5, 6, 7, 8 and 9 is used once to fill in one of the boxes in the equation below to make it correct. Of the three fractions being added, what is the value of the largest one?

$$\frac{1}{\square\times\square}+\frac{\square}{\square\times\square}+\frac{\square}{\square\times\square}=1$$

Answer：________________

11. Let x be a real number. Denote by $[x]$ the integer part of x and by $\{x\}$ the decimal part of x. Find the sum of all positive numbers satisfying $25\{x\}+[x]=125$.

Answer：________________

12. A positive integer n is said to be good if there exists a perfect square whose sum of digits in base 10 is equal to n. For instance, 13 is good because $7^2=49$ and $4+9=13$. How many good numbers are among 1, 2, 3, …, 2007?

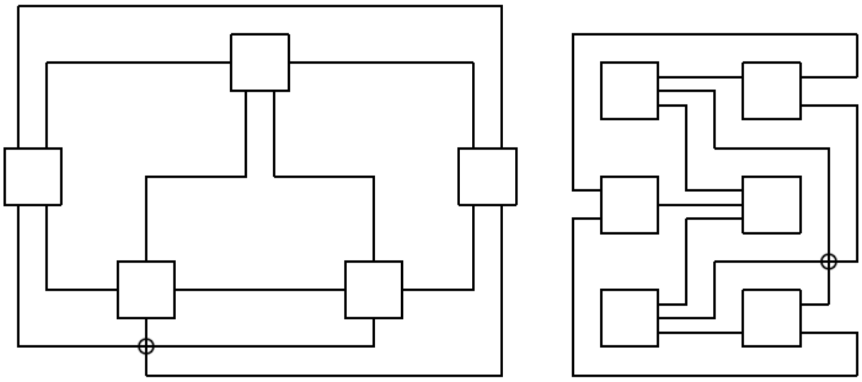
Answer：________________

Section II:

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. A 4×4 table has 18 lines, consisting of the 4 rows, the 4 columns, 5 diagonals running from southwest to northeast, and 5 diagonals running from northwest to southeast. A diagonal may have 2, 3 or 4 squares. Ten counters are to be placed, one in each of ten of the sixteen cells. Each line which contains an even number of counters scores a point. What is the largest possible score?

2. There are ten roads linking all possible pairs of five cities. It is known that there is at least one crossing of two roads, as illustrated in the diagram below on the left. There are nine roads linking each of three cities to each of three towns. It is known that there is also at least one crossing of two roads, as illustrated in the diagram below on the right. Of the fifteen roads linking all possible pairs of six cities, what is the minimum number of crossings of two roads?



3. A prime number is called an ***absolute prime*** if every permutation of its digits in base 10 is also a prime number. For example: 2, 3, 5, 7, 11, 13 (31), 17 (71), 37 (73) 79 (97), 113 (131, 311), 199 (919, 991) and 337 (373, 733) are absolute primes. Prove that no ***absolute prime*** contains all of the digits 1, 3, 7 and 9 in base 10.

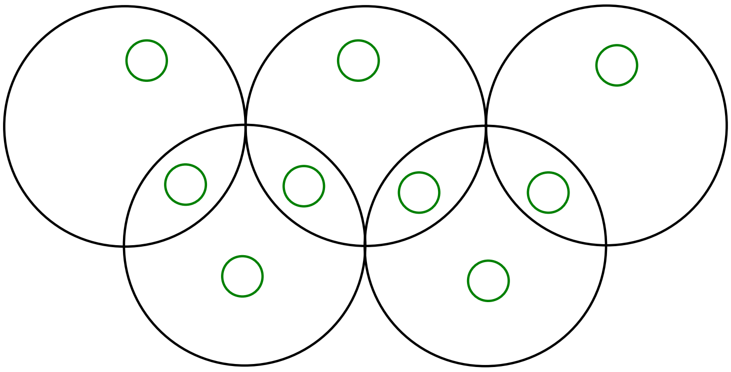
2007 Changchun Invitational World Youth Mathematics Intercity Competition

Team Contest

2007/7/23 Changchun, China

Team: ____________________________ **Score**: __________________

1. Use each of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 exactly once to fill in the nine small circles in the Olympic symbol below, so that the sum of all the numbers inside each large circle is 14. Write down the correct number in each small circle.



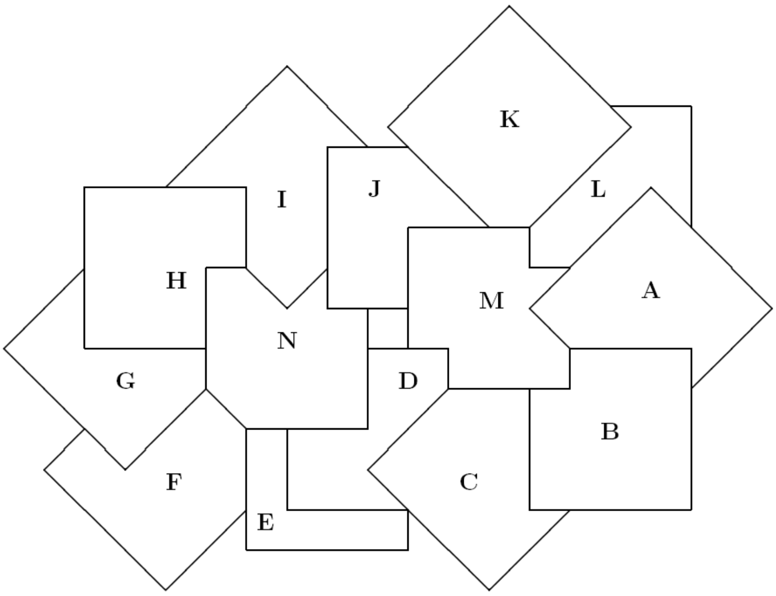
2007 Changchun Invitational World Youth Mathematics Intercity Competition

Team Contest

2007/7/23 Changchun, China

Team: ______________________ **Score**: ________________

2. The diagram below shows fourteen pieces of paper stacked on top of one another. Beginning on the pieces marked B, move from piece to adjacent piece in order to finish at the piece marked F. The path must alternately climb up to a piece of paper stacked higher and come down to a piece of paper stacked lower. The same piece may be visited more than once, and it is not necessary to visit every piece. List the pieces of paper in the order visited.



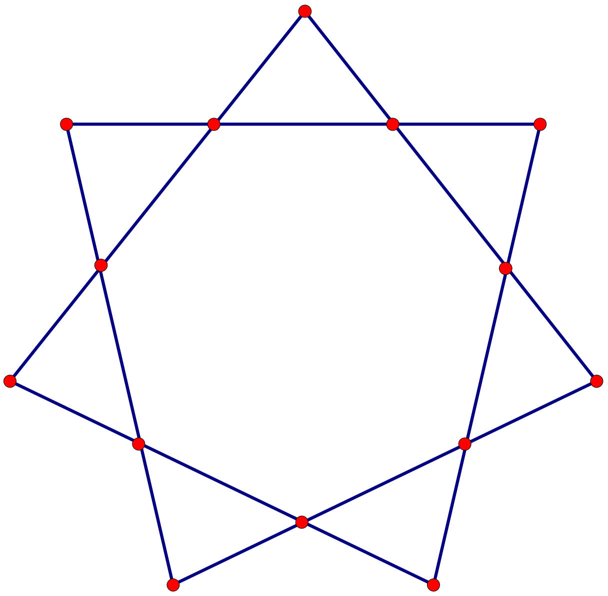
2007 Changchun Invitational World Youth Mathematics Intercity Competition

Team Contest

2007/7/23 Changchun, China

Team: ______________________ **Score**: ________________

3. There are 14 points of intersection in the seven-pointed star in the diagram on the below. Label these points with the numbers 1, 2, 3, …, 14 such that the sum of the labels of the four points on each line is the same. Give one set of solution, no explanation needed.



2007 Changchun Invitational World Youth Mathematics Intercity Competition

Team Contest

2007/7/23 Changchun, China

Team: ___________________________ **Score**: __________________

4. Mary found a 3-digit number that, when multiplied by itself, produced a number which ended in her 3-digit number. What is the sum of all the distinct 3-digit numbers which have this property?

2007 Changchun Invitational World Youth Mathematics Intercity Competition

Team Contest

2007/7/23   Changchun, China

Team: ___________________________   **Score**: __________________

5. Determine all positive integers m and n such that m^2+1 is a prime number and $10(m^2+1)=n^2+1$.

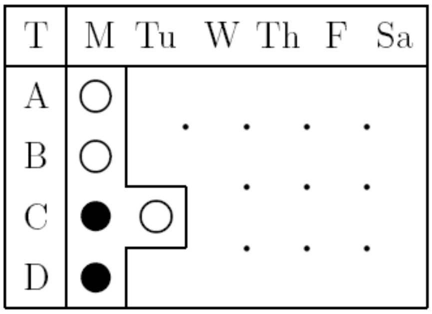
2007 Changchun Invitational World Youth Mathematics Intercity Competition

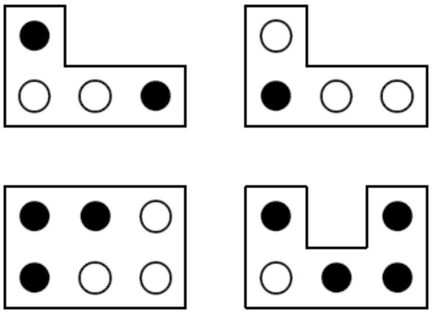
Team Contest

2007/7/23 Changchun, China

Team: ________________________ **Score**: ________________

6. Four teams take part in a week-long tournament in which every team plays every other team twice, and each team plays one game per day. The diagram below on the left shows the final scoreboard, part of which has broken off into four pieces, as shown on the diagram below on the right. These pieces are printed only on one side. A black circle indicates a victory and a white circle indicates a defeat. Which team wins the tournament?





2007 Changchun Invitational World Youth Mathematics Intercity Competition

Team Contest

2007/7/23 Changchun, China

Team: ______________________ **Score**: _______________

7. Let A be a 3 by 3 array consisting of the numbers 1, 2, 3, …, 9. Compute the sum of the three numbers on the i-th row of A and the sum of the three numbers on the j-th column of A. The number at the intersection of the i-th row and the j-th column of another 3 by 3 array B is equal to the absolute difference of the two sums of array A. For Example,

$$b_{12} = \left|(a_{11} + a_{12} + a_{13}) - (a_{12} + a_{22} + a_{32})\right|.$$

Is it possible to arrange the numbers in array A so that the numbers 1, 2, 3, …, 9 will also appear in array B?

a_{11}	a_{12}	a_{13}
a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}

A

b_{11}	b_{12}	b_{13}
b_{21}	b_{22}	b_{23}
b_{31}	b_{32}	b_{33}

B

2007 Changchun Invitational World Youth Mathematics Intercity Competition

Team Contest

2007/7/23 Changchun, China

Team: ____________________________ **Score**: __________________

8. The diagonals *AC* and *BD* of a convex quadrilateral are perpendicular to each other. Draw a line that passes through point *M*, the midpoint of *AB* and perpendicular to *CD;* draw another line through point *N*, the midpoint of *AD* and perpendicular to *CB*. Prove that the point of intersection of these two lines lies on the line *AC*.

2007 Changchun Invitational World Youth Mathematics Intercity Competition

Team Contest

2007/7/23 Changchun, China

Team: ___________________________ **Score**: __________________

9. The positive integers from 1 to *n* (where *n*>1) are arranged in a line such that the sum of any two adjacent numbers is a square. What is the minimum value of *n*?

2007 Changchun Invitational World Youth Mathematics Intercity Competition

Team Contest

2007/7/23 Changchun, China

Team: ___________________________ **Score**: __________________

10. Use one of the five colours (R represent red, Y represent yellow, B represent blue, G represent green and W represent white) to paint each square of an 8×8 chessboard, as shown in the diagram below. Then paint the rest of the squares so that all the squares of the same colour are connected to one another edge to edge. What is the largest number of squares of the same colour as compare to the other colours?

R							
						Y	
		B					
G							G
			R				
	W					W	
		B	Y				

2007 IWYMIC Answers

Individual

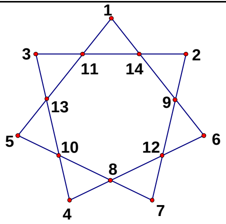
Part I

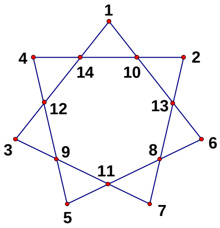
1.	A_5	**2.**	6	**3.**	18	**4.**	12
5.	2133	**6.**	108	**7.**	12	**8.**	18
9.	$\frac{1}{\sqrt{19}}$	**10.**	$\frac{7}{8}$	**11.**	2837	**12.**	992

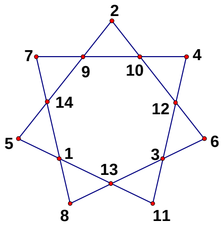
Part II

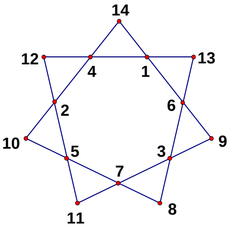
1.	17	**2.**	3

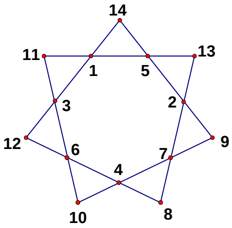
Team

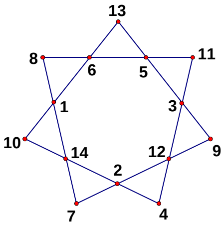
<table>
<tr><td>1.</td><td>861743295</td><td>2.</td><td>BMJKLAMDENGHIJNF</td></tr>
<tr><td>3.</td><td colspan="3">


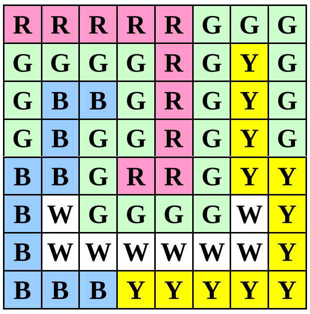










</td></tr>
<tr><td>4.</td><td>1001</td><td>5.</td><td>(2,7) or (4,13)</td></tr>
<tr><td>6.</td><td>C</td><td>7.</td><td>Not possible</td></tr>
<tr><td>9.</td><td>15</td><td>10.</td><td>24,

<table>
<tr><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>G</td><td>G</td><td>G</td></tr>
<tr><td>G</td><td>G</td><td>G</td><td>G</td><td>R</td><td>G</td><td>Y</td><td>G</td></tr>
<tr><td>G</td><td>B</td><td>B</td><td>G</td><td>R</td><td>G</td><td>Y</td><td>G</td></tr>
<tr><td>G</td><td>B</td><td>G</td><td>G</td><td>R</td><td>G</td><td>Y</td><td>G</td></tr>
<tr><td>B</td><td>B</td><td>G</td><td>R</td><td>R</td><td>G</td><td>Y</td><td>Y</td></tr>
<tr><td>B</td><td>W</td><td>G</td><td>G</td><td>G</td><td>G</td><td>W</td><td>Y</td></tr>
<tr><td>B</td><td>W</td><td>W</td><td>W</td><td>W</td><td>W</td><td>W</td><td>Y</td></tr>
<tr><td>B</td><td>B</td><td>B</td><td>Y</td><td>Y</td><td>Y</td><td>Y</td><td>Y</td></tr>
</table>
</td></tr>
</table>

International Mathematics Competition 2008
(IMC 2008)

World Youth Mathematics Intercity Competition

Individual Contest

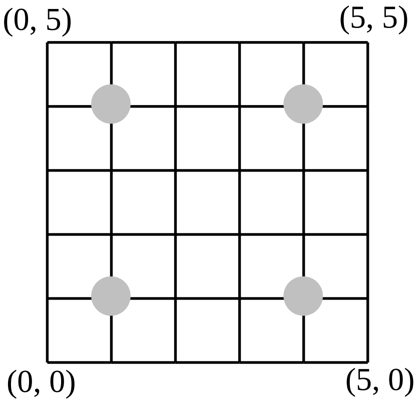
Time limit: 120 minutes 2008/10/28

Team:______________Name:_________________ Score:_______________

Section A.

In this section, there are 12 questions, fill in the correct answers in the spaces provided at the end of each question. Each correct answer is worth 5 points.

1. Starting from the southwest corner (0,0) of a 5×5 net, an ant crawls along the lines towards the northeast corner (5,5). It can only go east or north, but cannot get pass the four broken intersections at (1,1), (1,4), (4,1) and (4,4). What is the total number of different paths?



Answer：_________________________

2. The positive integer $a-2$ is a divisor of $3a^2-2a+10$. What is the sum of all possible values of a?

Answer：_________________________

3. Let a, b and c be real numbers such that $a+b+c=11$ and $\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{13}{17}$. What is the value of $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$?

Answer：_________________________

International Mathematics Competition 2008
(IMC 2008)

World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes   2008/10/28

Team:______________Name:_________________ Score:_______________

4. Let x be any real number. What is the maximum real value of $\sqrt{2008-x}+\sqrt{x-2000}$?

Answer：_________________________

5. How many ten-digit numbers are there in which every digit is 2 or 3, and no two 3s are adjacent?

Answer：_________________________

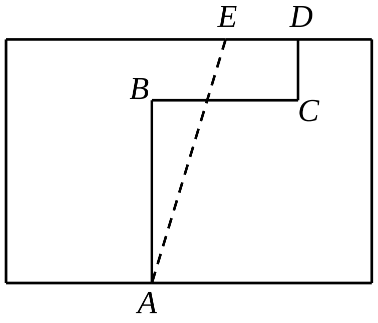
6. On a circle, there are n ($n > 3$) numbers with a total sum of 94, such that each number is equal to the absolute value of the difference between the two numbers which follow it in clockwise order. What is the possible value of n?

Answer：_________________________

7. If the thousands digit of a four-digit perfect square is decreased by 3 and its units digit is increased by 3, the result is another four-digit perfect square. What is the original number?

Answer：_________________________

8. Each segment of the broken line A-B-C-D is parallel to an edge of the rectangle, and it bisects the area of the rectangle. E is a point on the perimeter of the rectangle such that AE also bisects the area of the rectangle. If AB=30, BC=24 and CD=10, what is the length of DE?



Answer：_________________________

World Youth Mathematics Intercity Competition

Individual Contest

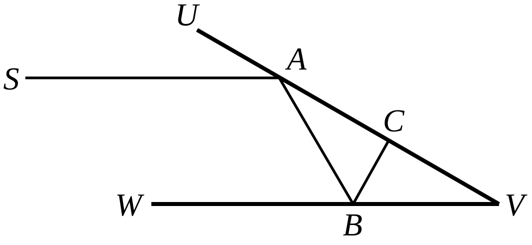
Time limit: 120 minutes   2008/10/28

***Team:*______________*Name:*________________ *Score:*______________**

9. Let $f(x)=ax^2-c$, where a and c are real numbers satisfying $-4\le f(1)\le -1$ and $-1\le f(2)\le 2$. What is the maximum value of $f(8)$?

Answer：________________________

10. Two vertical mirrors facing each other form a 30° angle. A horizontal light beam from source *S* parallel to the mirror *WV* strikes the mirror *UV* at *A*, reflects to strike the mirror *WV* at *B*, and reflects to strike the mirror *UV* at *C*. After that, it goes back to *S*. If *SA*=*AV*=1, what is the total distance covered by the light beam?

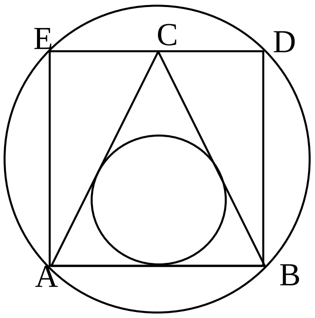


Answer：________________________

11. Let *n* be a positive integer such that n^2-n+11 is the product of four prime numbers, some of which may be the same. What is the minimum value of *n*?

Answer：________________________

12. *ABC* is an equilateral triangle, and *ABDE* is a rectangle with *DE* passing through *C*. If the circle touching all three sides of Δ*ABC* has radius 1, what is the diameter of the circle passing through *A*, *B*, *D* and *E*?



Answer：________________________

International Mathematics Competition 2008
(IMC 2008)

World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2008/10/28

Team:______________Name:__________________ Score:_______________

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. In the expression $\left[\sqrt{2008+\sqrt{2008+\sqrt{2008+\ldots+\sqrt{2008}}}}\right]$, the number 2008 appears 2008 times, and $[x]$ stands for the greatest integer not exceeding x. What is the value of this expression?

Answer：_________________________

International Mathematics Competition 2008
(IMC 2008)

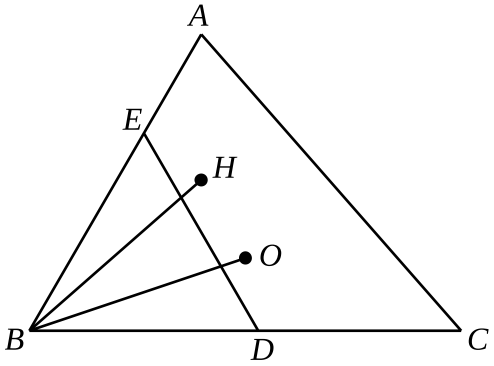
World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 2008/10/28

***Team:*______________*Name:*_________________ *Score:*_______________**

2. In the triangle ABC, $\angle ABC=60°$. O is its circumcentre and H is its orthocentre. D is a point on BC such that $BD=BH$. E is a point on AB such that $BE=BO$. If $BO=1$, what is the area of the triangle BDE? (The orthocenter is the intersection of the lines from each vertex of the triangle making a perpendicular with its opposite sides. The circumcenter is the center of the circle passing through each vertex of the triangle.)



World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes   2008/10/28

***Team:*_____________*Name:*________________ *Score:*______________**

3. Let t be a positive integer such that $2^t = a^b \pm 1$ for some integers a and b, each greater than 1. What are all the possible values of t?

Answer：________________________

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28
Chiang Mai, Thailand

Team: ________________________________ ***Score:*** _________________

1. The fraction $\frac{p}{q}$ is in the lowest form. Its decimal expansion has the form $0.abababab\ldots$ The digits a and b may be equal, except that not both can be 0. Determine the number of different values of p.

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28
Chiang Mai, Thailand

Team: **___________________________** ***Score:*** **_______________**

2. Cover up as few of the 64 squares in the following 8×8 table as possible so that neither two uncovered numbers in the same row nor in the same column are the same. Two squares sharing a common side cannot both be covered.

6	4	5	7	7	3	3	5
4	8	4	3	6	7	5	1
3	1	5	7	7	7	6	2
7	5	5	8	8	4	2	3
4	5	6	5	8	1	7	3
3	3	3	6	1	8	8	3
1	7	3	2	3	6	4	8
1	6	2	2	4	5	8	7

6	4	5	7	7	3	3	5
4	8	4	3	6	7	5	1
3	1	5	7	7	7	6	2
7	5	5	8	8	4	2	3
4	5	6	5	8	1	7	3
3	3	3	6	1	8	8	3
1	7	3	2	3	6	4	8
1	6	2	2	4	5	8	7

ANSWER :

World Youth Mathematics Intercity Competition

Team Contest　Time limit: 60 minutes　2008/10/28

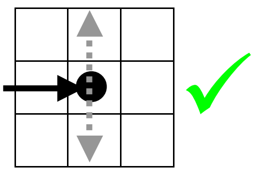
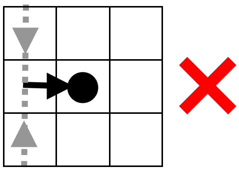
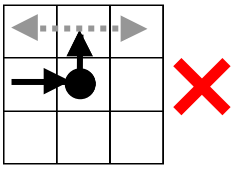
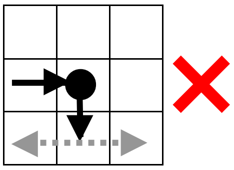
ChiangMai, Thailand

3. On the following 8×8 board, draw a single path going between squares with common sides so that

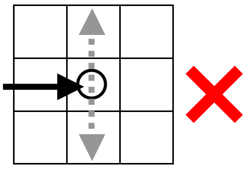
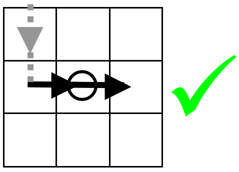
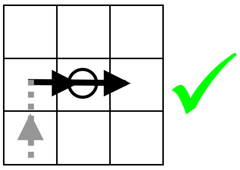
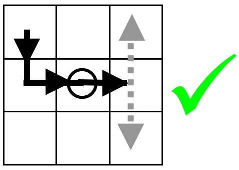
(a) it is closed and not self-intersecting;

(b) it passes through every square with a circle, though not necessarily every square;

(c) it turns at every square with a black circle, but does not do so on either the square before or the one after;

(d) it does not turn at any square with a white circle, but must do so on either the square before or the one after, or both.

		●		○		○	
				●			○
○	○			○	●		●
○							
	○						
	○					○	
				●		○	

		●		○		○	
				●			○
○	○			○	●		●
○							
	○						
	○					○	
				●		○	

World Youth Mathematics Intercity Competition
Team Contest   Time limit: 60 minutes   2008/10/28
Chiang Mai, Thailand

Team: ________________________________   ***Score:*** __________________

4. Consider all $a \times b \times c$ boxes where a, b and c are integers such that $1 \le a \le b \le c \le 5$. An $a_1 \times b_1 \times c_1$ box fits inside an $a_2 \times b_2 \times c_2$ box if and only if $a_1 \le a_2$, $b_1 \le b_2$ and $c_1 \le c_2$. Determine the largest number of the boxes under consideration such that none of them fits inside another.

ANSWER : _________________________

World Youth Mathematics Intercity Competition

Team Contest   Time limit: 60 minutes   2008/10/28
Chiang Mai, Thailand

Team: ________________________________   ***Score:*** _________________

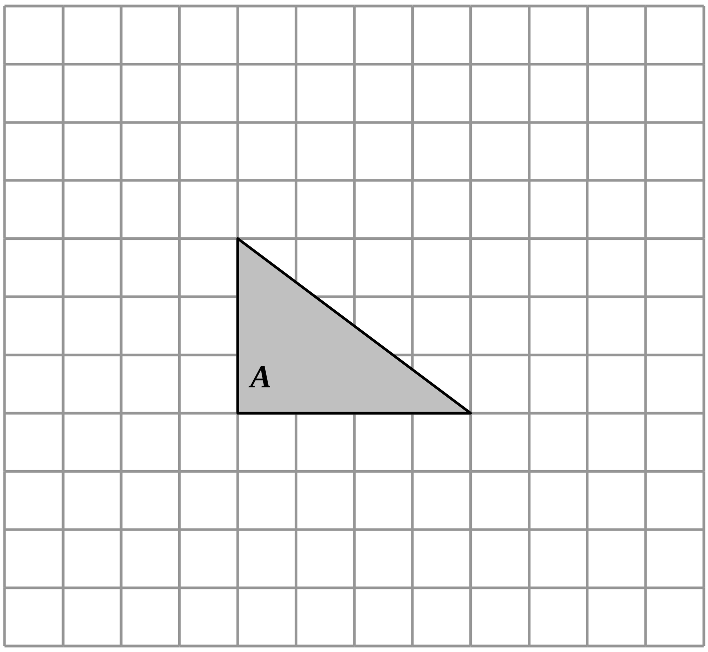
5. Initially, the numbers 0, 1 and 4 are on the blackboard. Our task is to add more numbers on the blackboard by using the following procedures: In each step, we select two numbers a and b on the blackboard and add the new number $c=ab+a+b$ on the blackboard. What is the smallest number not less than 2008 which can appear on the blackboard after repeating the same procedure for several times?

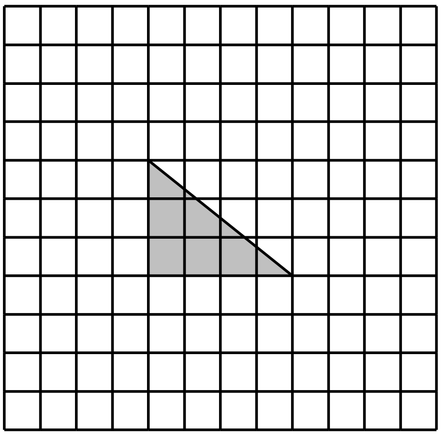
World Youth Mathematics Intercity Competition
Team Contest   Time limit: 60 minutes   2008/10/28

Chiang Mai, Thailand

Team: ________________________________ ***Score:*** _________________

6. Given a shaded triangle as below, find all possible ways of extending one of its sides to a new point so that the resulting triangle has two equal sides. Mark the points of extension on the space given below.





International Mathematics Competition 2008

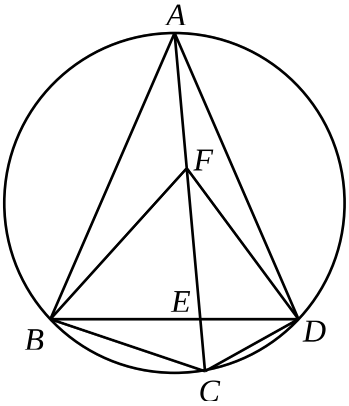
(IMC 2008)

World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Team: **__________________________________** ***Score:*** **__________________**

7. *ABCD* is a quadrilateral inscribed in a circle, with $AB=AD$. The diagonals intersect at *E*. *F* is a point on *AC* such that $\angle BFC=\angle BAD$. If $\angle BAD=2\angle DFC$, determine $\dfrac{BE}{DE}$.



Team: ________________________________ ***Score:*** _________________

8. How many five-digit numbers are there that contain the digit 3 at least once?

ANSWER : ________________________

***Team:* _________________________________ *Score:* __________________**

9. Among nine identically looking coins, one of them weighs a grams, seven of them b grams each and the last one c grams, where $a<b<c$. We wish to determine whether $a+c<2b$, $a+c=2b$ or $a+c>2b$ using only an unmarked beam balance four times.

ANSWER : __________________________

International Mathematics Competition 2008
(IMC 2008)

World Youth Mathematics Intercity Competition
Team Contest Time limit: 60 minutes 2008/10/28
Chiang Mai, Thailand

***Team:* _________________________________ *Score:* __________________**

10. Determine the sum of all positive integers n such that

$$1+n+\frac{n(n-1)}{2}+\frac{n(n-1)(n-2)}{6}=2^k$$ for some positive integer k.

ANSWER : ______________________

International Mathematics Competition 2008
(IMC 2008)

World Youth Mathematics Intercity Competition
Team Contest Time limit: 60 minutes 2008/10/28
Chiang Mai, Thailand

Team: ________________________________ ***Score:*** _________________

ANSWER : ______________________

2008 IWYMIC Answers

Individual

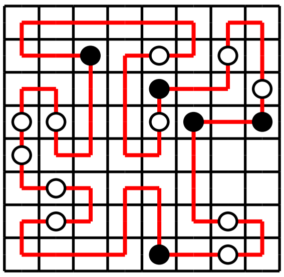
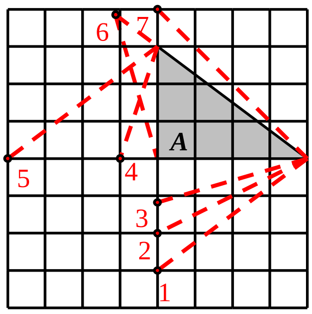
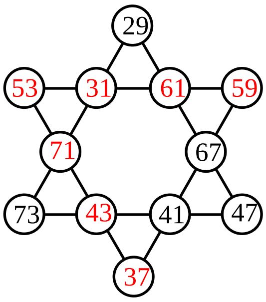
Part I

1.	34	**2.**	51	**3.**	$\frac{92}{17}$	**4.**	4
5.	144	**6.**	141	**7.**	4761	**8.**	12
9.	122	**10.**	$2+\sqrt{3}$	**11.**	132	**12.**	$\sqrt{21}$

Part II

1.	45	**2.**	$\frac{\sqrt{3}}{4}$	**3.**	3

Team

1.	63		
2.	6 4 ~~5~~ 7 ~~7~~ 3 ~~3~~ 5 ~~4~~ 8 4 3 6 7 5 1 3 1 5 ~~7~~ 7 ~~7~~ 6 2 7 5 ~~5~~ 8 ~~8~~ 4 2 ~~3~~ 4 ~~5~~ 6 5 8 1 7 3 ~~3~~ 3 ~~3~~ 6 1 ~~8~~ 8 ~~3~~ 1 7 3 2 ~~3~~ 6 4 8 ~~1~~ 6 2 ~~2~~ 4 5 ~~8~~ 7	**3.**	
4.	5	**5.**	2047
6.	7, 	**7.**	2
8.		**10.**	36

2009 Durban Invitational World Youth Mathematics Intercity Competition

10th IWYMIC

World Youth Mathematics Intercity Competition

Individual Contest

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and ID number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, watches or electronic devices are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All materials will be collected at the end of the competition.

2009 Durban Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 8th July 2009 Durban, South Africa

Team: ____________ Name: ___________________________ ID No.: ______________

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. If a, b and c are three consecutive odd numbers in increasing order, find the value of $a^2-2b^2+c^2$.

Answer **:__________**

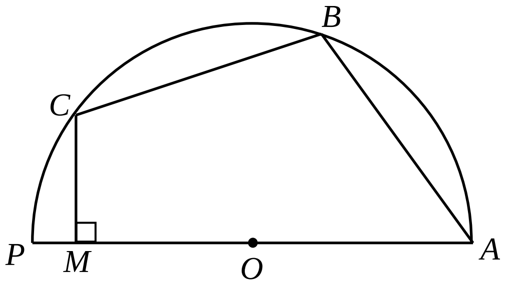
2. When the positive integer n is put into a machine, the positive integer $\frac{n(n+1)}{2}$ is produced. If we put 5 into this machine, and then put the produced number into the machine, what number will be produced?

Answer **:__________**

3. Children A, B and C collect mangos. A and B together collect 6 mangos less than C.
B and C together collect 16 mangos more than A.
C and A together collect 8 mangos more than B. What is the product of the number of mangos that A, B and C collect individually?

Answer **:__________**

4. The diagram shows a semicircle with centre O. A beam of light leaves the point M in a direction perpendicular to the diameter PA, bounces off the semicircle at C in such a way that *angle MCO* = *angle OCB* and then bounces off the semicircle at B in a similar way, hitting A. Determine *angle COM*， in degrees.



Answer **:__________**

5. Nineteen children, aged 1 to 19 respectively, are standing in a circle. The difference between the ages of each pair of adjacent children is recorded. What is the maximum value of the sum of these 19 positive integers?

***Answer* :__________**

6. Simplify as a fraction in lowest terms

$$\frac{(2^4+2^2+1)(4^4+4^2+1)(6^4+6^2+1)(8^4+8^2+1)(10^4+10^2+1)}{(3^4+3^2+1)(5^4+5^2+1)(7^4+7^2+1)(9^4+9^2+1)(11^4+11^2+1)}.$$

***Answer* :__________**

7. Given a quadrilateral *ABCD* not inscribed in a circle with *E*, *F*, *G* and *H* the circumcentres of triangles *ABD*, *ADC*, *BCD* and *ABC* respectively. If *I* is the intersection of *EG* and *FH*, and *AI* = 4 and *BI* = 3. Find *CI*.

***Answer* : __________**

8. To pass a certain test, 65 out of 100 is needed. The class average is 66. The average score of the students who pass the test is 71, and the average score of the students who fail the test is 56. It is decided to add 5 to every score, so that a few more students pass the test. Now the average score of the students who pass the test is 75, and the average score of the students who fail the test is 59. How many students are in this class, given that the number of students is between 15 and 30?

***Answer* :__________**

9. How many right angled triangles are there, all the sides of which are integers, having 2009^{12} as one of its shorter sides?
Note that a triangle with sides *a*, *b*, *c* is the same as a triangle with sides *b*, *a*, c; where *c* is the hypotenuse.

***Answer* :__________**

10. Find the smallest six-digit number such that the sum of its digits is divisible by 26, and the sum of the digits of the next positive number is also divisible by 26.

***Answer* :__________**

11. On a circle, there are 2009 blue points and 1 red point. Jordan counts the number of convex polygons that can be drawn by joining only blue vertices. Kiril counts the number of convex polygons which include the red point among its vertices. What is the difference between Jordan's number and Kiril's number?

***Answer* :__________**

12. Musa sold drinks at a sports match. He sold bottles of spring water at R4 each, and bottles of cold drink at R7 each. He started with a total of 350 bottles. Not all were sold and his total income was R2009. What was the minimum number of bottles of cold drink that Musa could have sold?

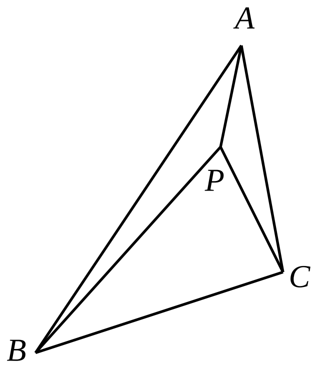
***Answer* :__________**

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. In a chess tournament, each of the 10 players plays each other player exactly once. After some games have been played, it is noticed that among any three players, there are at least two of them who have not yet played each other. What is the maximum number of games played so far?

2. *P* is a point inside triangle *ABC* such that *angle PBC* = 30°, *angle PBA* = 8° and *angle PAB* = *angle PAC* = 22°. Find *angle APC*, in degrees.



3. Find the smallest positive integer which can be expressed as the sum of four positive squares, not necessarily different, and divides $2^n + 15$ for some positive integer n.

2009 Durban Invitational World Youth Mathematics Intercity Competition

10th IWYMIC

World Youth Mathematics Intercity Competition

Team Contest

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on the first page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must solve at least one problem by themselves. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together. Each problem is worth 40 points and complete solutions of problem 1, 2, 6, 7, 8, 9 and 10 are required for full credits. Partial credits may be awarded.
- No calculator or calculating device or electronic devices are allowed.
- Answer in pencil or in blue or black ball point pen.
- Problems that required numerical answer must be filled in by Arabic numeral only.
- All materials will be collected at the end of the competition.

2009 Durban Invitational World Youth Mathematics Intercity Competition

Team Contest

Time limit: 60 minutes   8th July 2009   Durban, South Africa

Team: ______________________ Score: ______________________

1. The cards 1 to 15 are arranged in a deck, not in numerical order. The top card is placed on the table and the next card is transferred to the bottom of the deck. Now the new top card is placed on top of the card on the table and the next card is transferred to the bottom of the remaining deck. This process is repeated until all 15 cards are on the table. If the cards on the table are now in their natural order, 1 to 15, from top to bottom, what was the fourth card from the bottom in the original deck?

ANSWER: ______________________

Team Contest

Time limit: 60 minutes 8th July 2009 Durban, South Africa

Team: ________________________________ Score: ________________________

2. Find the smallest positive integer with at least one factor ending in each of the digits 0 to 9 i.e. at least one factor ends in 0, at least one factor ends in 1, …, at least one factor ends in 9.

ANSWER: ________________________

2009 Durban Invitational World Youth Mathematics Intercity Competition

10th IWYMIC

Team Contest

Time limit: 60 minutes 8th July 2009 Durban, South Africa

Team: ______________________________ Score: ________________________

3. Place the digits 1 to 6 in each of the rows and columns as well as the two diagonals such that no digit is repeated in a row, column or diagonal.

2			1		
					4
	2				
				6	
		5			1
3					

ANSWER:

2			1		
					4
	2				
				6	
		5			1
3					

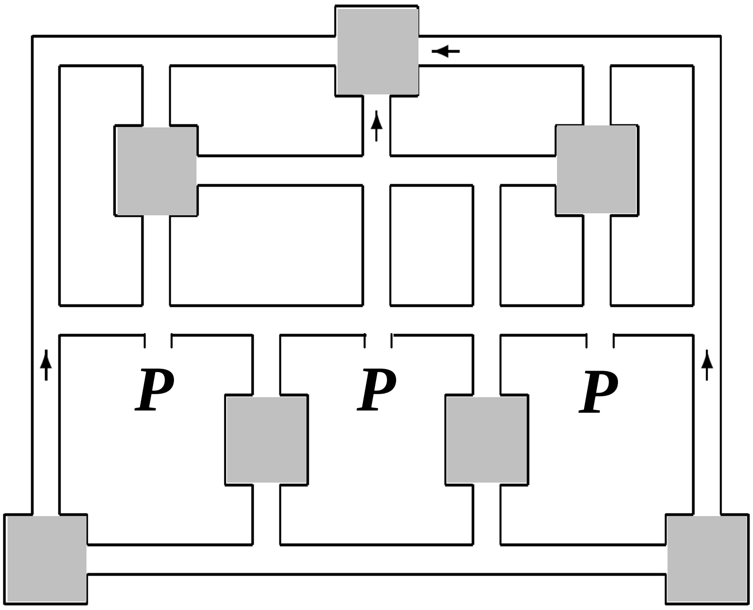
2009 Durban Invitational World Youth Mathematics Intercity Competition

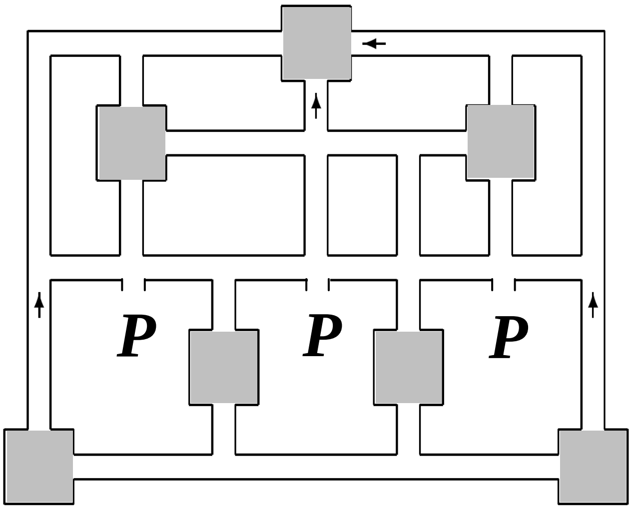
Team Contest

Time limit: 60 minutes　　8th July 2009　　Durban, South Africa

Team: ______________________ Score: ______________________

4. We have indicated the positions of three parking areas (indicated by the letter *P*) and seven squares (the shaded areas) on the map of this small town centre. Some of the streets only allow one-way traffic. This is shown by arrows which indicate the direction of traffic up to the first side street. Can you find a route that begins at one of the parking areas, passes through all the squares and ends at another parking area? Make sure that you do not visit any point, including intersection areas, on your route more than once.





ANSWER: ______________________

2009 Durban Invitational World Youth Mathematics Intercity Competition

Team Contest

Time limit: 60 minutes 8th July 2009 Durban, South Africa

Team: ________________________________ Score: ________________________

5. In the diagram below, draw a continuous path that begins and ends at the same place and runs through every square exactly once without crossing itself, so that between two consecutive circles on the path, if those circles are the same colour, then they must be joined by one straight line segment and if they are different colours, then they must be joined by two straight line segments which form a right angle. (You may only move horizontally or vertically.)

	○			○			
		●		●			
●	○	●				○	●
			○	●		●	
●				○			
	○		○			○	
		●		●	●		
		○		○		○	

	○			○			
		●		●			
●	○	●				○	●
			○	●		●	
●				○			
	○		○			○	
		●		●	●		
		○		○		○	

ANSWER: ______________________

2009 Durban Invitational World Youth Mathematics Intercity Competition

10th IWYMIC

Team Contest

Time limit: 60 minutes 8th July 2009 Durban, South Africa

Team: ______________________ Score: ______________________

6. Let $a_n = \dfrac{2^n}{2^{2n+1} - 2^{n+1} - 2^n + 1}$ for all positive integers *n*.

Prove that $a_1 + a_2 + \cdots + a_{2009} < 1$.

Proof

2009 Durban Invitational World Youth Mathematics Intercity Competition

10th IWYMIC

Team Contest

Time limit: 60 minutes 8th July 2009 Durban, South Africa

Team: ________________________________ Score: ________________________

7. Find **all the possible** ways of splitting the positive integers into cold numbers and hot numbers such that the sum of a hot number and a cold number is hot and their product is cold.

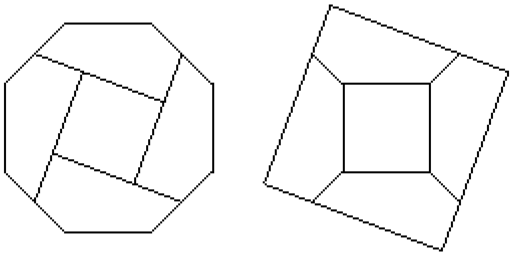
ANSWER: ________________________

Team Contest

Time limit: 60 minutes 8th July 2009 Durban, South Africa

Team: ________________________________ Score: ________________________

8. The diagram below shows how a regular octagon may be cut into a 1×1 square and four congruent pentagons which may be reassembled to form a square. Determine the perimeter of one of those pentagons.



ANSWER: ________________________

2009 Durban Invitational World Youth Mathematics Intercity Competition

10th IWYMIC

Team Contest

Time limit: 60 minutes 8th July 2009 Durban, South Africa

Team**:** ________________________________ Score**:** _______________________

9. A game of cards involves 4 players. In a contest, the total number of games played is equal to the total number of players entered in the contest. Every two players are together in at least one game. Determine the maximum number of players that can enter the contest.

ANSWER: ________________________

2009 Durban Invitational World Youth Mathematics Intercity Competition

10th IWYMIC

Team Contest

Time limit: 60 minutes 8th July 2009 Durban, South Africa

Team: ________________________________ Score: ________________________

10. Which of the numbers 2008, 2009 and 2010 may be expressed in the form $x^3 + y^3 + z^3 - 3xyz$, where x, y and z are positive integers?

ANSWER: ________________________

2009 IWYMIC Answers

Individual

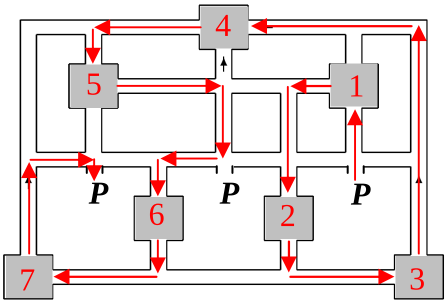
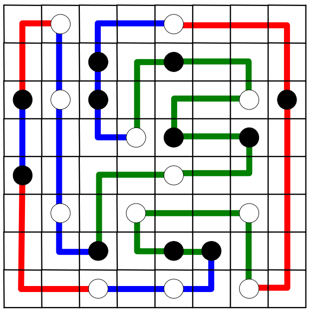
Part I

1.	8	**2.**	120	**3.**	60	**4.**	36°
5.	180	**6.**	$\frac{3}{133}$	**7.**	4	**8.**	24
9.	612	**10.**	898999	**11.**	2017036	**12.**	207

Part II

1.	25	**2.**	142°	**3.**	13

Team

1.	5	**2.**	270
3.	2 6 3 1 4 5 5 1 6 3 2 4 1 2 4 6 5 3 4 3 1 5 6 2 6 4 5 2 3 1 3 5 2 4 1 6	**4.**	
5.		**7.**	Let m be the smallest cold number. Then all the multiply of m are cold.
8.	$2+\sqrt{2+2\sqrt{2}}$	**9.**	13
10.	2008, 2009		

International Mathematics Competition,

25~29 July, 2010, Incheon, Korea,

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, watches or electronic devices are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

International Mathematics Competition,
25~29 July, 2010, Incheon, Korea,

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 27th July 2010 Incheon, Korea

Team**:** ____________ Name**:** ________________ No.**:** ____________ Score: ________

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Real numbers p, q, r satisfy the equations $p+q+r=26$ and $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=31$. Find the value of $\frac{p}{q}+\frac{q}{r}+\frac{r}{p}+\frac{p}{r}+\frac{r}{q}+\frac{q}{p}$.

   Answer : __________

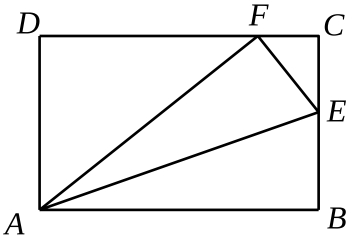
2. At a charity dinner, each person consumed half a plate of rice, a third of a plate of vegetables and a quarter of a plate of meat. Overall, 65 plates of food were served. What is the number of people at the charity dinner ?

   Answer : __________

3. How many triples (x, y, z) of positive integers satisfy $xyz = 3^{2010}$ and $x \le y \le z < x + y$?

   Answer : __________

4. E is a point on the side BC of a rectangle $ABCD$ such that if a fold is made along AE, as shown in the diagram below, the vertex B coincides with a point F on the side CD. If AD = 16 cm and BE = 10 cm, what is the length of AE, in cm ?



   Answer : ________cm

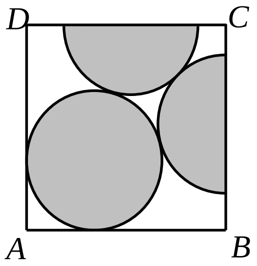
5. What is the smallest four-digit number which has exactly 14 positive divisors (including 1 and itself), such that the units digit of one of its prime divisors is 3?

   Answer : __________

6. Let $f(x)$ be a fourth-degree polynomial. $f(t)$ stands for the value of this polynomial while x=t. If $f(1)=f(2)=f(3)=0$, $f(4)=6$, $f(5)=72$, what's the last digit of the value of $f(2010)$?

Answer : _________

7. A square *ABCD* circumscribed a circle and two semicircles each with radius 1 cm. As shown in the diagram, the circle and two semicircles touch each other, and two sides of the square touch the circle also. Find, in cm^2, the area of the square *ABCD*.



Answer:_______cm^2

8. Let p and q be prime numbers such that $p^3+q^3+1=p^2q^2$. What is the maximum value of $p+q$?

Answer : _________

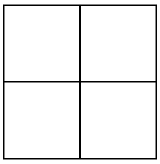
9. The sum of n positive integers, not necessarily distinct, is 100. The sum of any 7 of them is less than 15. What is the minimum value of n?

Answer : _________

10. P is a point inside triangle ABC such that $\angle ABP = 20^\circ$, $\angle PBC = 10^\circ$, $\angle ACP = 20^\circ$ and $\angle PCB = 30^\circ$. Determine $\angle CAP$, in degree.

Answer:________ °

11. A farmer has 100 pigs and 100 chickens. He has four yards each having square shape and forming together 2 × 2 grid. Farmer wants to distribute his animals into the yards in such way that first row has 120 heads, second row has 300 legs and first column has 100 heads, second column has 320 legs. How many different ways of doing this?



Answer : ______ways

12. An animal shelter consists of five cages in a row, labelled from left to right as shown in the diagram below. There is one animal in each cage.

Red Wolf	Silver Lion	Brown Fox	White Cow	Gray Horse

The five animals are indeed a wolf, a lion, a fox, a cow and a horse, and their colours are indeed red, silver, brown, white and gray. However, none of the labels matches any of the animals (For instance, the wolf is not red). Moreover, no animal is in or next to a cage whose label either matches its type or its colour. If the horse is not in the middle cage, what is the colour of the horse?

(Note：Write **R** for red, **S** for silver, **B** for Brown, **W** for white and **G** for Gray.)

Answer : __________

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. Point A and B lie on the sides of a square, segment AB divides the square into two polygons each of which has an inscribed circle. One of the circles has radius 6 cm while the other one is larger. What is the difference, in cm, between the side length of the square and twice the length of segment AB ?

2. A small bag of candy contains 6 pieces. A medium bag of candy contains 9 pieces. A large bag of candy contains 20 pieces. If we buy candy in bags only, what is the largest number of pieces of candies which we cannot obtain exactly?

3. There is a list of numbers a_1, a_2, …, a_{2010}. For $1 \le n \le 2010$, where n is positive integer, let $S_n = a_1 + a_2 + \cdots + a_n$. If $a_1 = 2010$ and $S_n = n^2 a_n$ for all n, what is the value of a_{2010} ?

International Mathematics Competition,

25~29 July, 2010, Incheon, Korea,

Invitational World Youth Mathematics Intercity Competition

Team Contest

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on the first page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. Each problem is worth 40 points and complete solutions of problem 1, 2, 3, 5, 6, 7 and 10 are required for full credits. Partial credits may be awarded. In case the spaces provided in each problem are not enough, you may continue you work at the back page of the paper. Only answers are required for Problem number 4, 8 and 9.The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must solve at least one problem by themselves. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator or calculating device or electronic devices are allowed.
- Answer must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

Team： ______________________________ Score： ________________

1. Solve the following system of equations for real numbers *w*, *x*, *y* and *z*:

$$\begin{cases} w+8x+3y+5z=20 \\ 4w+7x+2y+3z=-20 \\ 6w+3x+8y+7z=20 \\ 7w+2x+7y+3z=-20. \end{cases}$$

ANSWER: $w=$ $x=$ $y=$ $z=$

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

Team : ______________________________ Score : ________________

2. In the convex quadrilateral $ABCD$, AB is the shortest side and CD is the longest. Prove that $\angle A > \angle C$ and $\angle B > \angle D$.

ANSWER: ______________________________________

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

Team：___________________________ Score：______________

3. Let $m \ge n$ be integers such that $m^3 + n^3 + 1 = 4mn$. Determine the maximum value of $m - n$.

ANSWER: __________________________________

International Mathematics Competition,

25~29 July, 2010, Incheon, Korea,

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

Team：____________________________ Score：_______________

4. Arranged in an 8×8 array are 64 dots. The distance between adjacent dots on the same row or column is 1 cm. Determine the number of rectangles of area 12 cm^2 having all four vertices among these 64 dots.

ANSWER: ____________________________________

International Mathematics Competition,

25~29 July, 2010, Incheon, Korea,

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

Team : ______________________________ Score : ________________

5. Determine the largest positive integer n such that there exists a unique positive integer k satisfying $\frac{8}{15} < \frac{n}{n+k} < \frac{7}{13}$.

ANSWER: ______________________________________

TEAM CONTEST

Team：____________________________ Scroe：_______________

6. In a 9×9 table, every square contains a number. In each row and each column at most four different numbers appear. Determine the maximum number of different numbers that can appear in this table.

ANSWER: ___________________________________

TEAM CONTEST

Team：__________________________ Score：______________

7. In a convex quadrilateral *ABCD*, $\angle ABD = 16°$, $\angle DBC = 48°$, $\angle BCA = 58°$ and $\angle ACD = 30°$. Determine $\angle ADB$, in degree.

ANSWER: ______________________________

TEAM CONTEST

Team：________________________ Score：______________

8. Determine all ordered triples (x, y, z) of positive rational numbers such that each of $x+\frac{1}{y}$, $y+\frac{1}{z}$ and $z+\frac{1}{x}$ is an integer.

ANSWER: ______________________________

International Mathematics Competition,

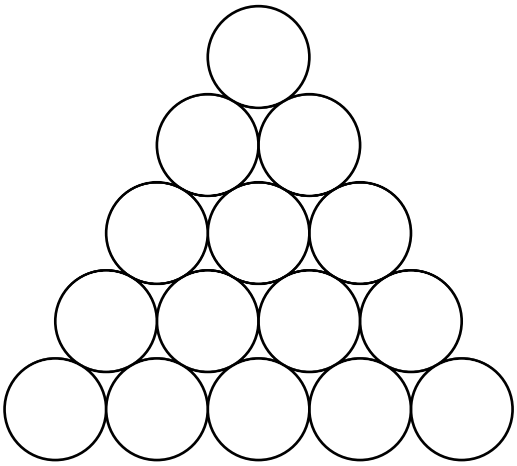
25~29 July, 2010, Incheon, Korea,

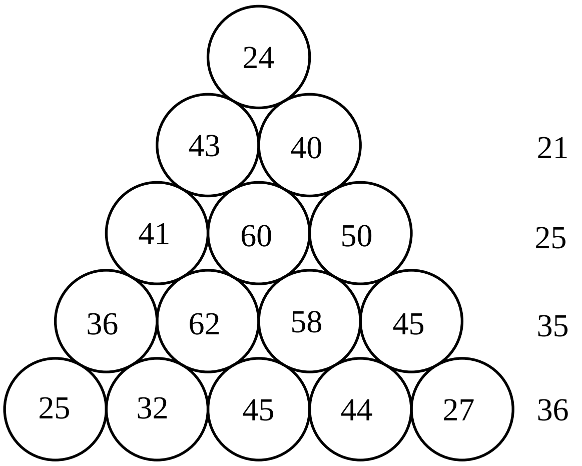
Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

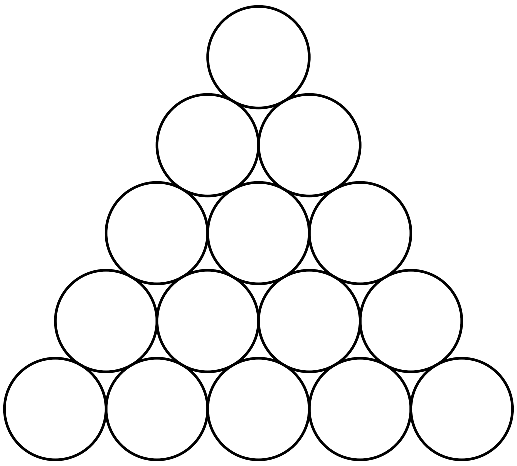
Team：__________________________ Score：______________

9. Assign each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15 into one of the fifteen different circles in the diagram shown below on the left, so that
   - (a) the number which appear in each circle in the diagram below on the right represents the sum of the numbers which will be in that particular circle and all circles touching it in the diagram below on the left;
   - (b) except the number in the first row, the sum of the numbers which will be in the circles in each row in the diagram below on the left is located at the rightmost column in the diagram below on the right.





ANSWER: ______________________________



International Mathematics Competition,

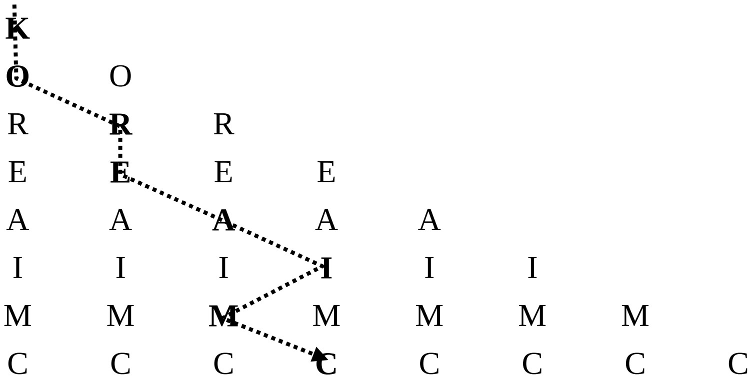
25~29 July, 2010, Incheon, Korea,

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

Team：____________________________ Score：_______________

10. The letters K, O, R, E, A, I, M and C are written in eight rows, with 1 K in the first row, 2 Os in the second row, and so on, up to 8 Cs in the last row. Starting with the lone K at the top, try to spell the "words" KOREA IMC by moving from row to row, going to the letter directly below or either of its neighbours, as illustrated by the path in boldface. It turns out that one of these 36 letters may not be used. As a result, the total number of ways of spelling KOREA IMC drops to 516. Circle the letter which may not be used.



K
O O
R R R
E E E E
A A A A A
I I I I I I
M M M M M M M
ANSWER: C C C C C C C C

2010 IWYMIC Answers

Individual

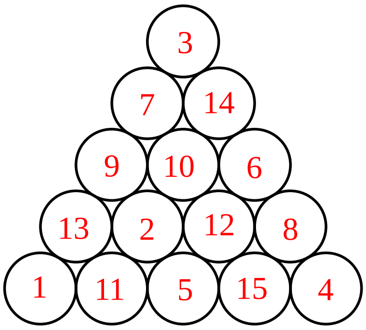
Part I

1.	803	**2.**	60	**3.**	336	**4.**	$10\sqrt{5}$
5.	1458	**6.**	2	**7.**	$3+\sqrt{2}+\sqrt{3}+\sqrt{6}$	**8.**	5
9.	50	**10.**	20°	**11.**	341	**12.**	W

Part II

1.	12	**2.**	43	**3.**	$\frac{2}{2011}$

Team

<table>
<tr><td>1.</td><td>$w=-8$ 、 $x=-\frac{12}{5}$ 、 $y=\frac{12}{5}$ 、 $z=8$</td><td>3.</td><td>1</td></tr>
<tr><td>4.</td><td>84</td><td>5.</td><td>112</td></tr>
<tr><td>6.</td><td>28,
<table>
<tr><td>1</td><td>2</td><td>3</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>7</td><td>8</td><td>9</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>10</td><td>11</td><td>12</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>13</td><td>14</td><td>15</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>16</td><td>17</td><td>18</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>19</td><td>20</td><td>21</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>22</td><td>23</td><td>24</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>25</td><td>26</td><td>27</td></tr>
</table>
<table>
<tr><td>1</td><td>10</td><td>19</td><td>28</td><td>28</td><td>28</td><td>28</td><td>28</td><td>28</td></tr>
<tr><td>28</td><td>2</td><td>11</td><td>20</td><td>28</td><td>28</td><td>28</td><td>28</td><td>28</td></tr>
<tr><td>28</td><td>28</td><td>3</td><td>12</td><td>21</td><td>28</td><td>28</td><td>28</td><td>28</td></tr>
<tr><td>28</td><td>28</td><td>28</td><td>4</td><td>13</td><td>22</td><td>28</td><td>28</td><td>28</td></tr>
<tr><td>28</td><td>28</td><td>28</td><td>28</td><td>5</td><td>14</td><td>23</td><td>28</td><td>28</td></tr>
<tr><td>28</td><td>28</td><td>28</td><td>28</td><td>28</td><td>6</td><td>15</td><td>24</td><td>28</td></tr>
<tr><td>28</td><td>28</td><td>28</td><td>28</td><td>28</td><td>28</td><td>7</td><td>16</td><td>25</td></tr>
<tr><td>26</td><td>28</td><td>28</td><td>28</td><td>28</td><td>28</td><td>28</td><td>8</td><td>17</td></tr>
<tr><td>18</td><td>27</td><td>28</td><td>28</td><td>28</td><td>28</td><td>28</td><td>28</td><td>9</td></tr>
</table>
</td><td>7.</td><td>30°</td></tr>
<tr><td>8.</td><td>$(1, 1, 1), (\frac{1}{2}, 2, 1), (2, 1, \frac{1}{2}),$
$(1, \frac{1}{2}, 2), (\frac{3}{2}, 3, \frac{1}{3}), (3, \frac{1}{3}, \frac{3}{2}),$
$(\frac{1}{3}, \frac{3}{2}, 3), (3, \frac{1}{2}, \frac{2}{3}), (\frac{1}{2}, \frac{2}{3}, 3)$
and $(\frac{2}{3}, 3, \frac{1}{2})$.</td><td>9.</td><td>
</td></tr>
<tr><td>10.</td><td colspan="3">The letter which may not be used is the third A in the fifth row from top</td></tr>
</table>

Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, watches or electronic devices are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes 20th July 2011 Bali, Indonesia

Team: ____________ Name: ________________ No.: ____________ Score: _________

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Let a, b, and c be positive integers such that

$$\begin{cases} ab+bc+ca+2(a+b+c)=8045, \\ abc-a-b-c=-2. \end{cases}$$

Find the value of $a+b+c$.

Answer : ________

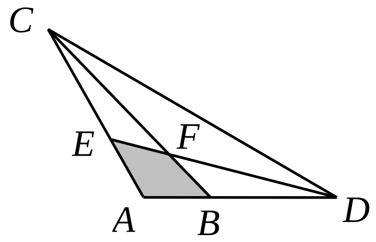
2. There are two kinds of students in a certain class, those who always lie and those who never lie. Each student knows what kind each of the other students is. In a meeting today, each student tells what kind each of the other students is. The answer "liar" is given 240 times. Yesterday a similar meeting took place, but one of the students did not attend. The answer "liar" was given 216 times then. How many students are present today?

Answer : _________

3. The product 1!×2! ×…×2011! ×2012! is written on the blackboard. Which factor, in term of a factorial of an integer, should be erased so that the product of the remaining factors is the square of an integer? (The factorial sign n! stands for the product of all positive integers less than or equal to n.)

Answer : _________

4. B and E are points on the sides AD and AC of triangles ACD such that BC and DE intersect at F. Triangles ABC and AED are congruent. Moreover, $AB=AE=1$ and $AC=AD=3$. Determine the ratio between the areas of the quadrilateral $ABFE$ and the triangle ADC.



Answer : _________

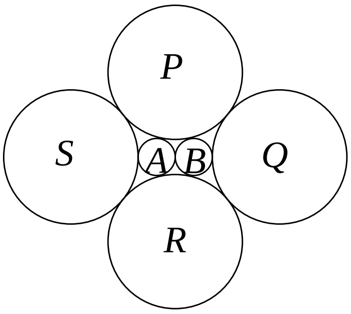
5. A positive integer n has exactly 4 positive divisors, including 1 and n. Furthermore, $n+1$ is four times the sum of the other two divisors. Find n.

Answer : _________

6. Jo tells Kate that the product of three positive integers is 36. Jo also tells her what the sum of the three numbers is, but Kate still does not know what the three numbers are. What is the sum of the three numbers?

Answer : _________

7. Two circles A and B, both with radius 1, touch each other externally. Four circles P, Q, R and S, all with the same radius r, are such that P touches A, B, Q and S externally; Q touches P, B and R externally; R touches A, B, Q and S externally; and S touches P, A and R externally. Calculate r.



Answer : _________

8. Find the smallest positive common multiple of 7 and 8 such that each digit is either 7 or 8, there is at least one 7 and there is at least one 8.

Answer : _________

9. The side lengths of a triangle are 50 cm, 120 cm and 130 cm. Find the area of the region consisting of all the points, inside and outside the triangle, whose distances from at least one point on the sides of the triangle are 2 cm. Take $\pi = \frac{22}{7}$.

Answer : _________

10. Find the number of positive integers which satisfy the following conditions:
    (1) It contains 8 digits each of which is 0 or 1.
    (2) The first digit is 1.
    (3) The sum of the digits on the even places equals the sum of the digits on the odd places.

Answer : ________

11. A checker is placed on a square of an infinite checkerboard, where each square is 1 cm by 1 cm. It moves according to the following rules:

- In the first move, the checker moves 1 square North.
- All odd numbered moves are North or South and all even numbered moves are East or West.
- In the n-th move, the checker makes a move of n squares in the same direction.

The checker makes 12 moves so that the distance between the centres of its initial and final squares is as small as possible. What is this minimum distance?

Answer : ________cm

12. Let a, b and c be three real numbers such that

$$\frac{a(b-c)}{b(c-a)} = \frac{b(c-a)}{c(b-a)} = k > 0$$

for some constant k. Find the greatest integer less than or equal to k.

Answer : _________

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. The diagonals AC and BD of a quadrilateral $ABCD$ intersect at a point E. If $AE=CE$ and $\angle ABC=\angle ADC$, does $ABCD$ have to be a parallelogram?

2. When a=1, 2, 3, …, 2010, 2011, the roots of the equation $x^2 - 2x - a^2 - a = 0$ are $(\alpha_1, \beta_1), (\alpha_2, \beta_2), (\alpha_3, \beta_3), \cdots, (\alpha_{2010}, \beta_{2010}), (\alpha_{2011}, \beta_{2011})$ respectively.

   Evaluate $\frac{1}{\alpha_1} + \frac{1}{\beta_1} + \frac{1}{\alpha_2} + \frac{1}{\beta_2} + \frac{1}{\alpha_3} + \frac{1}{\beta_3} + \cdots + \frac{1}{\alpha_{2010}} + \frac{1}{\beta_{2010}} + \frac{1}{\alpha_{2011}} + \frac{1}{\beta_{2011}}$.

3. Consider 15 rays that originate from one point. What is the maximum number of obtuse angles they can form? (The angle between any two rays is taken to be less than or equal to 180°)

Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

Invitational World Youth Mathematics Intercity Competition

Team Contest

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on every page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. Each problem is worth 40 points and complete solutions of problem 2, 4, 6, 8 and 10 are required for full credits. Partial credits may be awarded. In case the spaces provided in each problem are not enough, you may continue your work at the back page of the paper. Only answers are required for problem number 1, 3, 5, 7 and 9.
- The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must attempt at least one problem. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator or calculating device or electronic devices are allowed.
- Answer must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team：______________________________ Score：________________

1. Find all real solutions of the equation $x^2 - x + 1 = (x^2 + x + 1)(x^2 + 2x + 4)$.

ANSWER: ____________________________________

Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team：______________________________ Score：________________

2. A domino is a 1×2 or 2×1 piece. Seventeen dominoes are placed on a 5×8 board, leaving six vacant squares. Three of these squares are marked in the diagram below with white circles. The two squares marked with black circles are not vacant. The other three vacant squares are in the same vertical column. Which column contains them?

1	2	3	4	5	6	7	8
			●				
			○				
	○						○
							●

(For rough work)

1	2	3	4	5	6	7	8
◌	◌	◌	●	◌	◌	◌	◌
◌	◌	◌	○	◌	◌	◌	◌
◌	◌	◌	◌	◌	◌	◌	◌
◌	○	◌	◌	◌	◌	◌	○
◌	◌	◌	◌	◌	◌	◌	●

1	2	3	4	5	6	7	8
◌	◌	◌	●	◌	◌	◌	◌
◌	◌	◌	○	◌	◌	◌	◌
◌	◌	◌	◌	◌	◌	◌	◌
◌	○	◌	◌	◌	◌	◌	○
◌	◌	◌	◌	◌	◌	◌	●

ANSWER: Column______

Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

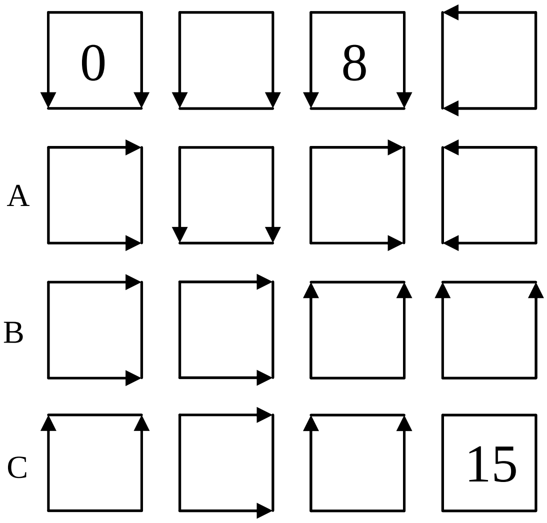
Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

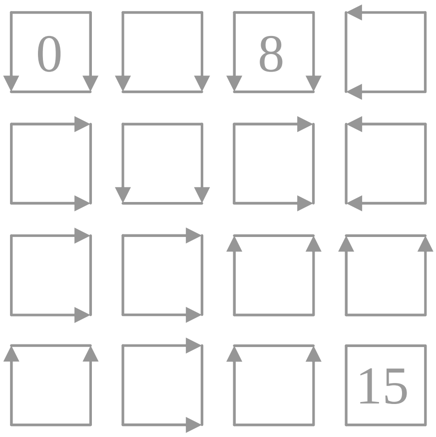
20th July 2011 Bali, Indonesia

Team： Score：

3. Place each of 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13 and 14 into a different vacant box in the diagram below, so that the arrows of the box containing 0 point to the box containing 1. For instance, 1 is in box A, B or C. Similarly, the arrows of the box containing 1 point to the box containing 2, and so on.



ANSWER:



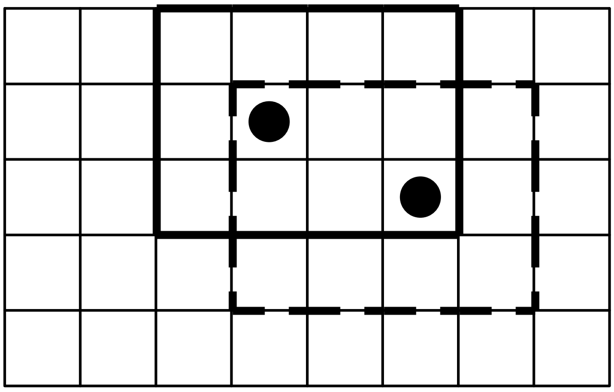
Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team : ______________________________ Score : ________________

4. The diagram below shows a 5×8 board with two of its squares marked with black circles, and the border of two 3×4 subboards which contain both marked squares. How many subboards (not necessarily 3×4) are there which contain at least one of the two marked squares?



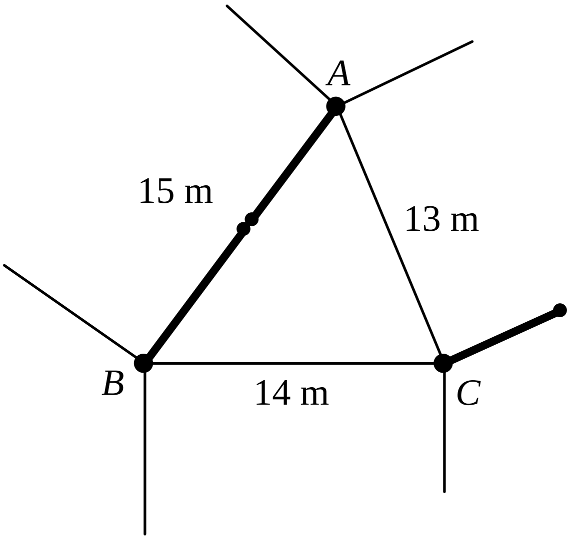
ANSWER: ______________________________________

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team : ______________________________ Score : ________________

5. Three avenues, of respective widths 15 m, 14 m and 13 m, converge on Red Triangle in the outskirt of Moscow. Traffic is regulated by three swinging gates hinged at the junction points of the three avenues. As shown in the diagram below, the gates at A and B close off one avenue while the gate at C is pushed aside to allow traffic between the other two avenues through the Red Triangle. Calculate the lengths of the three gates if each pair closes off one avenue exactly.



ANSWER: Gate at *A*=____m, at *B*=_____m, at *C*=_____m

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team：______________________________ Score：________________

6. Let $f(x)$ be a polynomial of degree 2010 such that $f(k)=-\frac{2}{k}$ where k is any of the first 2011 positive integers. Determine the value of $f(2012)$.

ANSWER: __________________________________

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team： ________________________ Score： _______________

7. A cat catches 81 mice, arrange them in a circle and numbers them from 1 to 81 in clockwise order. The cat counts them “One, Two, Three!” in clockwise order. On the count of three, the cat eats that poor mouse and counts “One, Two, Three!” starting with the next mouse. As the cat continues, the circle gets smaller, until only two mice are left. If the one with the higher number is 40, what is the number of the mouse from which the cat starts counting?

ANSWER: ______________________________

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team : ____________________ Score : ______________

8. In triangle ABC, $BC=AC$ and $\angle BCA=90°$. D and E are points on AC and AB respectively such that $AD = AE$ and $2CD = BE$. Let P be the point of intersection of BD with the bisector of $\angle CAB$. Determine $\angle PCB$.

ANSWER: ______________________________

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team：____________________________ Score：_______________

9. Paint 21 of the 49 squares of a 7×7 board so that no four painted squares form the four corners of any subboard.

(For rough work)

ANSWER: ____________________________________

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team : ______________________ Score : ______________

10. Arie, Bert and Caroline are given the positive integers a, b and c respectively. Each knows only his or her own number. They are told that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$, and are asked the following two questions:

(a) Do you know the value of $a+b+c$?

(b) Do you know the values of a, b and c?

Arie answers "No" to both questions. Upon hearing that, Bert answers "Yes" to the first question and "No" to the second. Caroline has heard everything so far. How does she answer these two questions?

ANSWER: (a) ____________ (b) ____________

2011 IWYMIC Answers

Individual

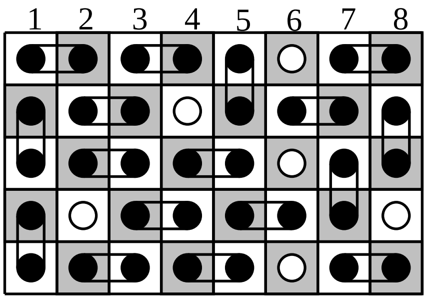
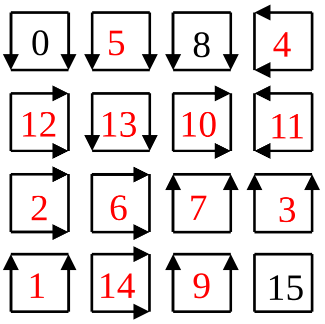
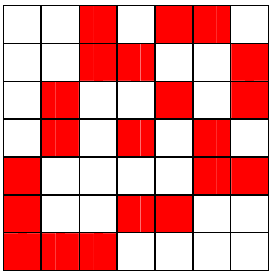
Part I

1.	2012	**2.**	22	**3.**	1006!	**4.**	$\frac{1}{6}$
5.	95	**6.**	13	**7.**	$\frac{3+\sqrt{17}}{2}$	**8.**	7888888
9.	$1182\frac{4}{7}$	**10.**	35	**11.**	2	**12.**	0

Part II

1.	Yes	**2.**	$-\frac{2011}{1006}$	**3.**	75

Team

1.	-1		
2.	6, 	**3.**	
4.	250	**5.**	7, 8 and 6
6.	$-\frac{1}{503}$	**7.**	7
8.	45°	**9.**	
10.	"Yes" to both questions		

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, watches or electronic devices are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Team:______________ Name:__________________ No.: _____________ Score:_________

For Juries Use Only

No.	Section A												Section B			Total	Sign by Jury
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**1**	**2**	**3**		
Score																	
Score																	

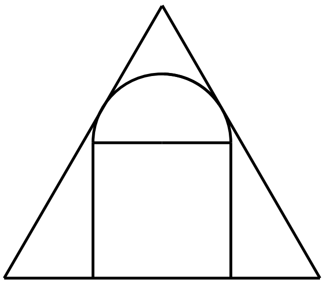
Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. Determine the maximum value of the difference of two positive integers whose sum is 2034 and whose product is a multiple of 2034.

Answer : _________

2. The diagram below shows a semicircle sitting on top of a square and tangent to two sides of an equilateral triangle whose base coincides with that of the square. If the length of each side of the equilateral triangle is 12 cm, what is the radius of the semicircle, in cm?



Answer : _______ cm

3. A four-digit number $\overline{abcd}$ is a multiple of 11, with $b + c = a$ and the two-digit number $\overline{bc}$ a square number. Find the number $\overline{abcd}$.

Answer : _________

4. The area of the equilateral triangle *ABC* is $8+4\sqrt{3}\,\text{cm}^2$. *M* is the midpoint of *BC*. The bisector of $\angle MAB$ intersects *BM* at a point *N*. What is the area of triangle *ABN*, in cm^2?

Answer: _______ cm^2

5. There is a 2×6 hole on a wall. It is to be filled in using 1×1 tiles which may be red, white or blue. No two tiles of the same colour may share a common side. Determine the number of all possible ways of filling the hole.

Answer : _________

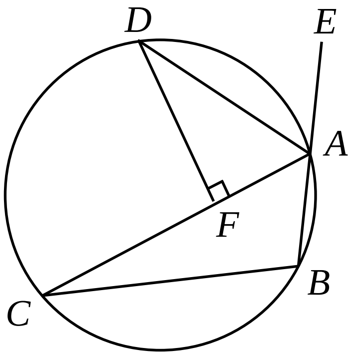
6. Let $N = 1^9 \times 2^8 \times 3^7 \times 4^6 \times 5^5 \times 6^4 \times 7^3 \times 8^2 \times 9^1$. How many perfect squares divide N?

Answer : _________

7. How many positive integers not greater than 20112012 use only the digits 0, 1 or 2?

Answer : _________

8. The diagram below shows four points *A*, *B*, *C* and *D* on a circle. *E* is a point on the extension of *BA* and *AD* is the bisector of $\angle CAE$. *F* is the point on *AC* such that *DF* is perpendicular to *AC*. If $BA = AF = 2$ cm, determine the length of *AC*, in cm.



Answer : ________ cm

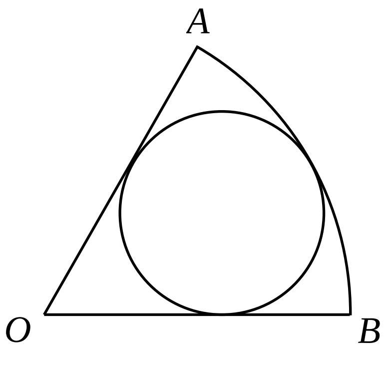
9. There are 256 different four-digit numbers $\overline{abcd}$ where each of *a*, *b*, *c* and *d* is 1, 2, 3 or 4. For how many of these numbers will $ad - bc$ be even?

Answer : __________

10. In a plane, given 24 evenly spaced points on a circle, how many equilateral triangles have at least two vertices among the given points?

Answer : __________

11. The diagram below shows a circular sector *OAB* which is one-sixth of a circle, and a circle which is tangent to *OA*, *OB* and the arc *AB*. What fraction of the area of the circular sector *OAB* is the area of this circle?



Answer : __________

12. An 8 × 8 chessboard is hung up on the wall as a target, and three identical darts are thrown in its direction. In how many different ways can each dart hit the center of a different square such that any two of these three squares share at least one common vertex?

Answer : __________

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. What is the integral part of M, if

$$M=\sqrt{2012\times\sqrt{2013\times\sqrt{2014\times\sqrt{\cdots\sqrt{(2012^2-3)\times\sqrt{(2012^2-2)\times\sqrt{(2012^2-1)\times\sqrt{2012^2}}}}}}}} \quad ?$$

Answer : _________

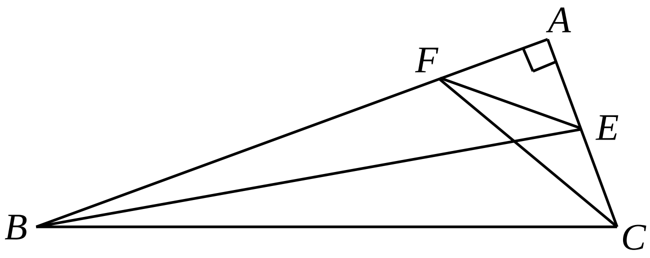
2. Let m and n be positive integers such that

$$n^2<8m<n^2+60(\sqrt{n+1}-\sqrt{n}) .$$

Determine the maximum possible value of n.

Answer : _________

3. Let ABC be a triangle with $\angle A = 90°$ and $\angle B = 20°$. Let E and F be points on AC and AB respectively such that $\angle ABE = 10°$ and $\angle ACF = 30°$. Determine $\angle CFE$.



$\circ$

Answer : ___________

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

Time：60 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on every page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. Each problem is worth 40 points and complete solutions of problem 2, 4, 6, 8 and 10 are required for full credits. Partial credits may be awarded. In case the spaces provided in each problem are not enough, you may continue your work at the back page of the paper. Only answers are required for problem number 1, 3, 5, 7 and 9.
- The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must attempt at least one problem. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator or calculating device or electronic devices are allowed.
- Answer must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

For Juries Use Only

No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**	Sign by Jury
Score												
Score												

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：__________________________ Score：_______________

1. A positive real number is given. In each move, we can do one of the following: add 3 to it, subtract 3 from it, multiply it by 3 and divide it by 3. Determine all the numbers such that after exactly three moves, the original number comes back.

Answer: ___________________________________

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：____________________ Score：____________

2. The average age of eight people is 15. The age of each is a prime number. There are more 19 year old among them than any other age. If they are lined up in order of age, the average age of the two in the middle of the line is 11. What is the maximum age of the oldest person among the eight?

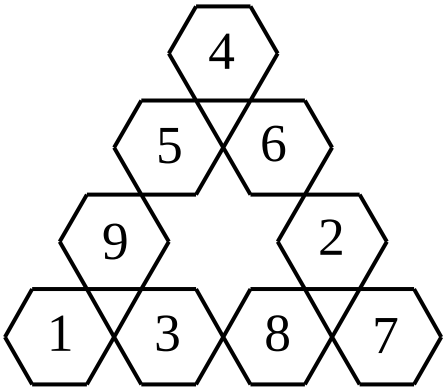
Answer: ________________________

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：________________ Score：________________

3. In the diagram below, the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 are placed one inside each hexagon, so that the sum of the numbers inside the four hexagons on each of the three sides of the triangle is 19. If you are allowed to rearrange the numbers but still have the same sum on each side, what is the smallest possible sum and what is the largest possible sum?



Answer:

The smallest possible sum is ________________

The largest possible sum is ________________

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team： ________________________ Score： ______________

4. There are 2012 evenly spaced points on a line. Each is to be painted orange or green. If three distinct points A, B and C are such that $AB = BC$, and if A and C are painted by the same color, so is B. Determine the number of all possible ways of painting these points.

Answer: ________________________________

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team： ____________________ Score： ______________

5. Consider the four-digit number 2012. We can divide it into two numbers in three ways, namely, 2|012, 20|12 and 201|2. If we multiply the two numbers in each pair and add the three products, we get 2×012+20×12+201×2=666. Find all other four-digit numbers which yield the answer 666 by this process.

Answer: ______________________________

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：____________________ Score：_______________

6. Let n be a positive integer such that $2n$ has 8 positive factors and $3n$ has 12 positive factors. Determine all possible numbers of positive factors of $12n$.

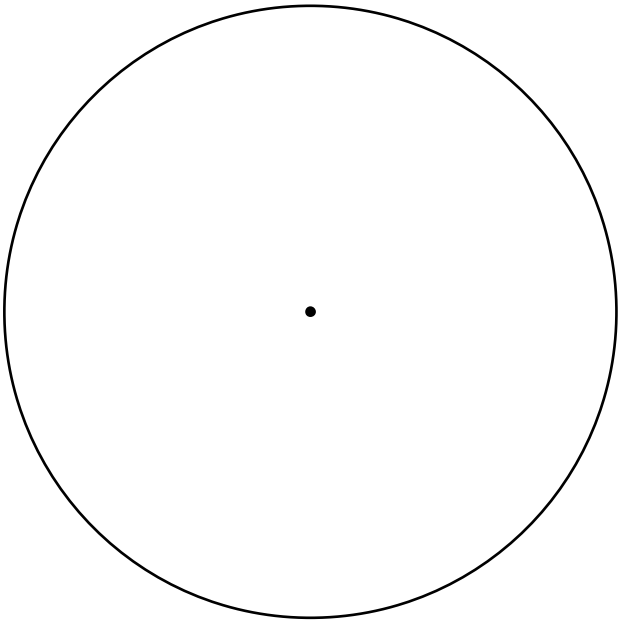
Answer: ____________________________

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：________________ Score：__________

7. Use straight and circular cuts to dissect a circle into congruent pieces. There must be at least one piece which does not contain the centre of the circle in its interior or on its perimeter.



Answer: ____________________

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：________________________ Score：_______________

8. A machine consists of three boxes each with a red light that is initially off. After putting objects into the boxes, the machine may be used to run a check. For each box, if the total weight in that box is strictly less than the total weight in each of the other two boxes, the red light of that box will go on. Otherwise, the red light will go off. Use this machine twice to find a fake ball among seven balls which is heavier than the other six. The other six are of equal weight.

Answer: ____

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

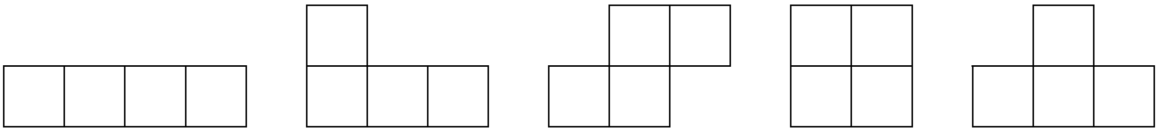
Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team : ______________________ Score : ______________

9. The diagram below shows all five pieces which can be formed of four unit squares. They are called the I-, L-, N-, O- and T-Tetrominoes.



(a) Use three different pieces to construct a figure with reflectional symmetry. Pieces can be rotated and reflected when used. Find five solutions.

(b) Use three different pieces to construct a figure with rotational symmetry. Pieces can be rotated and reflected when used. Find one solution.

(A figure consists of 12 connective unit squares joined edge to edge. Two figures are considered the same if one can be transformed into the other by rotation or reflection.)

(a)

(b)

Answer: ______________________

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：________________________ Score：______________

10. The digits in base 10 have been replaced in some order by the letters A, B, C, D, E, F, G, H, I and J. We have three clues.

(1) The two-digit number AB is the product of A, A and C.

(2) The two-digit number DE is the product of C and F.

(3) The two-digit number BG is the sum of H, I and the product of F and G.

Here A, B, and D are nonzero. Which digits may be represented by the letter J ?

Answer: ________________________________

Taiwan International Mathematics Competition 2012 (TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Section A. (5 points each)

Correct Answers:

1. **678**	2. $\frac{3\sqrt{3}}{2}$	3. **7161, 9361, 9812**	4. **4 cm^2**
5. **1458**	6. **672**	7. **4757**	8. **6 cm**
9. **160**	10. **536**	11. $\frac{2}{3}$	12. **196**

1. Determine the maximum value of the difference of two positive integers whose sum is 2034 and whose product is a multiple of 2034.

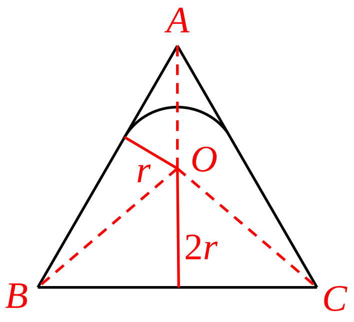
【Solution】

Let the two numbers be x and y. From $y = 2034 - x$, we have $xy = 2034x - x^2$. If this is divisible by 2034, then x^2 is divisible by 2034. Now $2034 = 2×32×113$. Hence $2×3×113 = 678$ must divide x, so that $678 \le x < 2034$. It follows that the only possible values for x are 678 and $2 × 678 = 1356$. The corresponding values for y are 1356 and 678 respectively. Hence $x - y = ±678$ and its maximum value is 678.

ANS : 678

2. The diagram below shows a semicircle sitting on top of a square and tangent to two sides of an equilateral triangle whose base coincides with that of the square. If the length of each side of the equilateral triangle is 12 cm, what is the radius of the semicircle, in cm?

【Solution】



Let the triangle be ABC and let O be the centre of the semicircle. Let r be the radius of the semicircle. With the sides of the triangle as bases, the heights of triangles OAB, OAC and OBC are r, r and $2r$ respectively. Their total area is equal to the area of triangle ABC. Since the height of triangle ABC is $12 \times \frac{\sqrt{3}}{2}$, we have $4r = 6\sqrt{3}$ and $r = \frac{3\sqrt{3}}{2}$.

ANS : $\frac{3\sqrt{3}}{2}$

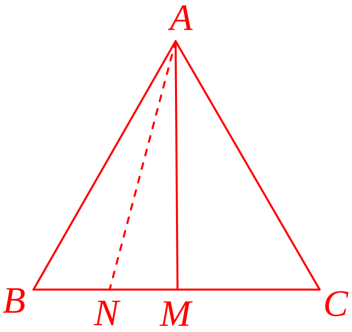
3. A four-digit number $\overline{abcd}$ is a multiple of 11, with $b + c = a$ and the two-digit number $\overline{bc}$ a square number. Find the number $\overline{abcd}$.

【Solution】

Since the two-digit number $\overline{bc}$ is a square number, and $b + c = a < 10$, we have $\overline{bc} = 16,25,36,81$. Since $\overline{abcd}$ is a multiple of 11, by trying possible digit d, we have $\overline{abcd} = 7161, 9361, 9812$

ANS：7161, 9361, 9812

4. The area of the equilateral triangle ABC is $8+4\sqrt{3}\,\text{cm}^2$. M is the midpoint of BC. The bisector of $\angle MAB$ intersects BM at a point N. What is the area of triangle ABN, in cm^2?



【Solution】

We have $\dfrac{NM}{BN} = \dfrac{AM}{AB} = \dfrac{\sqrt{3}}{2}$. Hence $\dfrac{BM}{BN} = \dfrac{2+\sqrt{3}}{2}$ and $\dfrac{BC}{BN} = 2+\sqrt{3}$. Denote the area of the triangle T by $[T]$.

Then $\dfrac{[ABC]}{[ABN]} = \dfrac{BC}{BN} = 2+\sqrt{3}$. It follows that $[ABN] = \dfrac{[ABC]}{2+\sqrt{3}} = \dfrac{8+4\sqrt{3}}{2+\sqrt{3}} = 4\,\text{cm}^2$.

ANS：4 cm^2

5. There is a 2×6 hole on a wall. It is to be filled in using 1×1 tiles which may be red, white or blue. No two tiles of the same colour may share a common side. Determine the number of all possible ways of filling the hole.

【Solution】

The top left space can be filled in 3 ways and the bottom left space can be filled in 2 ways, so that the first column from the left can be filled in 3×2=6 ways. In moving from column to column, we must retain at least one colour used in the preceding column. If we retain both colours, the only ways is to reverse the positions of the two tiles. If we retain just one colour, the tile with the repeated colour must be placed in a non-adjacent position, and the remaining space is filled with a tile of the third colour. Hence there are 3 ways to fill each subsequent column. It follows that the total number of ways is 6×3^5=1458.

ANS：1458

6. Let $N = 1^9 \times 2^8 \times 3^7 \times 4^6 \times 5^5 \times 6^4 \times 7^3 \times 8^2 \times 9^1$. How many perfect squares divide N?

【Solution】

The prime factorization of N is $2^{30} \times 3^{13} \times 5^5 \times 7^3$. Its largest square factor is $2^{30} \times 3^{12} \times 5^4 \times 7^2$. Its square factors are the squares of the factors of $2^{15} \times 3^6 \times 5^2 \times 7^1$. Their number is (15+1)(6+1)(2+1)(1+1)=672.

ANS：672

7. How many positive integers not greater than 20112012 use only the digits 0, 1 or 2?

【Solution】

The first few numbers are 1, 2, 10, 11, 12, 20, 21, 22, 100, 101 and so on. These are just numbers in base 3. The base 3 number 20112012 can be coverted to base 10 as follows.

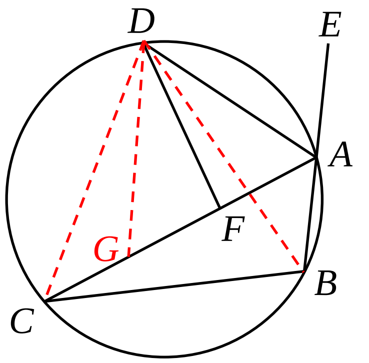
$$\begin{array}{rrrrrrrr}
2 & 0 & 1 & 1 & 2 & 0 & 1 & 2 \\
 & +\ 6 & +\ 18 & +\ 57 & +\ 174 & +\ 528 & +\ 1584 & +\ 4755 \\
\cline{2-8}
 & 6 & 19 & 58 & 176 & 528 & 1585 & 4757 \\
\times\ 3 & \times\ 3 & \times\ 3 & \times\ 3 & \times\ 3 & \times\ 3 & \times\ 3 & \\
\hline
6 & 18 & 57 & 174 & 528 & 1584 & 4755 &
\end{array}$$

Including the number 20112012 itself, there are 4757 positive integers which use only the digits 0, 1 and 2.

ANS : 4757

8. The diagram below shows four points *A*, *B*, *C* and *D* on a circle. *E* is a point on the extension of *BA* and *AD* is the bisector of $\angle CAE$. *F* is the point on *AC* such that *DF* is perpendicular to *AC*. If *BA* = *AF* = 2 cm, determine the length of *AC*, in cm.

【Solution】



Let *G* be the point on *AC* such that *FG* = *AF* = 2 cm. Then *GD* = *AD* and $\angle DAG = \angle DGA$. Sine *ABCD* is a cyclic quadrilateral, $\angle DCG = \angle DBA$. Moreover,

$$\begin{aligned}\angle DGC &= 180^\circ - \angle DGA \\ &= 180^\circ - \angle DAG \\ &= 180^\circ - \angle DAE = \angle DAB.\end{aligned}$$

It follows that triangles *DGC* and *DAB* are congruent, so that *GC* = *BA* = 2 cm. Hence *AC* = *AD* + *DG* + *GC* + 2 +2+2 = 6 cm.

ANS : 6 cm

9. There are 256 different four-digit numbers $\overline{abcd}$ where each of *a*, *b*, *c* and *d* is 1, 2, 3 or 4. For how many of these numbers will *ad* – *bc* be even?

【Solution】

Note that *ad* − *bc* is even if *ad* and *bc* are either both odd or both even. The former occurs when all four numbers are odd. The number of this case is $2^4 = 16$. The latter occurs when *a* and *d* are not both odd, and *b* and *c* are not both odd. The number of this case is $(16-2^2)^2 = 144$. Hence there are $16+144=160$ possible numbers.

ANS : 160

10. In a plane, given 24 evenly spaced points on a circle, how many equilateral triangles have at least two vertices among the given points?

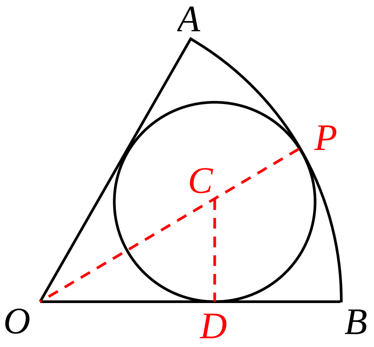
【Solution】

There are $\frac{24\times 23}{2} = 276$ pairs of given points. For each pair, we can have an equilateral triangle on each side of the line joining them. However, some of these 2×276 = 552 triangles have been counted 3 times, because all three vertices are among the given points. There are 24 ÷ 3 = 8 such triangles. Hence the final count is 552 − 2 × 8 = 536.

ANS : 536

11. The diagram below shows a circular sector *OAB* which is one-sixth of a circle, and a circle which is tangent to *OA*, *OB* and the arc *AB*. What fraction of the area of the circular sector *OAB* is the area of this circle?

【Solution】



Let *C* be the centre of the circle and let the extension of *OC* cut arc *AB* at a point *P*. Let *D* be the point on *OB* such that *CD* is perpendicular to *OB*. Let *CD* = *r*. Then *OC* = 2*r* and *CP* = *r*, so that *OP* = 3*r*. Hence the area of the sector is $\frac{1}{6}\pi(3r)^2=\frac{3}{2}\pi r^2$ while the area of the circle is πr^2. The desired fraction is $\frac{2}{3}$.

ANS : $\frac{2}{3}$

12. An 8 × 8 chessboard is hung up on the wall as a target, and three identical darts are thrown in its direction. In how many different ways can each dart hit a different square such that any two of these three squares share at least one common vertex?

【Solution】

There are 7 pairs of adjacent rows and 7 pairs of adjacent columns, so that the number of 2 × 2 subboards is 7 × 7 = 49. The three darts must all hit a different square of some 2 × 2 subboard, and the square they miss can be any of the 4 squares in the subboard. Hence the total number of ways is 4 × 49 = 196.

ANS : 196

Section B. (20 points each)

1. What is the integral part of *M*, if

$$M=\sqrt{2012\times\sqrt{2013\times\sqrt{2014\times\sqrt{\cdots\sqrt{(2012^2-3)\times\sqrt{(2012^2-2)\times\sqrt{(2012^2-1)\times\sqrt{2012^2}}}}}}}}$$

【Solution】

The required answer is 2012.

By using the inequality $\sqrt{(N-1)(N+1)}<N$, we arrived that

$$\sqrt{(2012^2-1)\times\sqrt{2012^2}}<\sqrt{(2012^2-1)(2012^2+1)}<2012^2,$$

It follows that

$$\sqrt{(2012^2-2)\times\sqrt{(2012^2-1)\times\sqrt{2012^2}}}<\sqrt{(2012^2-2)\times(2012^2)}<2012^2-1,$$

$$\sqrt{(2012^2-3)\times\sqrt{(2012^2-2)\times\sqrt{(2012^2-1)\times\sqrt{2012^2}}}} < \sqrt{(2012^2-3)\times(2012^2-1)}$$

$$< 2012^2-2$$

Repeating the same process, we conclude

$$\begin{aligned} M &= \sqrt{2012\times\sqrt{2013\times\sqrt{2014\times\sqrt{\cdots\sqrt{(2012^2-2)\times\sqrt{(2012^2-1)\times\sqrt{2012^2}}}}}}} \\ &< \sqrt{2012\times 2014} \\ &< 2013 \end{aligned}$$

This implies the integral part of *M* is less than 2013.

And Conversely,

$\sqrt{2012^2} \geq 2012$,

$\sqrt{(2012^2-1)\times\sqrt{2012^2}} > \sqrt{2012\times 2012} = 2012$,

$\sqrt{(2012^2-2)\times\sqrt{(2012^2-1)\times\sqrt{2012^2}}} > \sqrt{2012\times 2012} = 2012$,

$\sqrt{(2012^2-3)\times\sqrt{(2012^2-2)\times\sqrt{(2012^2-1)\times\sqrt{2012^2}}}} > \sqrt{2012\times 2012} = 2012$,

Continuing the same process, we have

$$\begin{aligned} M &= \sqrt{2012\times\sqrt{2013\times\sqrt{2014\times\sqrt{\cdots\sqrt{(2012^2-2)\times\sqrt{(2012^2-1)\times\sqrt{2012^2}}}}}}} \\ &> \sqrt{2012\times 2012} \\ &= 2012 \end{aligned}$$

In summary, the integral part of *M* is 2012.

ANS：2012

【Marking Scheme】

- ●.. Showing M is at least 2012..5 points
- ●.. Showing M is less than 2013 ....13 points
- ●.. Correct answer ....2 points

2. Let *m* and *n* be positive integers such that

$$n^2 < 8m < n^2 + 60(\sqrt{n+1}-\sqrt{n})$$

Determine the maximum possible value of *n*.

【Solution】

When divided by 8, n^2 leaves a remainder no greater than 4.

Hence if $60(\sqrt{n+1}-\sqrt{n}) < 4$, then there will not be a multiple of 8 between n^2 and $n^2 + 60(\sqrt{n+1}-\sqrt{n})$. It follows that we must have $60(\sqrt{n+1}-\sqrt{n}) \geq 4$.

Hence $15 \geq \frac{1}{\sqrt{n+1}-\sqrt{n}} = \sqrt{n+1}+\sqrt{n} > 2\sqrt{n}$, so that $n \leq 56$.

When n = 55 or 56, $60(\sqrt{n+1}-\sqrt{n}) < 5$, and the remainder when 55^2 or 56^2 is divided by 8 is no greater than 1. Hence the desired multiple of 8 cannot exist either. For n = 54, $60(\sqrt{55}-\sqrt{54}) = \frac{60}{\sqrt{55}+\sqrt{54}} \geq \frac{30}{\sqrt{55}} > 4$ since 30 × 30 = 900 > 880 = 4 × 4 × 55. Now 54^2 = 2916 so that $54^2 + 60(\sqrt{55}-\sqrt{54})$ > 2920. Since 2920 ÷8 = 365, we can take m = 365. It follows that the maximum value of n we seek is 54.

ANS : 54

【Marking Scheme】

● .... Show that $60(\sqrt{n+1}-\sqrt{n}) \geq 4$ .... 11 points

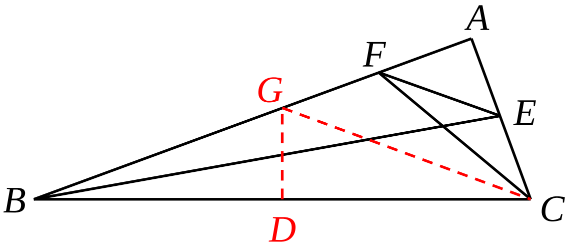
● .... Solve the inequality .... 7 points

● .... Correct answer .... 2 points

3. Let ABC be a triangle with $\angle A = 90°$ and $\angle B = 20°$. Let E and F be points on AC and AB respectively such that $\angle ABE = 10°$ and $\angle ACF = 30°$. Determine $\angle CFE$.

【Solution】

Note that $FC = 2AF$. Let D be the midpoint of BC and let G be the point on AB such that GD is perpendicular to BC. Then triangles ABC and DBG are similar, so that $\frac{BD}{BG} = \frac{BA}{BC}$. By symmetry, $\angle GCB = \angle GBC = 20°$, so that $\angle GCF = 20°$ also. Hence CG bisects $\angle BCF$ so that $\frac{FC}{FG} = \frac{BC}{BG}$. Since BE bisects $\angle ABC$, $\frac{BA}{BC} = \frac{AE}{CE}$. Now



$$\frac{AF}{FG} = \frac{\frac{1}{2}FC}{FG} = \frac{\frac{1}{2}BC}{BG} = \frac{BD}{BG} = \frac{BA}{BC} = \frac{AE}{EC}.$$

It follows that CG is parallel to EF, so that $\angle CFE = \angle GCF = 20°$.

ANS : 20°

【Marking Scheme】

● .... Draw the correct auxilary line CG .... 3 points

● .... List equations of ratios of lengths... 6 points

● .... Prove that CG is parallel to EF .... 9 points

- .... Correct answer .... 2 points

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Invitational World Youth Mathematics Intercity Competition

TEAM CONTEST

1. A positive real number is given. In each move, we can do one of the following: add 3 to it, subtract 3 from it, multiply it by 3 and divide it by 3. Determine all the numbers such that after exactly three moves, the original number comes back.

【Solution】

The operations of adding 3 and subtracting 3 are inverses of each other, as are the operations of multiplying by 3 and dividing by 3. If the same number is obtained after three operations, we can also achieve the same result by performing the inverses of these operations in reverse order. Sine the number of operations is odd, we cannot perform only additions and subtractions, nor can we perform only multiplications and divisions. Let the given number be x. We consider two cases.

Case I: Only one operation is multiplication or division.
By symmetry, we may assume that this operation is multiplication. There are three subcases.

Subcase I(a). The multiplication is the first operation.
The last two must both be subtractions. From $3x - 3 - 3 = x$, we have $x = 3$.

Subcase I(b). The multiplication is the second operation.
If the first operation is addition, then the third operation cannot bring the number back to x. Hence after two operations, we have $3(x - 3)$. If the third operation is addition, we have $3(x - 3) + 3 = x$ and we get $x = 3$ again. If the third operation is subtraction, we have $3(x - 3) - 3 = x$ so that $x = 6$.

Subcase I(c). The multiplication is the third operation.
The first two operations must both be subtractions. From $3(x - 3 - 3) = x$, we have $x = 9$.

Case II. Only one operation is additon or subtraction.
By symmetry, we may assume that this operation is subtraction. There are three subcases.

Subcase II(a). The subtraction is the first operation.
The last two operations must both be multiplications. From $3(3(x-3)) = x$, we have $x = \frac{27}{8}$.

Subcase II(b). The subtraction is the second operation.
If the first operation is division, then the third operation cannot bring the number back to x. Hence after two operations, we have $3x-3$. The third operation must also be multiplicaiton. From $3(3x-3)=x$, we have $x=\frac{9}{8}$.

Subcase II(c). The subtraction is the third operation.
The first two operations must both be multiplications. From $3(3x)-3=x$, we have $x=\frac{3}{8}$.

In summary, the possible values are $\frac{3}{8}$, $\frac{9}{8}$, $\frac{27}{8}$, 3, 6 and 9.

ANS : $\frac{3}{8}$, $\frac{9}{8}$, $\frac{27}{8}$, 3, 6 and 9

【Marking Scheme】

- Let k be the count of wrong/missing answers, $\text{Score}=40-\left\lceil\frac{20k}{3}\right\rceil$.

  (If the contestant missed the condition “positive”, and get answers $0,-3,-6,-9,-\frac{3}{8},-\frac{9}{8},-\frac{27}{8}$, they only counts as one wrong answer.)

2. The average age of eight people is 15. The age of each is a prime number. There are more 19 year old among them than any other age. If they are lined up in order of age, the average age of the two in the middle of the line is 11. What is the maximum age of the oldest person among the eight?

【Solution】

Note that the total age of the eight people is 8 × 15 = 120. The sum of the ages of the two people in the middle of the line is 22. There are only three ways of expressing 22 as a sum of two prime numbers.

Case I. 11 and 11.
Because 19 appears more often than 11, the oldest three must all be 19, so that none of the youngest three can be 11. Their ages must add up to $120-2\times11+3\times19=41$, but the sum of three prime numbers less than 11 is at most $7+7+7=21$. This case is impossible.

Case II. 5 and 17.
The ages of the youngest four must be among 2, 3 and 5. By the Pigeonhole Principle, two of them are of the same age. Hence the oldest three must all be 19, and we have the same contradiction as in Case I.

Case III. 3 and 19.
The the oldest three cannot all be 19. Hence there are at most three people who are 19. Now the ages of the youngest four must be among 2 and 3, so that exactly two of them are 2 and the other two are 3. The age of the oldest person is therefore

120−2×2−2×3−3×19 = 53, which happens to be a prime number.

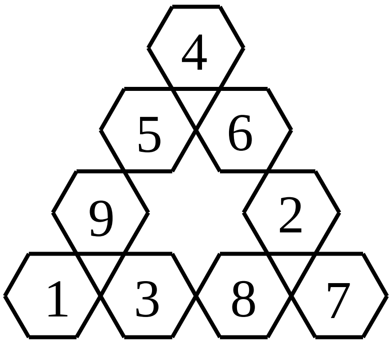
Thus the only possible age of the oldest person is 53.

ANS : 53

【**Marking Scheme**】

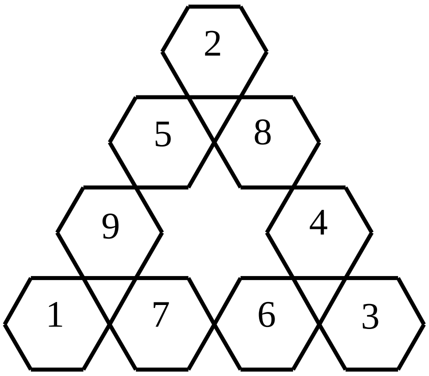
- ●.. Write out all possible case of the age of middle two people.....................................4 points
- ●.. Case I ....8 points
- ●.. Case II ....12 points
- ●.. Case III ....12 points
- ●.. Correct answer..4 points

3. In the diagram below, the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 are placed one inside each hexagon, so that the sum of the numbers inside the four hexagons on each of the three sides of the triangle is 19. If you are allowed to rearrange the numbers but still have the same sum on each side, what is the smallest possible sum and what is the largest possible sum?

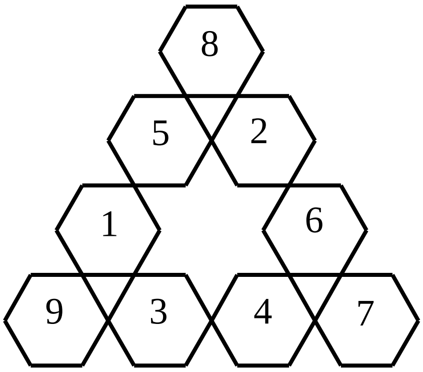


【**Solution**】

Each number is counted once except for the three at the corners of the triangle.
To minimize the constant sum, we put 1, 2 and 3 there. Since 1+4+7−1−2−3 = 6 and 6÷ 3 = 2, the minimum sum is 19 −2 = 17. The diagram below shows that the value may be attained.



To maximize the constant sum, we put 7, 8 and 9 there. Since 7+8+9−1−4−7 = 12 and 12÷3 = 4, the maximum sum is 19+4=23. The diagram below shows that the value may be attained.



<u>ANS：The minimum sum is 17 and the maximum sum is 23</u>

【Marking Scheme】

- ●.. Minimum sum.. 20 points
- ●.. Maximum sum... 20 points

4. There are 2012 evenly spaced points on a line. Each is to be painted orange or green. If three points *A*, *B* and *C* are such that *AB* = *BC*, then if *A* and *C* are colored by the same color, so is *B*. Determine the number of all possible ways of painting these points.

【Solution】

Let these points from left to right be $p_1, \cdots, p_{2012}$. WLOG assume that p_1 is green. If not all points are green, we consider the minimum x such that p_{x+1} is orange, then p_x is green. Since $\overline{p_x p_{x+1}} = \overline{p_{x+1} p_{x+2}}$, p_{x+2} must also be orange. We have the following lemma:

Lemma 1:

If p_x is green, p_{x+1} is orange, then for every $k > 0$ that satisfy $x + 2k \leq 2012$, points $p_{x+1}, \cdots, p_{x+2k}$ are all orange.

Proof of Lemma 1:

We prove it by induction on k.

When $k = 1$, We've already proved that both p_{x+1} and p_{x+2} are both orange.

If the lemma is correct for $k = k_0$, that is, all points $p_{x+1}, \cdots, p_{x+2k_0}$ are all orange.

When $k = k_0 + 1$, we have to prove that p_{x+2k_0+1} and p_{x+2k_0+2} are both orange.

Since $\overline{p_x p_{x+k_0+1}} = \overline{p_{x+k_0+1} p_{x+2k_0+2}}$, and p_x is green, $k_0 + 1 \leq 2k_0$, so by

induction hypothesis, p_{x+k_0+1} is orange. So p_{x+2k_0+2} must be orange too. Since

$\overline{p_{x+2k_0} p_{x+2k_0+1}} = \overline{p_{x+2k_0+1} p_{x+2k_0+2}}$, and both p_{x+2k_0} and p_{x+2k_0+2} are orange,

p_{x+2k_0+1} must be orange too. So the lemma also holds at $k = k_0 + 1$.

By induction, the lemma is proved.

Back to the original problem, we consider the following 2 cases:
Case I. $x = 1$, then by lemma 1, we'll have $p_2, \dots, p_{2011}$ are all orange, and p_{2012} can be green or orange.
Case II. $x > 1$, then by lemma 1, we'll have $p_{x+1}, \dots, p_{2011}$ are all orange, and since either the midpoint of $\overline{p_{x-1}p_{2012}}$ or $\overline{p_x p_{2012}}$ would be one of orange points, we must have p_{2012} is orange.

In conclusion, all possible solutions are of form OO…OGG…G, GG…GOO…O, OGG…GO, GOO…OG, where O stands for Orange, and G stands for Green. The first 2 cases both have 2012 possible solutions, so there are in total 2012×2+2 = 4026 solutions.

ANS：4026

【Marking Scheme】

- Prove lemma 1 or its equivalent .... 28 points
- Show solution of the form OO…OGG…G .... 4 points
- Show solution of the form OG…GO .... 4 points
- Correct answer .... 4 points

5. Consider the four-digit number 2012. We can divide it into two numbers in three ways, namely, 2|012, 20|12 and 201|2. If we multiply the two numbers in each pair and add the three products, we get 2×012+20×12+201×2=666. Find all other four-digit numbers which yield the answer 666 by this process.

【Solution】

Let the number of $1000a+100b+10c+d$, where a is a non-zero digit while b, c and d are any digits. Then $100ab+110ac+111ad+10bc+11bd+cd=666$. Note that $d\neq 0$ as otherwise $100ab+110ac+10bc\not\equiv 6 \pmod{10}$. We consider six cases.

Case I. $ad = 6$.
Then $111ad = 666$ so that all other terms must be 0, which means $b=c=0$. Hence we have 1006, 2003, 3002 and 6001.

Case II. $ad = 5$.
Then we have either $511b+551c+10bc=111$ or $155b+115c+10bc=111$. We must also have $b=c=0$, but the equation is not satisfied.

Case III. $ad = 4$.
We have three subcases.

Subcase III(a). $a = 4$ and $d = 1$.

Then $411b+441c+10bc = 222$, which forces $b = c = 0$. The equation is not satisfied.

Subcase III(b). $a = 1$ and $d = 4$.
Then $144b+114c+10bc = 222$, which forces $b = 0$ or $c = 0$. The equation is not satisfied.

Subcase III(c). $a = d = 2$.
Then $222b+222c+10bc = 222$. We have either $b = 0$ and $c = 1$ or $b = 1$ and $c = 0$, yielding 2012 or 2102.

Case IV. $ad = 3$.
If $a = 3$ and $d = 1$, then $311b+331c+10bc = 333$. Hence one of b and c is 1 and the other is 0, but the equation is not satisfied. If $a = 1$ and $d = 3$, then $133b+113c+10bc = 333$ so that $b + c \equiv 1 \pmod{10}$. The equation cannot be satisfied.

Case V. $ad =2$.
If $a =2$ and $d =1$, then $211b+221c+10bc =444$ so that $b+c\equiv 4 \pmod{10}$. The equation cannot be satisfied. If $a=1$ and $d=2$, then $122b+112c+10bc = 444$ so that $b + c\equiv 2 \pmod{5}$. The equation cannot be satisfied.

Case VI. $ad =1$.
Then $a = d =1$ and $111b+111c+10bc =555$. This is only possible for $b =0$ and $c =5$ or $b =5$ and $c =0$, yielding 1051 and 1501.

In summary, apart from 2012, the other numbers with the desired property are 1006, 1051, 1501, 2003, 2102, 3002 and 6001.

ANS：1006, 1051, 1501, 2003, 2102, 3002 and 6001

【Marking Scheme】

●.. Each wrong/missing answer ....-6 points

6. Let n be a positive integer such that $2n$ has 8 positive factors and $3n$ has 12 positive factors. Determine all possible numbers of positive factors of $12n$.

【Solution】

Note that $8 = 7 + 1 = (3 + 1)(1 + 1) = (1 + 1)(1 + 1)(1 + 1)$. Since $2n$ has 8 positive factors, it is either the 7th power of a prime, the product of a prime and the cube of another prime, or the product of 3 different primes. We consider these cases separately.

Case I. $2n = p^7$ for some prime p.
We must have $p = 2$, but then $3n = 3\times 2^6$ has $(1 + 1)(6 + 1) = 14$ positive factors instead of 12. This is impossible.

Case II. $2n = p^3q$ where p and q are different primes.
Suppose $q = 2 < p$. If $p = 3$, then $3n = 3^4$ has $4+1=5$ positive factors. If $p > 3$, then $3n = 3q^3$ has $(1 + 1)(3 + 1) = 8$ positive factors. Neither is possible. Suppose $p = 2 < q$. If $q = 3$, then $3n = 2^2 \times 3^2$ has $(2 + 1)(2 + 1) = 9$ positive factors. This is also impossible. If $q > 3$, then $3n = 2^2 \times 3q$ has $(2 + 1)(1 + 1)(1 + 1) = 12$ positive factors, which satisfies the given condition. Hence $12n = 2^4 \times 3q$ has $(4 + 1)(1 + 1)(1 + 1) = 20$

positive factors.

Case III. $2n = pqr$ where p, q and r are different primes.
By symmetry, we may take $r = 2$ and $q < p$. If $q = 3$, then $3n = 3^3p$ has $(2 + 1)(1 + 1) = 6$ positive factors. If $q > 3$, Then $3n = 3pq$ has $(1+1)(1+1)(1+1) = 8$ positive factors. Neither is possible.

In summary, we must have $n = 48q$ for some prime $q > 3$, and it has 20 positive factors.

ANS : 20

【Marking Scheme】

●.. Consider factorization and write out the relation with the number of divisors. ....8 points

●.. Case I ....8 points

●.. Case II ....8 points

●.. Case III ....8 points

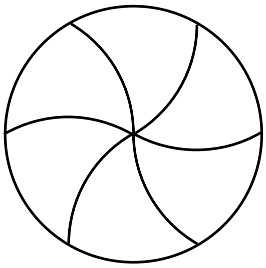
●.. Existence of such n..4 points

●.. Correct answer ....4 points

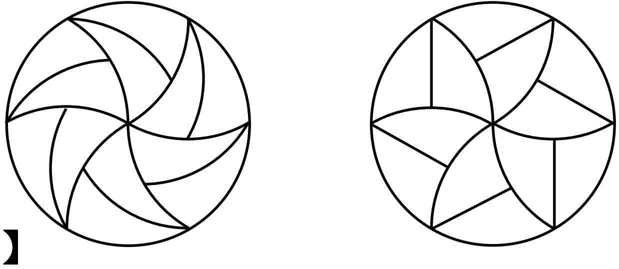
7. Use straight and circular cuts to dissect a circle into at least two congruent pieces. There must be at least one piece which does not contain the centre of the circle in its interior or on its perimeter.

【Solution】

The diagram on the right shows a six-piece dissection in which every piece contains the centre of the circle on its perimeters. Thus it is not a solution. However, it is a good first step towards a solution.



The diagram below shows the second step of two different twelve-piece dissections which satisfy the problem.



【Marking Scheme】

●.. Correct answer ....40 points

8. A machine consists of three boxes each with a red light that is initially off. When objects are put into the boxes, the machine checks the total weight in each box. If the total weight in one box is strictly less than the total weight in each of the other two boxes, the red light of that box will go on. Otherwise, all red lights remain off. Use this machine twice to find a fake ball among seven balls which is heavier than the other six. The other six are of equal weight.

【Solution】

Label the balls 1 to 7. In the first weighing, put balls 1 and 2 in the first box, balls 3 and 4 in the second box and balls 5, 6 and 7 into the third box. The red light of the third box cannot go on. There are three cases.

Case I. No red lights go on.
Then one of ball 5, 6 and 7 is heavy. In the second weighing, put ball 5 in the first box, ball 6 is in the second box and put two of the other five balls in the third box. Again, the red light of the third box cannot go on. If no red lights go on, then ball 7 is heavy. If the red light of the first box goes on, then ball 6 is heavy. If the red light on the second box goes on, then ball 5 is heavy.

Case II. The red light of the first box goes on.
Then one of balls 3 and 4 is heavy. In the second weighing, put ball 3 in the first box, ball 4 is in the second box and put two of the other five balls in the third box. The red light of either the first box or the second box must go on, and the heavy ball can be found.

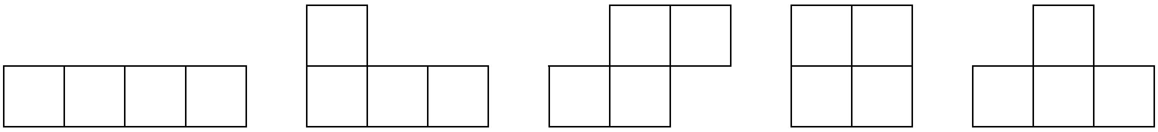
Case III. The red light of the second box goes on.
This is analogous to Case II, with one of balls 1 and 2 being heavy.

【Marking Scheme】

●... Consider
put 2, 2, 3 balls into each box and explain the result .... 24 points

●... Consider
put 1, 1, 2 balls into each box and explain the result .... 16 points

9. The diagram below shows all five pieces which can be formed of four unit squares joined edge to edge. They are called the I-, L-, N-, O- and T-Tetrominoes.



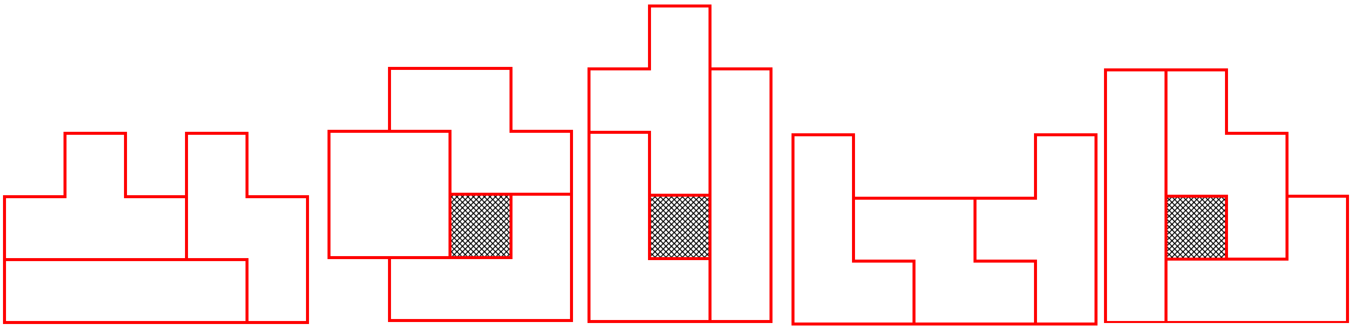
(a) Use three different pieces to construct a figure with reflectional symmetry. Find five solutions.

(b) Use three different pieces to construct a figure with rotational symmetry. Find one solution.

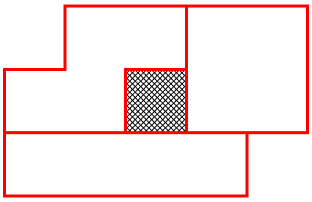
【Solution】

(a) Five constructions are shown in the diagram below, using the combinations TIN, LON, LIT, LNT and NIL For some combinations, there are other constructions

.



(b) A solution is shown in the diagram below, using the combination ION.



【Marking Scheme】

●... Correct answer of (a) ....3 points each(15 max)

●... Correct answer of (b) ....25 points

10. The digits in base 10 have been replaced in some order by the letters *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I* and *J*. We have three clues.

(1) The two-digit number *AB* is the product of *A*, *A* and *C*.

(2) The two-digit number *DE* is the product of *C* and *F*.

(3) The two-digit number *BG* is the sum of *H*, *I* and the product of *F* and *G*.

What digit is replaced by the letter *J* ?

【Solution】

Note that none of *A*, *B* and *D* can be 0. We have

$$10A+B=A^2C \qquad (1)$$

$$10D+E=CF \qquad (2)$$

$$10B+G=H+I+FG \qquad (3)$$

Consider (1). If $A = 1$, $10\le 10+B = C\le 9$, which is a contradiction. If A=2. $20+B = 4C$. We have B =4 and C6, or B = 8 and C =7. If A=3, $30+B$ =9C. We must have B =6 and

C =4. If A = 4, 40 +B =16C. We must have B =8 and C = 3. Suppose B = 8. Then H +I≤7+9 = 16 and FG≤7×9 = 63. By (3), $80\le 80+G = H+I+FG\le 16+63 = 79$, which is a contradiction. We now have two cases.

Case 1. B = 4.
Then A = 2 and C = 6. Now (2) becomes $10D + E = 6F$. Hence E is even. We have two subcases.

Subcase 1(a). E = 0.
From (2), F = 5 and D = 3, so that only the digits 1, 7, 8 and 9 are left. Now (3) becomes $40 = H + I + 4G$. The only solution here is G = 8 and $\{H, I\} = \{1, 7\}$.

Subcase 1(b). E = 8.
From (2), F = 3 and D = 1, so that only the digits 0, 5, 7 and 9 are left. Now (3) becomes $40 = H + I + 2G < 40$, which is a contradiction.

Case 2. B = 6.
Then A = 3 and C = 4. Now (2) becomes $10D + E = 4F$. Again E is even. We have three subcases.

> **Subcase 2(a).** E = 0.
> From (2), F = 5 and D = 2, so that only the digits 1, 7, 8 and 9 are left. Now (3) becomes $60 = H + I + 4G < 60$, which is a contradiction.
>
> **Subcase 2(b).** E = 2.
> From (2), we have either F = 3 and D = 1 or F = 8 and D = 3. Neither is possible as we already have A = 3.
>
> **Subcase 2(c).** E = 8.
> From (2), we have two possibilities. If F = 2 and D = 1, then (3) becomes $60 = H+I+G < 30$, which is a contradiction. Suppose F = 7 and D = 2, so that only the digits 1, 5, 8 and 9 are left. Now (3) becomes $60 = H + I + 6G$. There is a solution G = 9 and $\{H, I\} = \{1, 5\}$.

In conclusion, there are two solutions:

$A = 2, B = 4, C = 6, D = 3, E = 0, F = 5, G = 8, \{H, I\} = \{1, 7\}$ and $J = 9$.

$A = 3, B = 6, C = 4, D = 2, E = 8, F = 7, G = 9, \{H, I\} = \{1, 5\}$ and $J = 0$.

ANS : 0, 9

【Marking Scheme】

●.. List all
case of (A, B, C)..8 points

●.. Prove
cases for B=8 is impossible...8 points

●.. Case 1
and solution 1...12 points

●.. Case 2 and
solution 2...12 points

Thailand 1st Elementary Mathematics International Contest 2003 (TEMIC)

September 8, 2003

Rose Garden Aprime Resort, Nakhon Pathom, Thailand.

---

1. M sold some apples and received an amount of money. If M had sold 10 more apples for the same amount of money, the price of one apple would be 2 baht less than the original price. If M had sold 10 less apples for the same amount of money, the price of one apple would be 4 baht more than the original price. (Note: *Baht is the Thai Currency*)

   a) How many apples did M sell ?

   b) What was the price of one apple?

2. Bag A has twice the number of beads in bag B. 12% of beads in bag A are removed and transferred to bag C. 20% of beads in bag B are removed and transferred to bag C. After removing and transferring beads, there are now 488 beads in bag C which is 22% more than the original number of beads in bag C. How many beads were there in the bag A at the beginning?

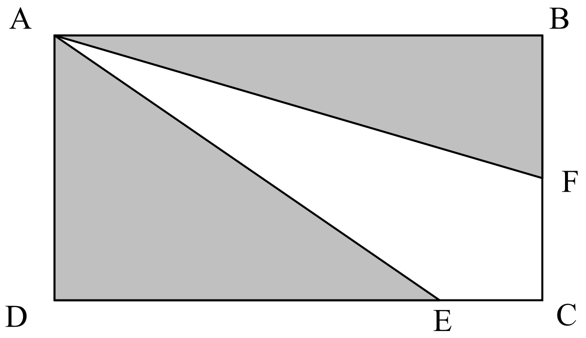
3. City P is 625 kilometers from City Q. M departed from City P at 5:30 a.m. travelling at 100 kilometers per hour, and arrived at City Q. Fifteen minutes after M left, N departed from City Q and arrived at City P travelling at 80 kilometers per hour. At what time did M and N meet together?

4. Alan has 80% more stamps than Billy. Billy has $\frac{3}{5}$ of the number of Charlie's stamps. If Billy gave 150 stamps to Charlie, then Charlie would now have three times the number of Billy’s remaining stamps. What is the total number of stamps they have altogether?

5. A boat is 50 kilometers away from the port. The boat is leaky, so water flows into the boat at the rate of 2 tons per 5 minutes. If there were 90 tons of water in the boat, the boat would sink. If there is a pump in the boat, pumping out 12 tons of water per hour, what should be the minimum speed of the boat in km/h to avoid the boat from sinking?

6. X is a 2-digit number whose value is $\frac{13}{4}$ of the sum of its digits. If 36 is added to X, the result will contain the same digits but in reverse order. Find X.

7.



Given; ABCD is a rectangle

BF = FC

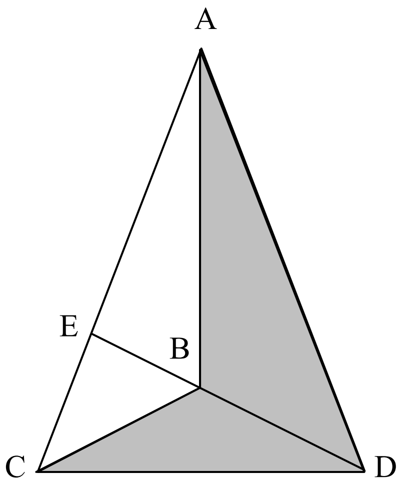
DE = 6EC

What is the ratio between the unshaded area and the shaded area?

8. Find all 2-digit numbers such that when the number is divided by the sum of its digits the quotient is 4 with a remainder of 3.

9. Calculate the result of $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 2001^2 - 2002^2 + 2003^2$.

10.



In the figure above, $\frac{EB}{BD} = \frac{1}{2}$ and the area of the shaded part is 42 cm^2. Find the area of ABC.

11. A, B and C worked together and received a total wage of 52400 baht. A received 125% of B's wage, but 90% of C's wage.
(Baht = Currency of Thailand)

a) Determine who received more: B or C?

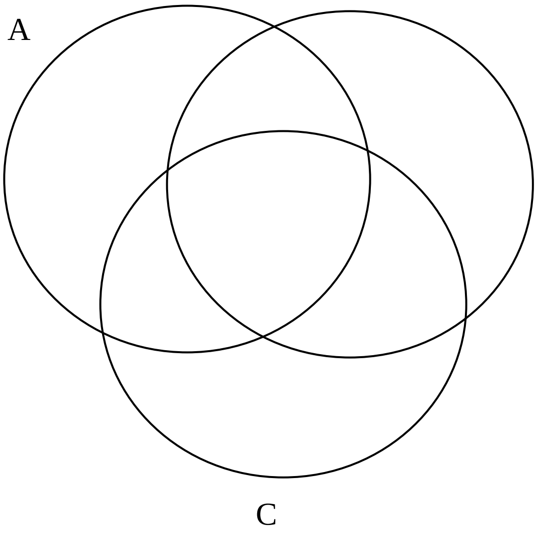
b) What is the difference between the wages of B and C?

12. There are 20 red marbles, 30 white marbles and some blue marbles in a box. If you draw one marble from the box, the probability or chance of drawing one blue marble is $\frac{9}{11}$. How many blue marbles are there in the box?

13. When 31513 and 34369 are each divided by a certain three-digit number, the remainders are equal. Find this remainder.

14. Fill in **all** the numbers below into circles A, B, C, such that all numbers in circle A are divisible by 5, all numbers in circle B are divisible by 2, all numbers in circle C are divisible by 3.

1749, 3250, 7893, 2025, 1348, 2001, 112, 102, 48, 2030, 930, 207, 750, 1605



15. Fill the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 into the boxes
□□□□□ × □□□ × □ , so that the expression will produce the **largest** product. (Each digit can be used only once)

Thailand 1st Elementary Mathematics International Contest 2003 (TEMIC)

September 8, 2003

Rose Garden Aprime Resort, Nakhon Pathom, Thailand.

(TEAM Contest)

Name..**Team**....................**Country**.....................

1. On quadrilateral ABCD, points M, N, P and Q are located on AB, BC, CD and DA, respectively. The ratios of distance are as follows:

AM : MB = 3 : 5
BN : NC = 1 : 3
CP : PD = 4 : 5
DQ : QA = 1 : 8

What is ratio of the area of MBNPDQ to the area of ABCD?

2. Peter had 144 books and donated them to four schools. When Peter checked the number of books given to each school, he found out that the difference of the number of books between School A and School B was 4; between School B and School C was 3; between School C and School D was 2.

School A had the most number of books, but received less than 40 books.

(a) In how many ways could Peter allot the books to School B and School D, according to all conditions?

(b) How many books will School B and School D each get?

3. The area of quadrilateral ABCD is 6174 cm^2. Points E and F are the midpoints of AB and CD, while G and H are the points on BC and AD respectively, such that CG = 2GB and AH = 2HD. What is the area of EGFH?

4. How many trailing zeros are there in the product of 1 x 2 x 3 x 4 x 5 x . . . x 2003? (Example: *10200000 has 5 trailing zeros*)

5. Alloy M is composed of 95% bronze, 4% tin and 1% zinc. Alloy N is composed of bronze and tin only. If alloy M is mixed with alloy N in equal proportion, a new alloy is formed, which has 86% bronze, 13.6% tin and 0.4% zinc.

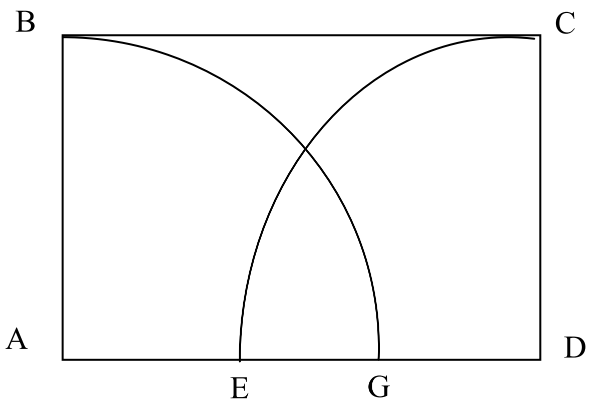
What is the percentage of bronze in alloy N?
(Note: *alloy is a mixture of metals*)

6. An uncovered tank of water has the capacity 43.12 m^3. The inner diameter of the tank is 2.8 meters. The walls and the base of the tank have a uniform thickness of 10 cm. If it costs 80 baht per square meter to paint the tank, calculate the cost of painting the total surface area. (Note: Baht is the Thai currency) (Given $\pi = \frac{22}{7}$ and answer to 2 decimals places.)

   (Hint: Remember to include all surfaces)

7. There are three numbers: 3945, 4686 and 5598. When they are divided by X, the remainder is the same for each. What is the sum of the X and the common remainder?

8. ABCD is a rectangle, with AB = 4 cm. The area of rectangle ABCD is equal to the area of the semicircle with radius AB. Find the length EG. ($\pi =$ 3.14)



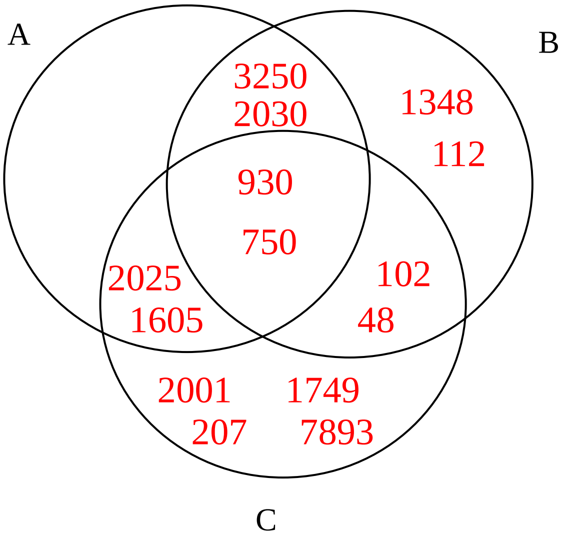
9. In a box of 12 different colored crayons, one of them is black. In how many different ways can the teacher give these crayons to a student so that the student receives at least one black?

(Note: A student may receive from 1 – 12 crayons)

10. How many seven-digit numbers contain the digit '7' at least once?

2003 EMIC Answers

Individual

1.	(a) 30 (b) 8	**2.**	400	**3.**	9:05	**4.**	2010	**5.**	$\frac{20}{3}$
6.	26	**7.**	$\frac{9}{19}$	**8.**	23,35, 47,59	**9.**	2007006	**10.**	21
11.	(a) C (b) 5600	**12.**	225	**13.**	97				
14.						**15.**	76421x853x9		

Team

1.	2:3	**2.**	(a) 1 (b) 34, 35
3.	3087	**4.**	499
5.	80	**6.**	11414.40 or 11415.20 or 11414.86
7.	69	**8.**	1.72
9.	2048	**10.**	4748472

India 2nd Elementary Mathematics International Contest

Individual Contest

Time Limit – 90 Minutes   10th September 2004   Lucknow, India

Team ________________   Contestant No. ____________   Score __________

Name ______________________________

Q1. There are 5 trucks. Trucks ***A*** and ***B*** each carry 3 tons. Trucks ***C*** and ***D*** each carry 4.5 tons. Truck ***E*** carries 1 ton more than the average load of all the trucks. How many tons does truck ***E*** carry?

Q2. Let $\boldsymbol{A}$ = 200320032003 × 2004200420042004 and

$\boldsymbol{B}$ = 200420042004 ×2003200320032003.

Find $\boldsymbol{A} - \boldsymbol{B}$.

Q3. There are 5 boxes. Each box contains either green or red marbles only. The numbers of marbles in the boxes are 110, 105, 100, 115 and 130 respectively. If one box is taken away, the number of green marbles in the remaining boxes will be 3 times the number of red marbles. How many marbles are there in the box that is taken away?

Q4. Find the smallest natural number which when multiplied by 123 will yield a product that ends in 2004.

Q5. Peter has a weigh balance with two pans. He also has one 200 g weight and one 1000 g weight. He wants to take 600 g of sugar out of a pack containing 2000 g of sugar. What is the minimum number of moves to accomplish this task?

India 2nd Elementary Mathematics International Contest

Q6. It takes 6 minutes to fry each side of a fish in a frying pan. Only 4 fish can be fried at a time. What is the minimum number of minutes needed to fry 5 fish on both sides?

Q7. John and Carlson take turns to pick candies from a bag. John picks 1 candy, Carlson 2 candies, John 3, Carlson 4 and so forth. After a while there are too few candies to continue and so the boy whose turn it is, takes all the remaining candies. When all the candies are picked, John has 1012 candies in total. What was the original number of candies in the bag?

Q8. There are five positive numbers. The sum of the first and the fifth number is 13. The second number is one-third of the sum of these five numbers, the third number is one-fourth of this sum and the fourth number is one-fifth of this sum. What is the value of the largest number?

Q9. In a class of students, 80% participated in basketball, 85% participated in football, 74% participated in baseball, 68% participated in volleyball. What is the minimum percent of the students who participated in all the four sports events?

Q10. Three digit numbers such as 986, 852 and 741 have digits in decreasing order. But 342, 551, 622 are not in decreasing order.

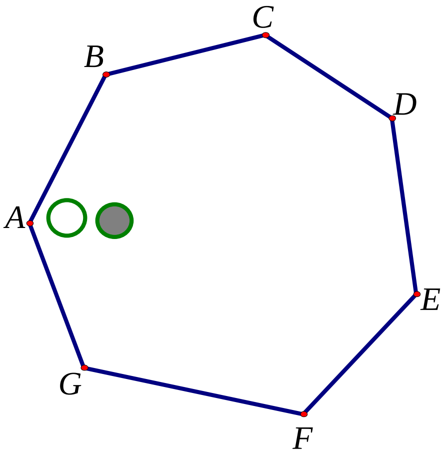
Each number in the following sequence is composed of three digits:

100, 101, 102, 103, …, 997, 998, 999.

How many three digit numbers in the given sequence have digits in decreasing order?

India 2nd Elementary Mathematics International Contest

Q 11. In the following figure, the black ball moves one position at a time clockwise. The white ball moves two positions at a time counter–clockwise. In how many moves will they meet again?



Q12. Compute: $1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots\ldots.. - 2002^2 + 2003^2 - 2004^2 + 2005^2$.

Q13. During recess one of the five pupils wrote something nasty on the blackboard. When questioned by the class teacher, they answered in following order:

A: "It was ***B*** and ***C***."

B: "Neither ***E*** nor I did it."

C: "***A*** and ***B*** are both lying."

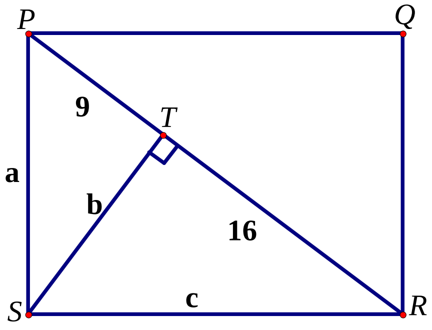
D: "Either ***A*** or ***B*** is telling the truth."

E: "***D*** is not telling the truth."

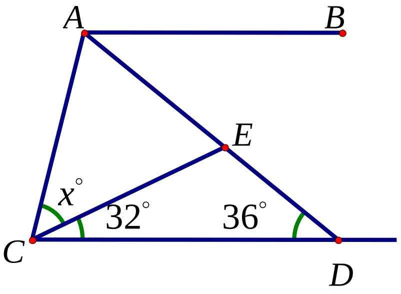
The class teacher knows that three of them never lie while the other two may lie. Who wrote it?

India 2^{nd} Elementary Mathematics International Contest

Q14. In the figure below, ***PQRS*** is a rectangle. What is the value of ***a*** **+** ***b*** **+** ***c***?



Q15. In the following figure, if ***CA*** **=** ***CE***, what is the value of *x*?



India 2nd Elementary Mathematics International Contest

Team Contest

Date- 10th September 2004 Place – Lucknow, India

Team _______________ Name ____________________________Score __________

T1. There are three people: grandfather, father and son. The grandfather’s age is an even number. If you invert the order of the digits of the grandfather’s age, you get the father’s age. When adding the digits of the father’s age together, you get the son’s age. The sum of the three people’s ages is 144. The grandfather’s age is less than 100. How old is the grandfather?

Answer : ____

India 2nd Elementary Mathematics International Contest

Team Contest

Date- 10th September 2004 Place – Lucknow, India

Team _______________ Name ____________________________Score __________

T2. Three cubes of volume 1 cm^3, 8 cm^3 and 27 cm^3 are glued together at their faces. Find the smallest possible surface area of the resulting configuration.

Answer : ____

India 2nd Elementary Mathematics International Contest

Team Contest

Date- 10th September 2004 Place – Lucknow, India

Team _______________ Name ____________________________Score __________

T3. A rectangle is 324 m in length and 141 m in width. Divide it into squares with sides of 141 m, and leave one rectangle with a side less than 141 m. Then divide this new rectangle into smaller squares with sides of the new rectangle's width, leaving a smaller rectangle as before. Repeat until all the figures are squares. What is the length of the side of the smallest square?

Answer : ____

India 2nd Elementary Mathematics International Contest

Team Contest

Date- 10th September 2004 Place – Lucknow, India

Team _______________ Name ____________________________Score __________

T4. We have assigned different whole numbers to different letters and then multiplied their values together to make the values of words. For example, if $F = 5$, $O = 3$ and $X = 2$, then $FOX = 30$.

Given that $TEEN = 52$, $TILT = 77$ and $TALL = 363$, what is the value of $TATTLE$?

Answer : ____

India 2nd Elementary Mathematics International Contest

Team Contest

Date- 10th September 2004 Place – Lucknow, India

Team _______________ Name ____________________________Score __________

T5. If $A = 1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots\ldots\ldots\ldots\ldots\ldots + 98 \times 99$ and

$B = 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + 97^2 + 98^2$,

what is the value of $A + B$?

Answer : ____

India 2nd Elementary Mathematics International Contest

Team Contest

Date- 10th September 2004 Place – Lucknow, India

Team ______________ Name __________________________Score __________

T6. Nine chairs in a row are to be occupied by six students and Professors Alpha, Beta and Gamma. These three professors arrive before the six students and decide to choose their chairs so that each professor will be between two students. In how many ways can Professors Alpha, Beta and Gamma choose their chairs?

Answer : ____

India 2nd Elementary Mathematics International Contest

Team Contest

Date- 10th September 2004 Place – Lucknow, India

Team ______________ Name __________________________Score __________

T7. Compute: $\frac{3}{1}+\frac{3}{1+2}+\frac{3}{1+2+3}+...+\frac{3}{1+2+3+...+100}$

Answer : ____

India 2nd Elementary Mathematics International Contest

Team Contest

Date- 10th September 2004 Place – Lucknow, India

Team _______________ Name ____________________________Score __________

T8. How many different three–digit numbers can satisfy the following multiplication problem?

$$\begin{array}{r} \square\ \square\ \square \\ \times\quad 9\ \square \\ \hline \square\ 2\ \square\ \square \end{array}$$

Answer : ____

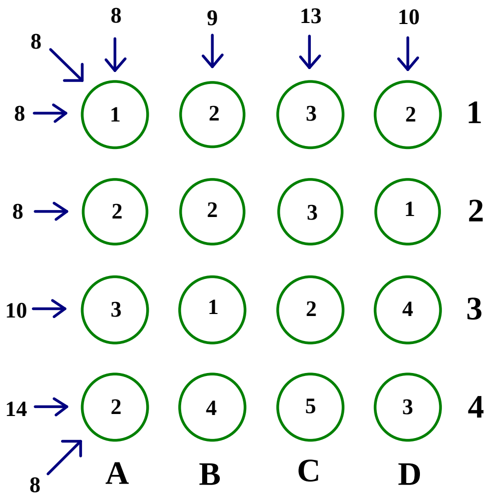
India 2nd Elementary Mathematics International Contest

Team Contest

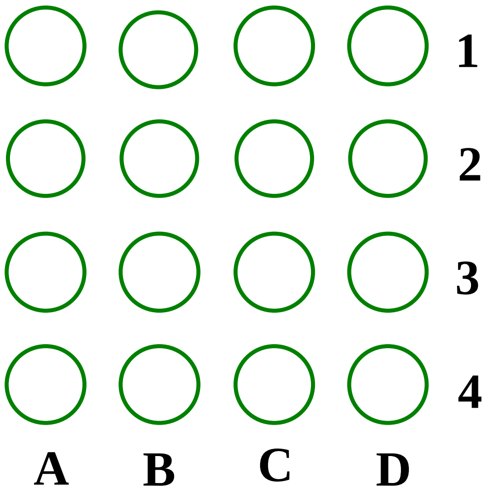
Date- 10th September 2004 Place – Lucknow, India

Team _______________ Name ____________________________Score ___________

T9. There are 16 containers of various shapes in the 4 × 4 array below. Each container has a capacity of 5 litres, but only contains the number of litres as shown in the diagram. The numbers on the sides indicate the total amount of water in the corresponding line of containers. Redistribute the water from only one container to make all the totals equal.



Show your answer by writing the new number of litres in each of the containers in the diagram below.



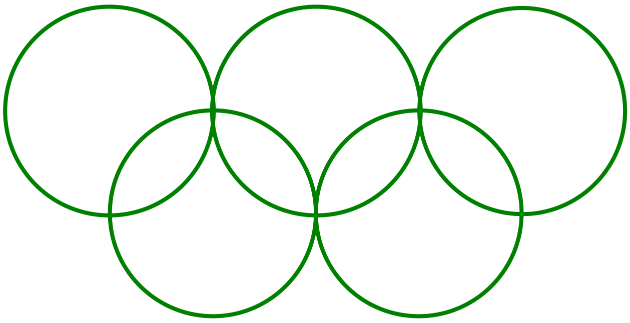
India 2nd Elementary Mathematics International Contest

Team Contest

Date- 10th September 2004 Place – Lucknow, India

Team _______________ Name ____________________________Score __________

T10. How many possible solutions are there in arranging the digits 1 to 9 into each closed area so that the sum of the digits inside every circle is the same. Each closed area contains only one digit and no digits are repeated. Draw all the possible solutions.



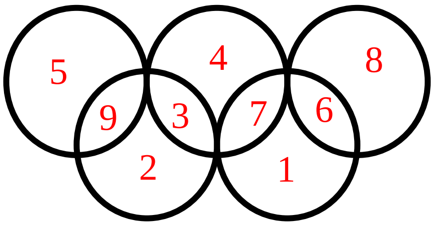
Answer: ____

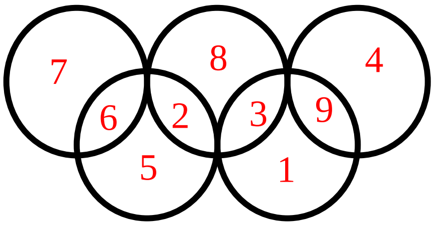
2004 EMIC Answers

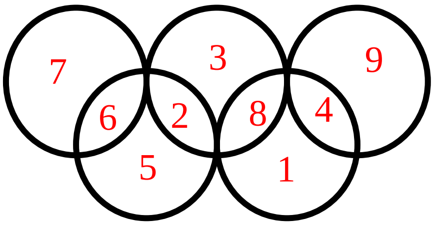
Individual

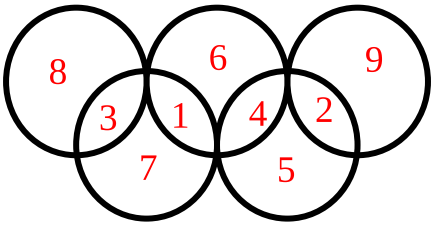
1.	5	**2.**	0	**3.**	100	**4.**	748	**5.**	1
6.	15 or 18	**7.**	2004	**8.**	20	**9.**	7	**10.**	120
11.	7	**12.**	2011015	**13.**	C	**14.**	47	**15.**	44°

Team

<table>
<tr><td>1.</td><td>84</td><td>2.</td><td>72</td></tr>
<tr><td>3.</td><td>3</td><td>4.</td><td>66</td></tr>
<tr><td>5.</td><td>641949</td><td>6.</td><td>60</td></tr>
<tr><td>7.</td><td>$5\frac{95}{101}$</td><td>8.</td><td>4</td></tr>
<tr><td>9.</td><td>③ ② ③ ② 1
② ② ⑤ ① 2
③ ① ② ④ 3
② ⑤ ⓪ ③ 4
A B C D</td><td></td><td>② ② ④ ② 1
② ③ ④ ① 2
③ ① ② ④ 3
③ ④ ⓪ ③ 4
A B C D</td></tr>
<tr><td>10.</td><td colspan="3">






</td></tr>
</table>

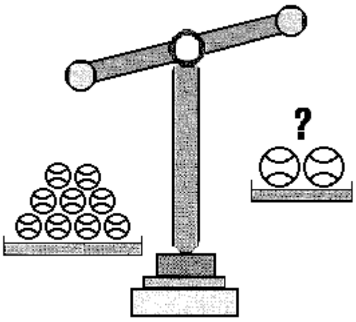
PEMIC PROBLEMS – Individual Contest

1. The numbers 4, 7, 10, 13, 16, …, where each number is three greater than the number preceding it, are written in order in a book, one hundred to a page. The first group of one hundred numbers begins on page 526. On which page will the number 2005 be located?
2. The numbers *a*, *b*, *c*, *d*, *e*, *f* and *g* are consecutive non-zero whole numbers arranged in increasing order. If $a + b + c + d + e + f + g$ is a perfect cube and $c + d + e$ is a perfect square, find the smallest possible value of *d*?

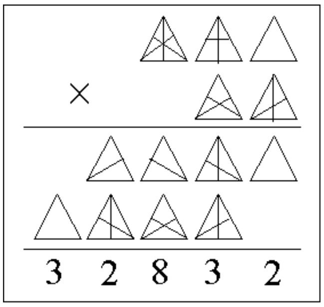
   (An example of a perfect cube is 8 because $8 = 2^3$.)

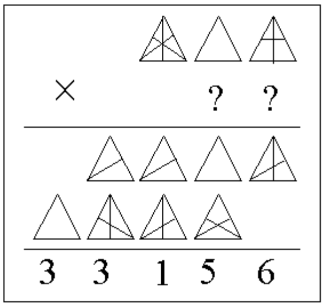
   (An example of a perfect square is 9 because $9 = 3^2$.)

3. If each large ball weighs $1\frac{1}{3}$ times the weight of each little ball, what is the minimum number of balls that need to be added to the right-hand side to make the scale balance? You may not remove balls, but only add small and/or large balls to the right-hand side.



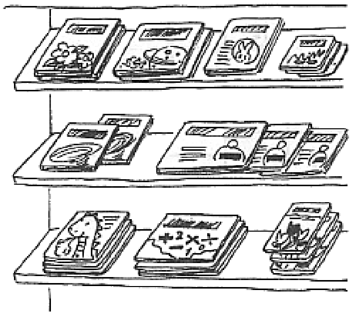
4. The different triangular symbols represent different digits from 1 to 9. The symbols represent the same digits in both examples. Find the two-digit number represented by **??.**





5. The following table shows the number of mathematics books sold over a period of five days. Find the number of books sold on Tuesday.

MATHEMATICS BOOKS

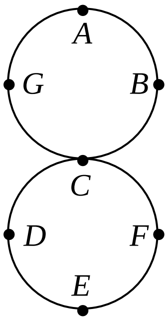


Monday, Tuesday & Wednesday	115
Wednesday & Thursday	85
Tuesday & Thursday	90
Monday & Friday	70
Thursday & Friday	80

6. Fractions in the form $\frac{a}{b}$ are created such that a and b are positive whole numbers and $a + b = 333$. How many such fractions are less than one and cannot be simplified?
(Cannot be simplified means that the numerator and denominator have no common factor)

7. Four friends were racing side by side down a dusty staircase. Peter went down two steps at a time, Bruce three steps at a time, Jessica four steps at a time, and Maitreyi five steps at a time. If the only steps with all four footprints were at the top and the bottom, how many steps had only one person's footprint?

8. In the diagram, there are two touching circles, each of radius 2 cm. An ant starts at point *A* and walks around the figure 8 path ABCDEFCGA in that order. The ant repeats the figure 8 walk, again and again. After the ant has walked a distance of 2005π cm it becomes tired and stops. The ant stops at a point in the path. What letter point is it?



9. A basket and 16 potatoes are placed in a straight line at equal intervals of 6 meters, with the basket fixed at one end. What is the shortest possible time for Jose to bring the potatoes one by one into the basket, if he starts from where the basket is and runs at an average speed of 3 meters per second?

10. A sequence of digits is formed by writing the digits from the natural numbers in the order that they appear. The sequence starts:

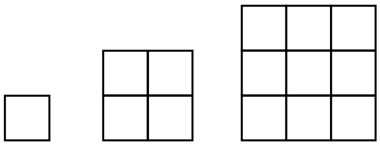
123456789101112 …

What is the 2005th digit in the sequence?

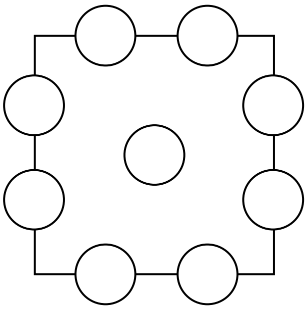
11. While B is riding a bicycle from Point X to Point Y, C is driving a car from Point Y to Point X, each at a steady speed along the same road. They start at the same time and, after passing each other, B takes 25 times longer to complete the journey as C. Find the ratio of the speed of the bicycle to the speed of the car.

12. Ten whole numbers (not necessarily all different) have the property that if all but one of them are added, the possible sums (depending on which one is omitted) are: 82, 83, 84, 85, 87, 89, 90, 91, 92. The 10th sum is a repetition of one of these. What is the sum of the ten whole numbers?

13. A sequence of squares is made of identical square tiles. The edge of each square is one tile length longer than the edge of the previous square. The first three squares are shown. How many more tiles does the 2005th square have than the 2004th?



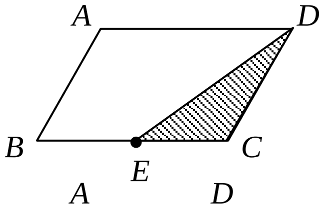
14. Lucky, Michael, Nelson and Obet were good friends. Obet had no money. Michael gave one-fifth of his money to Obet. Lucky gave one-fourth of his money to Obet. Finally, Nelson gave one-third of his money to Obet. Obet received the same amount of money from each of them. What fraction of the group's total money did Obet have at the end?



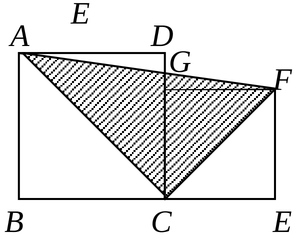
15. Each of the numbers from 1 to 9 is placed, one per circle, into the pattern shown. The sums along each of the four sides are equal. How many different numbers can be placed in the middle circle to satisfy these conditions?

PEMIC PROBLEMS – Team Contest

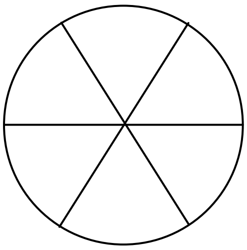
1. In parallelogram *ABCD*, *BE* = *EC*. The area of the shaded region is 2 cm^2. What is the area of parallelogram *ABCD*, in cm^2?



2. Refer to the diagram at the right. The length of one side of the large square is 4 cm and the length of one side of the small square is 3 cm. Find the area of the shaded region, in cm^2.



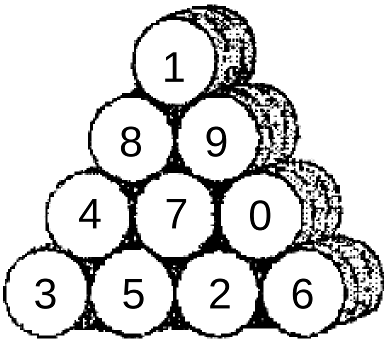
3. The circle below is divided into six equal parts.

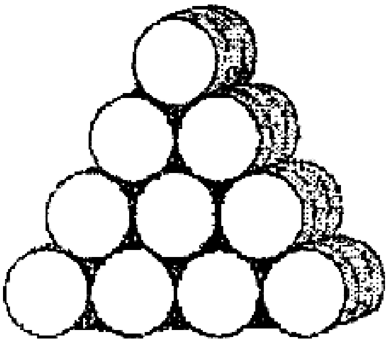


Suppose you paint one or more of these parts black, how many **different** patterns can you form? Any rotation of a pattern will be counted once.

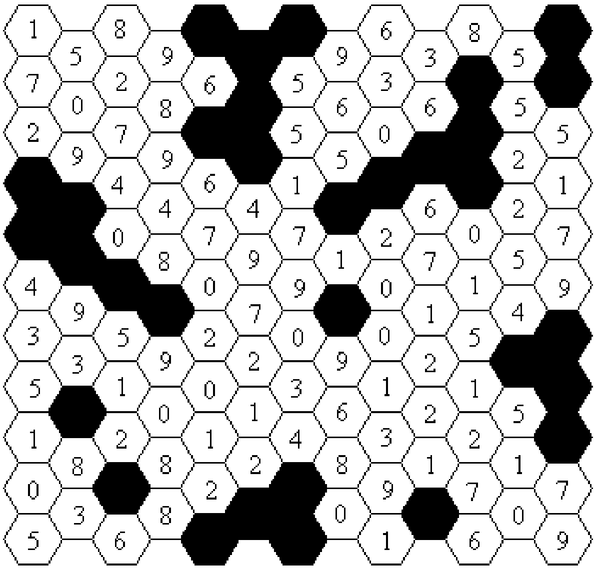
4. Let $n = 9 + 99 + 999 + \cdots + 99999\cdots 9$, where the last number to be added consists of 2005 digits of 9. How many times will the digit 1 appear in n?

5. A merchant had ten barrels of oil which he arranged as a pyramid, as shown. Every barrel bore a different number. You can see that he had accidentally arranged them so that for each side the numbers add up to 16. Rearrange them so that for each side, the numbers add up to the smallest sum possible. The sum must be the same for all three sides.

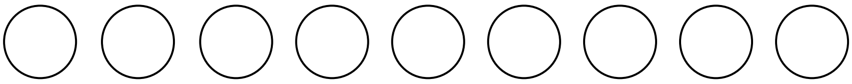




6. Find a route from a top cell to a bottom cell of this puzzle that gives 175 as a total. When your route passes any cell adjacent to zero, your total reduces to zero. Each cell may be used only once.



7.



Arrange the digits 1 – 9 in the circles in such a way that:

1 and 2 and all the digits between them add up to 9.

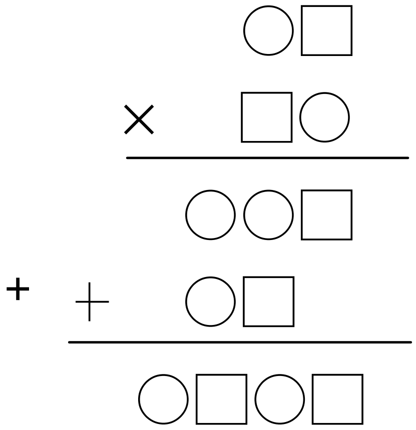
2 and 3 and all the digits between them add up to 19.

3 and 4 and all the digits between them add up to 45.

4 and 5 and all the digits between them add up to 18.

8. During a recent census, a man told the census taker that he had three children all having their birthdays today. When asked about their ages, he replied, “The product of their ages is 72. The sum of their ages is the same as my house number.” The census taker ran towards the door and looked at the house number. “I still can’t tell” the census taker complained. The man replied, “Oh, that’s right. I forgot to tell you that the oldest one likes ice cream.” The census taker promptly wrote down the ages of the three children. How old were they?

9. Digits of the multiplication operation below have been replaced by either a circle or a square. Circles hide odd digits, and squares hide even digits. Fill in the squares and the circles with the missing digits.



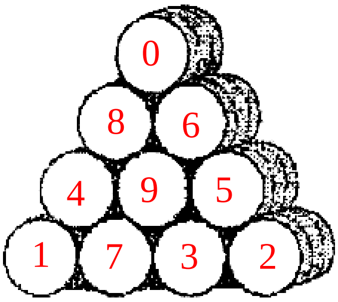
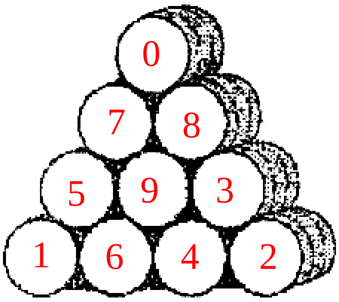
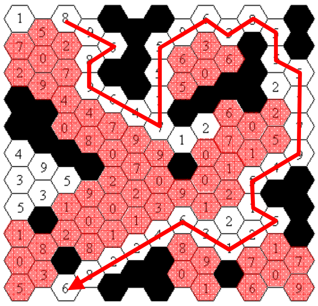
10. Donuts are sold only in boxes of 7, 13, or 25. To buy 14 donuts you must order two boxes of 7, but you cannot buy exactly 15 since no combination of boxes contains 15 donuts. What is the largest number of donuts that **cannot** be ordered using combinations of these boxes?

2005 EMIC Answers

Individual

1.	532	**2.**	1323	**3.**	5	**4.**	86	**5.**	40
6.	108	**7.**	20	**8.**	F	**9.**	544	**10.**	7
11.	1:5	**12.**	97	**13.**	4009	**14.**	$\frac{1}{4}$	**15.**	3

Team

1.	8	**2.**	12
3.	13	**4.**	2002
5.	 		
6.		**7.**	3,7,1,6,2,8,5,9,4, or 4,9,5,8,2,6,1,7,3
8.	3, 3 and 8	**9.**	$\begin{array}{rrrrr} & & & 1 & 6 \\ \times & & & 6 & 7 \\ \hline & & 1 & 1 & 2 \\ + & & 9 & 6 & \\ \hline & 1 & 0 & 7 & 2 \end{array}$
10.	44		

Indonesia Elementary Mathematics International Contest (INAEMIC) 2006

Individual Test Problems

Bali, May 26-31, 2006

Instructions:

- Write down your name on the answer sheet.
- Write your answer on the answer sheet.
- Answer all 15 questions.
- You have 90 minutes to work on this test.

1. When Anura was 8 years old his father was 31 years old. Now his father is twice as old as Anura is. How old is Anura now?

2. Nelly correctly measures three sides of a rectangle and gets a total of 88 cm. Her brother Raffy correctly measures three sides of the same rectangle and gets a total of 80 cm. What is the perimeter of the rectangle, in cm?

3. Which number should be removed from: $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ and 11 so that the average of the remaining numbers is 6.1?

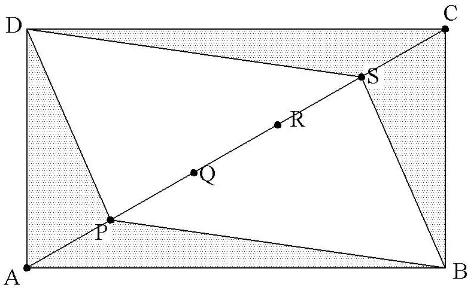
4. The houses in a street are located in such a way that each house is directly opposite another house. The houses are numbered $1, 2, 3, ...$ up one side, continuing down the other side of the street. If number 37 is opposite number 64, how many houses are there in the street altogether?

5. There are 6 basketball players and 14 cheerleaders. The total weight of the 6 basketball players is 540 kg. The average weight of the 14 cheerleaders is 40 kg. What is the average weight of all 20 people?

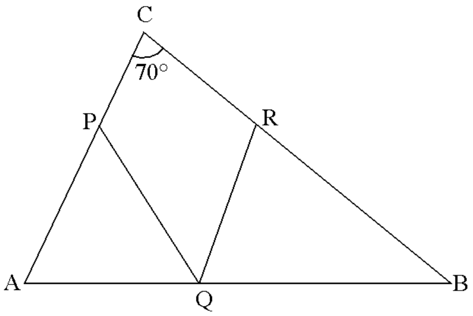
6. How many natural numbers less than 1000 are there, so that the sum of its first digit and last digit is 13?

7. Two bikers A and B were 370 km apart traveling towards each other at a constant speed. They started at the same time, meeting after 4 hours. If biker B started $\frac{1}{2}$ hour later than biker A, they would be 20 km apart 4 hours after A started. At what speed was biker A traveling?

8. In rectangle $ABCD$, $AB = 12$ and $AD = 5$. Points P, Q, R and S are all on diagonal AC, so that $AP = PQ = QR = RS = SC$. What is the total area of the shaded region?



9. In triangle ABC, $AP = AQ$ and $BQ = BR$. Determine angle PQR, in degrees.



10. In the equation below, N is a positive whole number.

$$N = \square + \square - \square$$

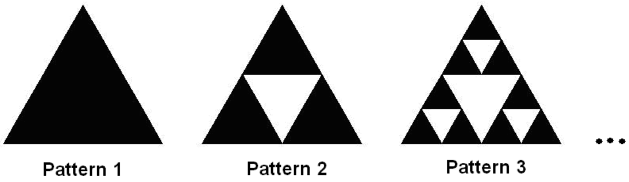
A numbered card is placed in each box. If three cards numbered $1, 2, 3$ are used, we get 2 different answers for N, that is 2 and 4. How many different answers for N can we get if four cards numbered $1, 2, 3,$ and 5 are used?

11. A mathematics exam consists of 20 problems. A student gets 5 points for a correct answer, a deduction of 1 point for an incorrect answer and no points for a blank answer. Jolie gets 31 points in the exam. What is the most number of problems she could have answered (including correct and incorrect answers)?

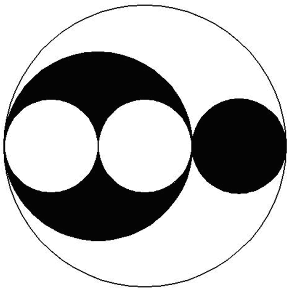
12. Joni and Dini work at the same factory. After every nine days of work, Joni gets one day off. After every six days of work, Dini gets one day off. Today is Joni's day off and tomorrow will be Dini's day off. At least how many days from today they will have the same day off?

13. In a bank, Bava, Juan and Suren hold a distinct position of director (D), manager (M) and teller (T). The teller, who is the only child in his family, earns the least. Suren, who is married to Bava's sister, earns more than the manager. What position does Juan hold? Give your answer in terms of D, M or T.

14. The following figures show a sequence of equilateral triangles of 1 square unit. The unshaded triangle in Pattern 2 has its vertices at the midpoint of each side of the larger triangle. If the pattern is continued as indicated by Pattern 3, what is the total area of the shaded triangles in Pattern 5, in square units?



15. There are five circles with 3 different diameters. Some of the circles touch each other as shown in the figure below. If the total area of the unshaded parts is 20 cm^2, find the total area of the shaded parts, in cm^2.



Indonesia Elementary Mathematics International Contest (INAEMIC) 2006

Team Test Problems

Bali, May 26-31, 2006

Instructions:

- * Ten minute discussion in the beginning to distribute problems to team members.
- * No more discussion or exchange of problems allowed after the ten-minute discussion.
- * Each student must solve at least one problem.
- * Write down your team name on the sheet.
- * Write only your answer in the box on the sheet. No explanation is needed.
- * After 10-minute discussion, you have 50 minutes to work on this test.

Name : ....
Team : ....
Country : ....

---

1. Four different natural numbers, all larger than 3, are placed in the four boxes below.

$$\square + \square + \square + \square = 27$$

The four numbers are arranged from the smallest to the largest. How many different ways can we fill the four boxes?

Answer : ☐

Name : ....

Team : ....

Country : ....

---

2. The number 22 has the following property: the sum of its digits is equal to the product of its digits. Find the smallest 8-digit natural number that satisfies the given condition.

Answer : ☐

Name : ....
Team : ....
Country : ....

3. A number X consists of 4 non-zero digits. A number Y is obtained from X reversing the order of its digits. If the sum of X and Y is 14773 and the difference between them is 3177, determine the larger of these two numbers.

Answer :

Name : ....

Team : ....

Country : ....

4. $ABCD$ is a parallelogram. $P, Q, R,$ and S are points on the sides AB, BC, CD and DA respectively so that $AP = DR$. The area of parallelogram $ABCD$ is 16 cm^2. Find the area of the quadrilateral $PQRS$.

Answer : ☐ **cm^2**

Name : ....

Team : ....

Country : ....

---

5. Adi has written a number of mathematical exams. In order to obtain an overall average of 90 points/percentage, he needed to score 100 points/percentage in the final exam. Unfortunately, he achieved only 75 points/percentage in the final exam, resulting in an overall average of 85 points/percentage. How many exams did he write altogether?

Answer : ☐

Name : ....

Team : ....

Country : ....

6. Annisa used 120 unit cubes to make a parallelepiped (rectangular prism). She painted all six faces of the parallelepiped. Once the paint had dried, she disassembled the cubes and found that 24 of the cubes had not been painted on any face. What is the surface area of the parallelepiped?

Answer : ☐ **square unit**

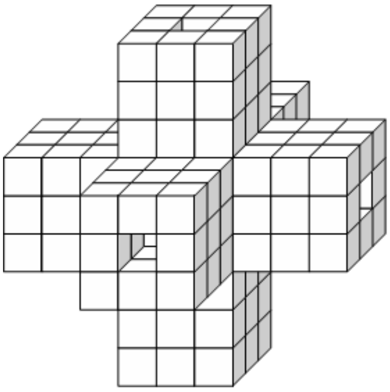
Name : ....

Team : ....

Country : ....

---

7. A number of unit cubes are arranged to build a tower-like shape as shown in the figure below. Note that there is a hole across from the left to the right, from the top to the bottom, and from the front to the back. How many unit cubes are there altogether?



Answer :

Name : ....

Team : ....

Country : ....

8. When 31513 and 34369 are divided by the same three-digit number, the remainders are equal.
What is the remainder?

Answer :

Name : ....

Team : ....

Country : ....

9. Place any four digits from 1 to 5 in a 2×2 square so that:

   (a) in the same row, the digit on the left is greater than that on the right, and

   (b) in the same column, the digit in the top is greater than that at the bottom.

   The diagrams below show two different ways of arranging the digits. How many different ways are there in total?

5	**3**
4	**2**

EXAMPLE 1

5	**3**
2	**1**

EXAMPLE 2

Answer :

Name : ....

Team : ....

Country : ....

10. Peter uses a remote control to move his robot. The remote control has 3 buttons on it. One button moves the robot 1 step forward, another button moves it 2 steps forward and the third button moves it 3 steps forward. How many different ways are possible to move the robot 8 steps forward?

Answer :

2006 EMIC Answers

Individual

1.	23	**2.**	112	**3.**	5	**4.**	100	**5.**	55
6.	66	**7.**	52.5	**8.**	24	**9.**	55	**10.**	5
11.	17	**12.**	50	**13.**	T	**14.**	$\frac{81}{256}$	**15.**	10

Team

1.	6	**2.**	11111128
3.	8975	**4.**	8
5.	5	**6.**	148
7.	164	**8.**	97
9.	10	**10.**	81

International Youth Mathematics Contest 2007

Hongkong Elementary Mathematics International Contest (HEMIC)

Individual Competition

Time allowed : 1 hour 30 minutes

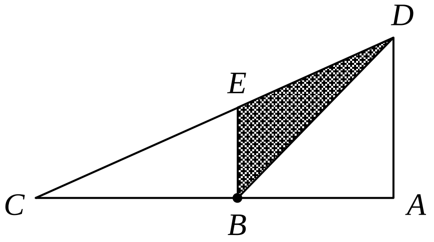
Hong Kong : 29 July – 2 August 2007

Instructions:

- Write down your name, team name and candidate number on the answer sheet.
- Write down all answers on the answer sheet.
- Answer all 15 problems. Problems are in ascending order of level of difficulty. Only NUMERICAL answers are needed.
- Each problem is worth 6 points and the total is 90 points.
- For problems involving more than one answer, points are given only when ALL answers are correct.
- Take π = 3.14 if necessary.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All materials will be collected at the end of the competition.

1. The product of two three-digit numbers $\overline{abc}$ and $\overline{cba}$ is 396396, where a > c. Find the value of $\overline{abc}$.

2. In a right-angled triangle *ACD*, the area of shaded region is 10 cm^2, as shown in the figure below. *AD* = 5 cm, *AB = BC, DE = EC.* Find the length of *AB*.



3. A wooden rectangular block, 4 cm × 5 cm × 6 cm, is painted red and then cut into several 1 cm × 1 cm × 1 cm cubes. What is the ratio of the number of cubes with two red faces to the number of cubes with three red faces?

4. Eve said to her mother, "If I reverse the two-digits of my age, I will get your age." Her mother said, "Tomorrow is my birthday, and my age will then be twice your age." It is known that their birthdays are not on the same day. How old is Eve?

5. Find how many three-digit numbers satisfy all the following conditions:
if it is divided by 2, the remainder is 1,
if it is divided by 3, the remainder is 2,
if it is divided by 4, the remainder is 3,
if it is divided by 5, the remainder is 4,
if it is divided by 8, the remainder is 7.

6. A giraffe lives in an area shaped in the form of a right-angled triangle. The base and the height of the triangle are 12 m and 16 m respectively. The area is surrounded by a fence. The giraffe can eat the grass outside the fence at a maximum distance of 2 m. What is the maximum area outside the fence, in which the grass can be eaten by the giraffe?

7. Mary and Peter are running around a circular track of 400 m. Mary's speed equals $\frac{3}{5}$ of Peter's. They start running at the same point and the same time, but in opposite directions. 200 seconds later, they have met four times. How many metres per second does Peter run faster than Mary?

8. Evaluate $2^{2007}-\left(2^{2006}+2^{2005}+2^{2004}+......+2^3+2^2+2+1\right)$

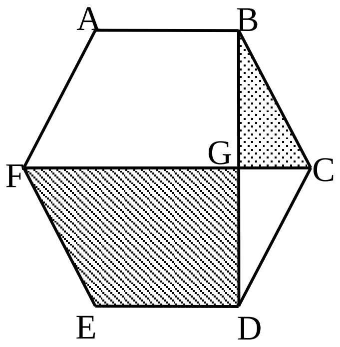
9. A, B and C are stamp-collectors. A has 18 stamps more than B. The ratio of the number of stamps of B to that of C is 7:5. The ratio of the sum of B's and C's stamps to that of A's is 6:5. How many stamps does C have?

10. What is the smallest amount of numbers in the product $1\times2\times3\times4\times...\times26\times27$ that should be removed so that the product of the remaining numbers is a perfect square?

11. Train A and Train B travel towards each other from Town A and Town B respectively, at a constant speed. The two towns are 1320 kilometers apart. After the two trains meet, Train A takes 5 hours to reach Town B while Train B takes 7.2 hours to reach Town A. How many kilometers does Train A run per hour?

12. Balls of the same size and weight are placed in a container. There are 8 different colors and 90 balls in each color. What is the minimum number of balls that must be drawn from the container in order to get balls of 4 different colors with at least 9 balls for each color?

13. In a regular hexagon *ABCDEF*, two diagonals, *FC* and *BD*, intersect at *G*. What is the ratio of the area of ΔBCG to that of quadrilateral *FEDG?*



14. There are three prime numbers. If the sum of their squares is 5070, what is the product of these three numbers?

15. Let *ABCDEF* be a regular hexagon. *O* is the centre of the hexagon. *M* and *N* are the mid-points of *DE* and *OB* respectively. If the sum of areas of ΔFNO and ΔFME is 3 cm^2, find the area of the hexagon.

~ End of Paper ~

International Youth Mathematics Contest 2007

Hongkong Elementary Mathematics International Contest (HEMIC)

Team Competition

Time allowed : 1 hour 30 minutes

Hong Kong : 29 July – 2 August 2007

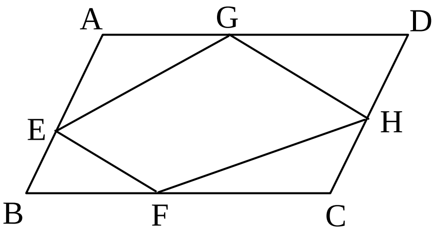
Instructions:

- Write down the team name and the name and candidate number of each team member on the answer sheet.
- Discussion among the team members is allowed.
- Write down all answers on the answer sheet.
- Answer all 10 problems. Problems are in ascending order of level of difficulty. Only NUMERICAL answers are needed.
- Each problem is worth 20 points and the total is 200 points.
- For problems involving more than one answer, points are given only when ALL answers are correct.
- Take π = 3.14 if necessary.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All materials will be collected at the end of the competition.

1. Town A and Town B are connected by a highway which consists of an uphill and a downhill section. A car’s speed is 20 km/hr and 35 km/hr for the uphill and downhill sections respectively. It takes 9 hours from A to B but $7\frac{1}{2}$ hours from B to A. What is the downhill distance (in km) from A to B?

2. The houses on one side of a street are numbered using consecutive odd numbers, starting from 1. On the other side, the houses are numbered using consecutive even numbers starting from 2. In total 256 digits are used on the side with even numbers and 404 digits on the side with odd numbers. Find the difference between the largest odd number and the largest even number.

3. As shown in the figure below, *ABCD* is a parallelogram with area of 10. If *AB*=3, *BC*=5, *AE*=*BF*=*AG*=2, *GH* is parallel to *EF*, find the area of *EFHG*.



4. Find the two smallest integers which satisfy the following conditions:
   (1) The difference between the integers is 3.
   (2) In each number, the sum of the digits is a multiple of 11.

5. A four-digit number can be formed by linking two different two-digit prime numbers together. For example, 13 & 17 can be linked together to form a four-digit number 1317 or 1713. Some four-digit numbers formed in this way can be divided by the average of the two prime numbers. Give one possible four-digit number that fulfills the requirement. (Please be reminded that 1317 and 1713 in the example above do not fulfill the requirement, because they are not divisible by 15.)

6. How many prime factors does the number $2 + 2^2 + 2^3 + \ldots + 2^{15} + 2^{16}$ have?

7. A pencil, an easer, and a notebook together cost 100 dollars. A notebook costs more than two pencils, three pencils cost more than four erasers, and three easers cost more than a notebook. How much does each item cost (assuming that the cost of each item is a whole number of dollars)?

8. There are 8 pairs of natural numbers which satisfy the following condition:
The product of the sum of the numbers and the difference of the numbers is 1995.
Which pair of numbers has the greatest difference?

9. A land with a dimension 52 m × 24 m is surrounded by fence. An agricultural scientist wants to divide the land into identical square sections for testing, using fence with total length 1172 m. The sides of the square sections must be parallel to the sides of the land. What is the maximum number of square testing sections that can be formed?

10. Find the total number of ways that 270 can be written as a sum of consecutive positive integers.

~ End of Paper ~

2007 EMIC Answers

Individual

1.	924	**2.**	8	**3.**	3:1	**4.**	37	**5.**	8
6.	108.56	**7.**	2	**8.**	1	**9.**	30	**10.**	5
11.	120	**12.**	311	**13.**	1:5	**14.**	710	**15.**	18

Team

1.	70	**2.**	97
3.	5	**4.**	89999、90002
5.	1353、5313、1947、4719、2343、4323、3937、3729	**6.**	5
7.	Pencil : $55 Easer : $26 Notebook : $19	**8.**	998、–997
9.	312	**10.**	7

International Mathematics Competition 2008
(IMC 2008)

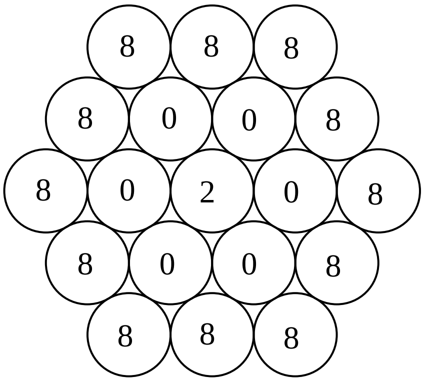
Elementary Mathematics International Contest

Individual Contest   Time limit: 90 minutes   2008/10/28

Instructions:

- Write down your name, team name and candidate number on the answer sheet.
- Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.
- Answer all 15 problems. Each problem is worth 1 point and the total is 15 points.
- For problems involving more than one answer, points are given only when ALL answers are corrected.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All materials will be collected at the end of the competition.

1. Starting from the central circle, move between two tangent circles. What is the number of ways of covering four circles with the numbers 2, 0, 0 and 8 inside, in that order?



2. Each duck weighs the same, and each duckling weighs the same. If the total weight of 3 ducks and 2 ducklings is 32 kilograms, the total weight of 4 ducks and 3 ducklings is 44 kilograms, what is the total weight, in kilograms, of 2 ducks and 1 duckling?

3. If 25% of the people who were sitting stand up, and 25% of the people who were standing sit down, then 70% of the people are standing. How many percent of the people were standing initially?

4. A sedan of length 3 metres is chasing a truck of length 17 metres. The sedan is travelling at a constant speed of 110 kilometres per hour, while the truck is travelling at a constant speed of 100 kilometres per hour. From the moment when the front of the sedan is level with the back of the truck to the moment when the front of the truck is level with the back of the sedan, how many seconds would it take?

5. Consider all six-digit numbers consisting of each of the digits '0', '1', '2', '3', '4' and '5' exactly once in some order. If they are arranged in ascending order, what is the 502nd number?

6. How many seven-digit numbers are there in which every digit is '2' or '3', and no two '3's are adjacent?

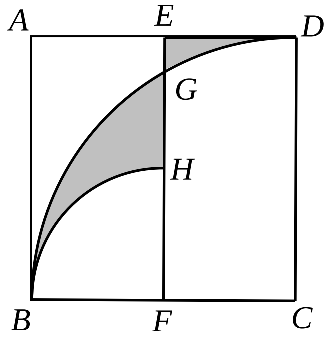
7. The six-digit number $\overline{abcabc}$ has exactly 16 positive divisors. What is the smallest value of such numbers?

8. How many five-digit multiples of 3 have at least one of its digits equal to '3'?

9. *ABCD* is a parallelogram. *M* is a point on *AD* such that $AM=2MD$, *N* is a point on *AB* such that $AN=2NB$. The segments *BM* and *DN* intersect at *O*. If the area of *ABCD* is 60 cm^2, what is the total area of triangles *BON* and *DOM*?

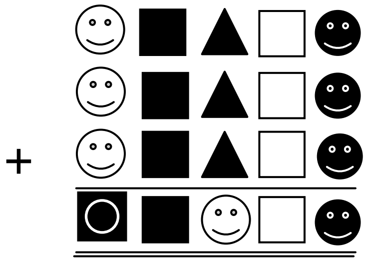
10. The four-digit number $\overline{ACCC}$ is $\frac{2}{5}$ of the four-digit number $\overline{CCCB}$. What is the value of the product of the digits *A*, *B* and *C*?

11. *ABCD* is a square of side length 4 cm. *E* is the midpoint of *AD* and *F* is the midpoint of *BC*. An arc with centre *C* and radius 4 cm cuts *EF* at *G*, and an arc with centre *F* and radius 2 cm cuts *EF* at *H*. The difference between the areas of the region bounded by *GH* and the arcs *BG* and *BH* and the region bounded by *EG*, *DE* and the arc *DG* is of the form $m\pi - n$ cm^2, where *m* and *n* are integers. What is the value of $m+n$?



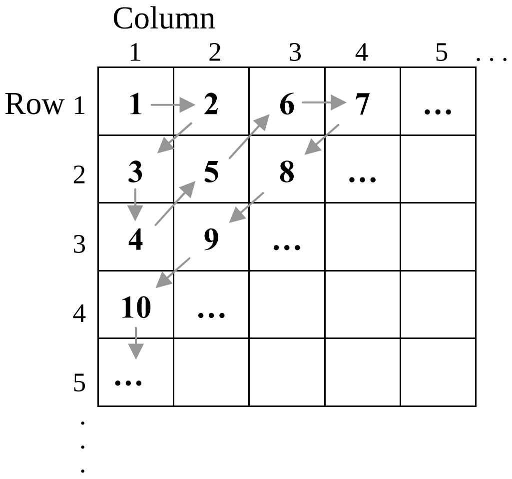
12. In a chess tournament, the number of boy participants is double the number of girl participants. Every two participants play exactly one game against each other. At the end of the tournament, no games were drawn. The ratio between the number of wins by the girls and the number of wins by the boys is 7:5. How many boys were there in the tournament?

13. In the puzzle every different symbol stands for a different digit.



What is the answer of this expression which is a five-digit number?

14. In the figure below, the positive numbers are arranged in the grid follow by the arrows' direction.



For example,

"8"is placed in Row 2, Column 3.
"9" is placed in Row 3, Column 2.

Which Row and which Column that "2008" is placed?

15. As I arrived at home in the afternoon. The 24-hour digital clock shows the time as below (HH:MM:SS). I noticed instantly that the first three digits on the platform clock were the same as the last three, and in the same order. How many times in twenty four hours does this happen?

13:21:32

Note: The clock shows time from 00:00:00 to 23:59:59.

2008 Thailand Elementary Mathematics International Contest (TEMIC)

Team Contest

2008/10/28 Chiang Mai, Thailand

Team: ______________________ ***Score:*** ________________

1. *N* is a five-digit positive integer. *P* is a six–digit integer constructed by placing a digit '1' at the right-hand end of *N*. *Q* is a six–digit integer constructed by placing a digit '1' at the left–hand end of *N*. If $P = 3Q$, find the five-digit number *N*.

ANSWER：________________

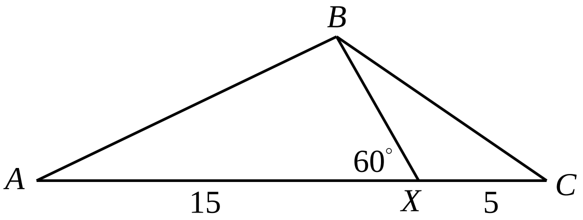
2008 Thailand Elementary Mathematics International Contest (TEMIC)

Team Contest

2008/10/28 Chiang Mai, Thailand

Team: ____________________________ ***Score:*** ____________________

2. In a triangle ABC, X is a point on AC such that AX=15 cm, XC=5 cm, $\angle AXB$=60° and $\angle ABC = 2\angle AXB$. Find the length of BC, in cm.



ANSWER : ____________________ **cm.**

2008 Thailand Elementary Mathematics International Contest (TEMIC)

Team Contest

2008/10/28 Chiang Mai, Thailand

Team: __________________________ ***Score:*** __________________

3. A track *AB* is of length 950 metres. Todd and Steven run for 90 minutes on this track, starting from *A* at the same time. Todd's speed is 40 metres per minute while Steven's speed is 150 metres per minute. They meet a number of times, running towards each other from opposite directions. At which meeting are they closest to *B*?

ANSWER： ____________________

2008 Thailand Elementary Mathematics International Contest (TEMIC)

Team Contest

2008/10/28 Chiang Mai, Thailand

Team: ______________________ ***Score:*** ________________

4. The numbers in group A are $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{20}$, $\frac{1}{30}$ and $\frac{1}{42}$. The numbers in group B are $\frac{1}{8}$, $\frac{1}{24}$, $\frac{1}{48}$ and $\frac{1}{80}$. The numbers in group C are 2.82, 2.76, 2.18 and 2.24. One number from each group is chosen and their product is computed.

What is the sum of all 80 products?

ANSWER：____________________

2008 Thailand Elementary Mathematics International Contest (TEMIC)

Team Contest

2008/10/28 Chiang Mai, Thailand

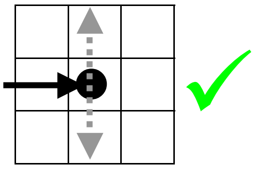
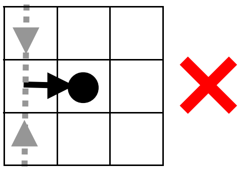
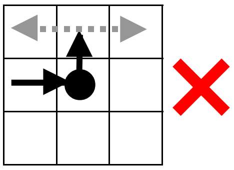
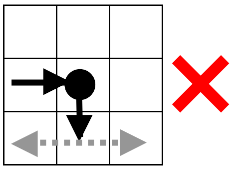
Team: ______________________ ***Score:*** ________________

5. On the following 8×8 board, draw a single path going between squares with common sides so that

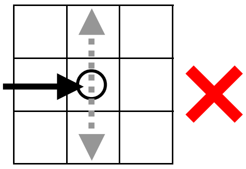
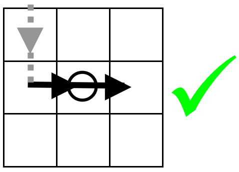
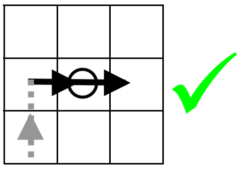
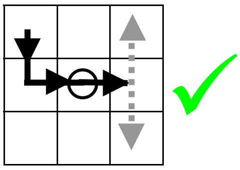
(a) it is closed and not self-intersecting;

(b) it passes through every square with a circle, though not necessarily every square;

(c) it turns (left or right) at every square with a black circle, but does not do so on either the square before or the one after;

(d) it does not turn (left or right) at any square with a white circle, but must do so on either the square before or the one after, or both.

						○	○
	●		●	●			
●					●		○
	○		○				
	○						
	○				○	○	
						○	

						○	○
	●		●	●			
●					●		○
	○		○				
	○						
	○				○	○	
						○	

ANSWER： ______________________

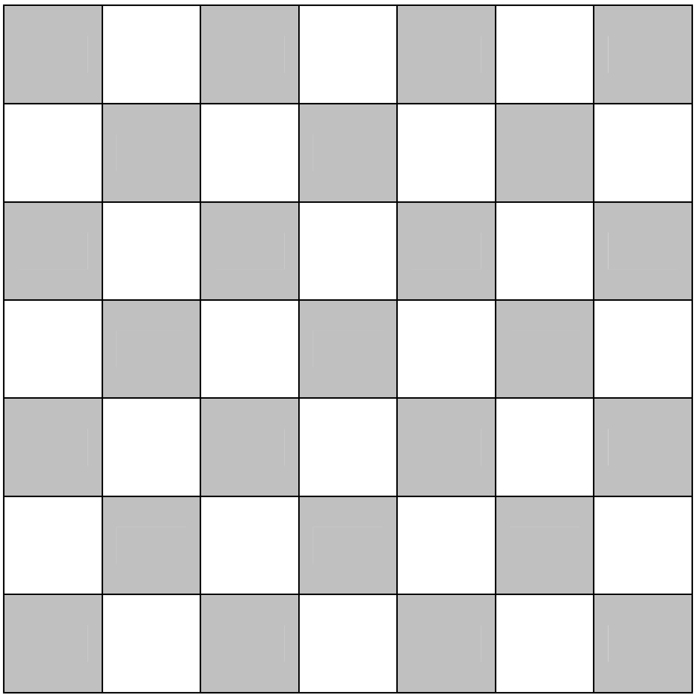
2008 Thailand Elementary Mathematics International Contest (TEMIC)

Team Contest

2008/10/28 Chiang Mai, Thailand

Team: ______________________ ***Score:*** ________________

6. The diagram below shows a 7×7 checkerboard with black squares at the corners. How many ways can we place 6 checkers on squares of the same colour, so that no two checkers are in the same row or the same column?



ANSWER：________________

2008 Thailand Elementary Mathematics International Contest (TEMIC)

Team Contest

2008/10/28 Chiang Mai, Thailand

Team: ________________________ ***Score:*** ________________

7. How many different positive integers not exceeding 2008 can be chosen at most such that the sum of any two of them is not divisible by their difference?

ANSWER： ____________________

2008 Thailand Elementary Mathematics International Contest (TEMIC)

Team Contest

2008/10/28 Chiang Mai, Thailand

Team: __________________________ ***Score:*** ___________________

8. A 7×7×7 cube is cut into any 4×4×4, 3×3×3, 2×2×2, or 1×1×1 cubes. What is the minimum number of cubes which must be cut out?

ANSWER：_____________________

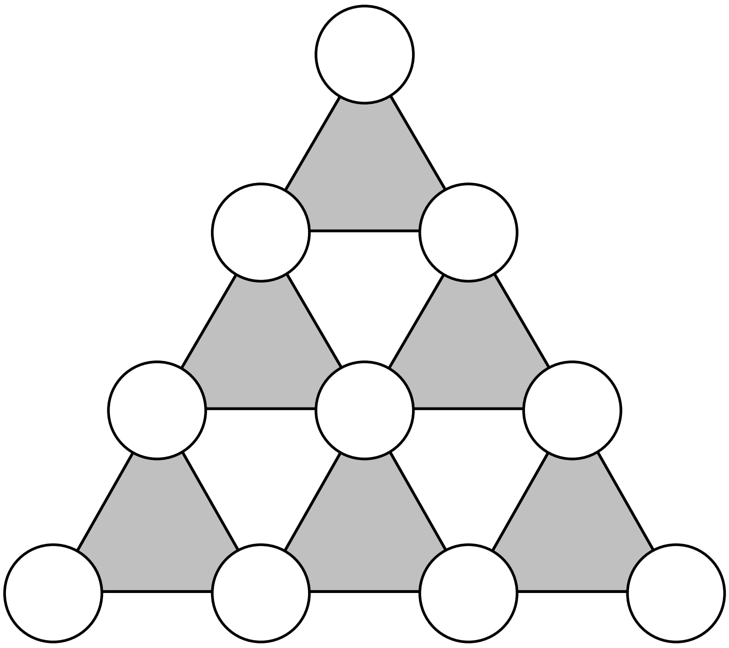
2008 Thailand Elementary Mathematics International Contest (TEMIC)

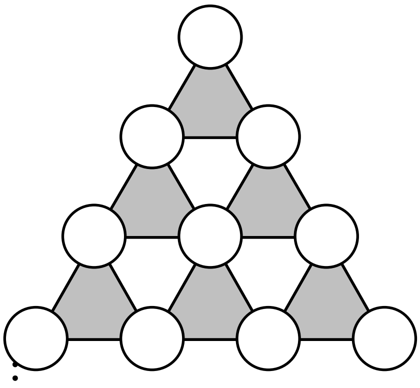
Team Contest

2008/10/28 Chiang Mai, Thailand

Team: ______________________ ***Score:*** ________________

9. Place the numbers 0 through 9 in the circles in the diagram below without repetitions, so that for each of the six small triangles which are pointing up (shaded triangles), the sum of the numbers in its vertices is the same.





ANSWER : ________________

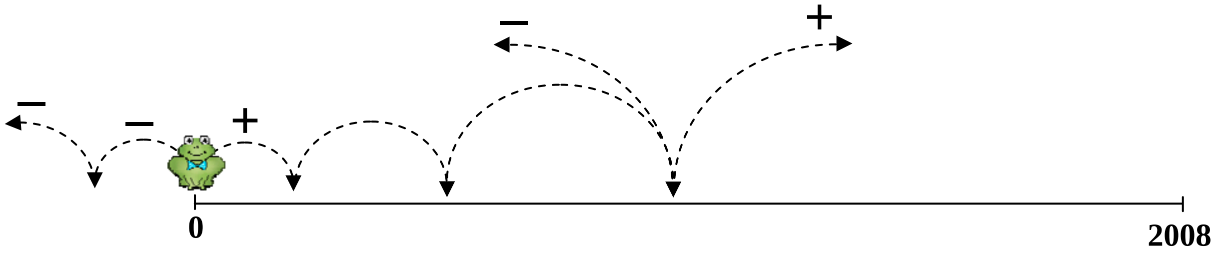
2008 Thailand Elementary Mathematics International Contest (TEMIC)

Team Contest

2008/10/28 Chiang Mai, Thailand

Team: ______________________ ***Score:*** ________________

10. A frog is positioned at the origin (which label as 0) of a straight line. He can move in either positive(+) or negative(–) direction. Starting from 0, the frog must get to 2008 in exactly 19 jumps. The lengths of his jump are 1^2, 2^2, …, 19^2 respectively (i.e. 1st jump $=1^2$, 2nd jump $=2^2$, . . ., and so on). At which jump is the smallest last negative jump?



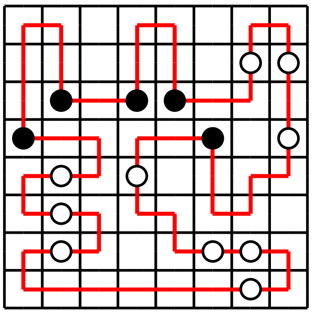
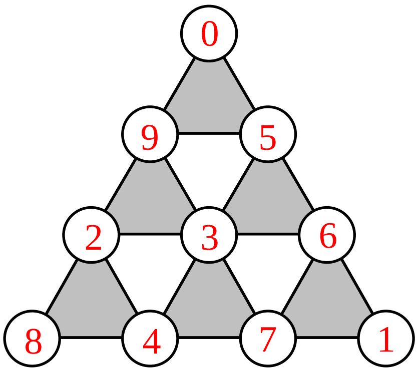
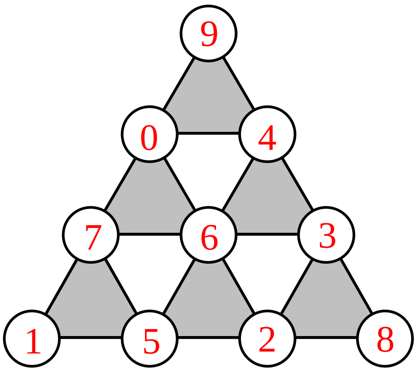
ANSWER： ________________

2008 EMIC Answers

Individual

1.	36	**2.**	20	**3.**	90	**4.**	7.2	**5.**	504231
6.	34	**7.**	101101	**8.**	12504	**9.**	8	**10.**	60
11.	11	**12.**	6	**13.**	89250	**14.**	Row 9, Column 55	**15.**	96

Team

1.	42857	**2.**	10
3.	7	**4.**	$\frac{5}{7}$
5.		**6.**	1584
7.	670	**8.**	71
9.			
10.	9		

2009
Philippine
Elementary
Mathematics
International
Contest

2009
Philippine
Elementary
Mathematics
International
Contest

Individual Contest

Time limit: 90 minutes 2009/11/30

Instructions:

- Do not turn to the first page until you are told to do so.
- Write down your name, your contestant number and your team's name on the answer sheet.
- Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.
- Answer all 15 problems. Each problem is worth 10 points and the total is 150 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given.
- Diagrams shown may not be drawn to scale.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.

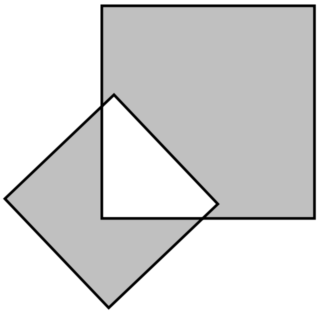
English Version

1. Find the smallest positive integer whose product after multiplication by 543 ends in 2009.

2. Linda was delighted on her tenth birthday, 13 July 1991 (13/7/91), when she realized that the product of the day of the month together with the month in the year was equal to the year in the century: 13×7 = 91. She started thinking about other occasions in the century when such an event might occur, and imagine her surprise when she realized that the numbers in her two younger brothers' tenth birthdays would also have a similar relationship. Given that the birthdays of the two boys are on consecutive days, when was Linda's youngest brother born?

3. Philip arranged the number 1, 2, 3, ... , 11, 12 into six pairs so that the sum of the numbers in any pair is prime and no two of these primes are equal. Find the largest of these primes.

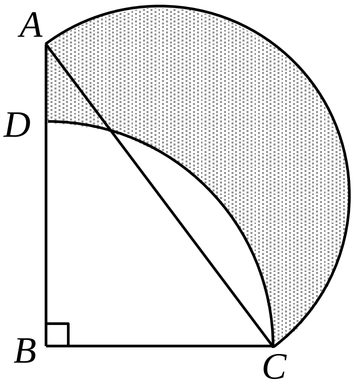
4. In the figure, $\frac{3}{4}$ of the larger square is shaded and $\frac{5}{7}$ of the smaller square is shaded. What is the ratio of the shaded area of the larger square to the shaded area of the smaller square?



5. Observe the sequence 1, 1, 2, 3, 5, 8, 13, … . Starting from the third number, each number is the sum of the two previous numbers. What is the remainder when the 2009th number in this sequence is divided by 8?

6. Ampang Street has no more than 15 houses, numbered 1, 2, 3 and so on. Mrs. Lau lives in one of the houses, but not in the first house. The product of all the house numbers before Mrs. Lau's house, is the same as that of the house numbers after her house. How many houses are on Ampang Street?

7. In the given figure, *ABC* is a right-angled triangle, where $\angle B = 90°$, $BC = 42$ cm and $AB = 56$ cm. A semicircle with *AC* as a diameter and a quarter-circle with *BC* as radius are drawn. Find the area of the shaded portion, in cm^2. (Use $\pi = \frac{22}{7}$)

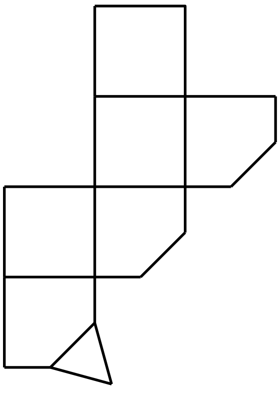


8. A number consists of three different digits. If the difference between the largest and the smallest numbers obtained by rearranging these three digits is equal to the original number, what is the original three-digit number?

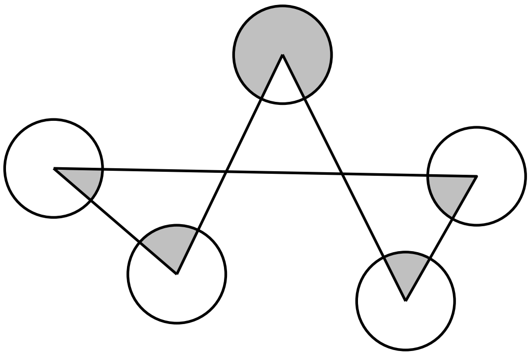
9. The last 3 digits of some perfect squares are the same and non-zero. What is the smallest possible value of such a perfect square?

10. Lynn is walking from town *A* to town *B*, and Mike is riding a bike from town *B* to town *A* along the same road. They started out at the same time and met 1 hour after. When Mike reaches town *A*, he turns around immediately. Forty minutes after they first met, he catches up with Lynn, still on her way to town *B*. When Mike reaches town *B*, he turns around immediately. Find the ratio of the distances between their third meeting point and the towns *A* and *B*.

11. The figure shows the net of a polyhedron. How many edges does this polyhedron have?



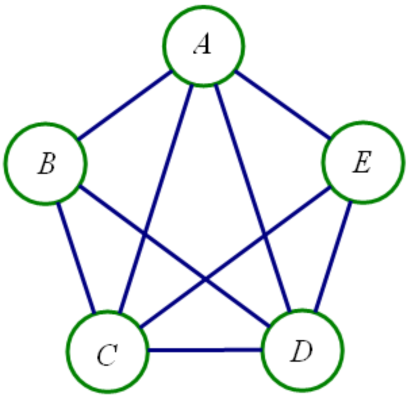
12. In the figure, the centers of the five circles, of same radius 1 cm, are the vertices of the triangles. What is the total area, in cm^2, of the shaded regions? (Use $\pi = \frac{22}{7}$)



13. There are 10 steps from the ground level to the top of a platform. The 6th step is under repair and can only be crossed over but not stepped on. Michael walks up the steps with one or two steps only at a time. How many different ways can he use to walk up to the top of the platform?

14. For four different positive integers a, b, c and d, where $a < b < c < d$, if the product $(d - c) \times (c - b) \times (b - a)$ is divisible by 2009, then we call this group of four integers a “friendly group”. How many “friendly groups” are there from 1 to 60?

15. The figure shows five circles A, B, C, D and E. They are to be painted, each in one color. Two circles joined by a line segment must have different colors. If five colors are available, how many different ways of painting are there?



2009
Philippine
Elementary
Mathematics
International
Contest

2009
Philippine
Elementary
Mathematics
International
Contest

TEAM CONTEST

Time：60 minutes 2009/11/30

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on every page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each problem is worth 40 points and the total is 400 points. Each question is printed on a separate sheet of paper. Complete solutions of problems 1, 2, 3, 5, 6, 7, 8 and 9 are required. Partial credits may be given depending on the solutions written down. Only final answers are required for Problem number 4 and 10.
- The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each team member must solve at least one problem. Each will then have 35 minutes to write the solutions of the assigned problem/s independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator or calculating device or electronic devices are allowed.
- Answer in pencil or in blue or black ball point pen.
- All papers will be collected at the end of the competition.

English Version

2009
Philippine
Elementary
Mathematics
International
Contest

2009
Philippine
Elementary
Mathematics
International
Contest

TEAM CONTEST

Team：______________________________ Score：________________

1. Below is a 3×60 table. Each row is filled with digits following its own particular sequence. For each column, a sum is obtained by adding the three digits in each column. How many times is the most frequent sum obtained?

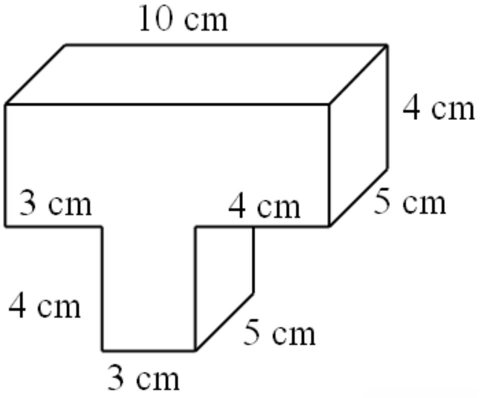
Row A	1	2	3	4	5	1	2	3	4	5	…	4	5
Row B	1	2	3	4	1	2	3	4	1	2	…	3	4
Row C	1	2	1	2	1	2	1	2	1	2	…	1	2

Answer：________________ times

TEAM CONTEST

Team： ______________________________ Score： ________________

2. All surfaces of the T-shape block below is painted red. It is then cut into $1cm\times1cm\times1cm$ cubes. Find the number of $1cm\times1cm\times1cm$ cubes with all six faces unpainted.



Answer： ____________ cubes

TEAM CONTEST

Team：____________________________ Score：_______________

3. Kiran and his younger brother Babu are walking on a beach with Babu walking in front. Each of Kiran's step measures 0.8 meter while each of Babu's step measures 0.6 meter. If both of them begin their walk along a straight line from the same starting point (where the first footprint is marked) and cover a 100 meter stretch, how many foot-prints are left along the path? (If a footprint is imprinted on the 100 meter point, it should be counted. Consider two foot-prints as recognizable and distinct if one does not overlap exactly on top of the other.)

Answer：__________________ foot-prints

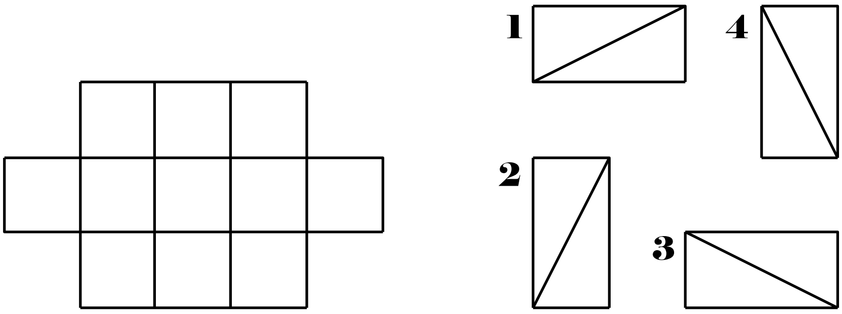
2009
Philippine
Elementary
Mathematics
International
Contest

2009
Philippine
Elementary
Mathematics
International
Contest

TEAM CONTEST

Team：________________________ Score：______________

4. Four 2 ×1 cards, shown on the right in the following figure, are to be placed on the board shown on the left below, without overlapping and such that the marked diagonals of any two cards do not meet at a corner. The cards may not be rotated nor flipped over. Find all the ways of arranging these cards that satisfy the given conditions.



Answer：

TEAM CONTEST

Team：

Score：

5. Water is leaking out continuously from a large reservoir at a constant rate. To facilitate repair, the workers have to first drain-off the water in the reservoir with the help of water pumps. If 20 pumps are used, it takes 5 full hours to completely drain-off the water from the reservoir. If only 15 pumps are used, it will take an hour longer. If the workers are given 10 hours to complete the job of draining-off the water, what is the minimum number of water pumps required for the job?

Answer： water pumps

TEAM CONTEST

Team： ______________________________ Score： ________________

6. As shown in the following figure, we arranged the positive integers into a triangular shape so that the numbers above or on the left must be less than the numbers below or on the right and each line has one more number than those above. Let us suppose a_{ij} stands for the number which is in the i-th line from the top and j is the count from the left in the triangular figure(e.g. a_{43}=9). If a_{ij} is 2009, what is the value of i+j?

1
2 3
4 5 6
7 8 9 10
…

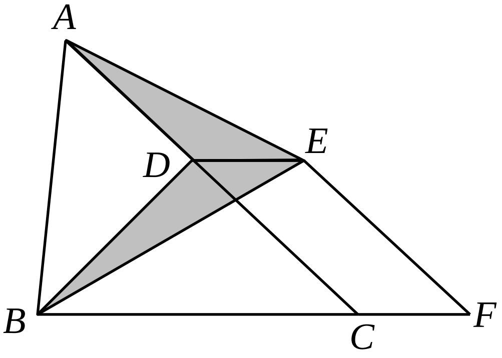
Answer： ______________________

2009
Philippine
Elementary
Mathematics
International
Contest

TEAM CONTEST

Team：______________________ Score：____________

7. In the figure below, the area of triangle *ABC* is 12 cm^2. *DCFE* is a parallelogram with vertex *D* on the line segment *AC* and *F* is on the extension of line segment *BC*. If $BC = 3CF$, find the area of the shaded region, in cm^2.

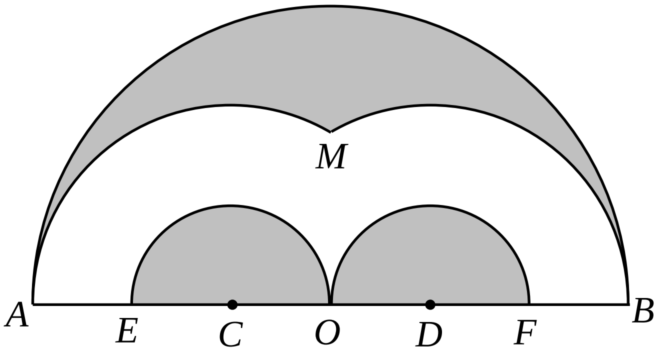


Answer：____________ cm^2

TEAM CONTEST

Team：____________________ Score：__________

8. In the figure, the diameter *AB* of semi-circle *O* is 12 cm long. Points *C* and *D* trisect line segment *AB*. An arc centered at *C* and with *CA* as radius meets another arc centered at *D* and with *DB* as radius at point *M*. Take the distance from point *M* to *AB* as 3.464 cm. Using *C* as center and *CO* as radius, a semi-circle is constructed to meet *AB* at point *E*. Using *D* as center and *DO* as radius, another semi-circle is constructed to meet *AB* at point *F*. Find the area of the shaded region. (Use $\pi = 3.14$ and give your answer correct to 3 decimal places.)

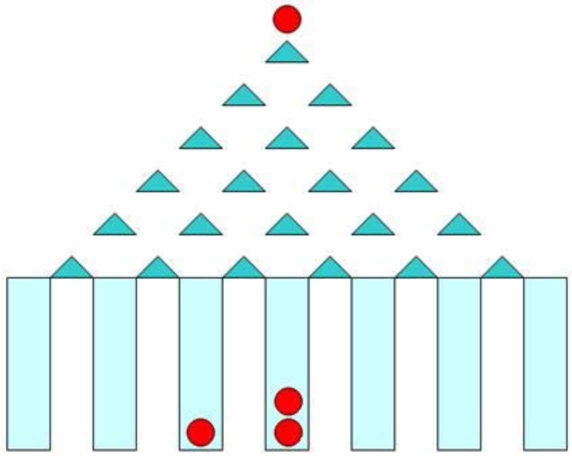


Answer：__________ cm^2

TEAM CONTEST

Team： ______________________ Score： ____________

9. The following figure shows a famous model, designed by Galton, a British biostatistician, to test the stability of frequency. Some wooden blocks with cross-sections in the shape of isosceles triangles are affixed to a wooden board. There are 7 bottles below the board and a small ball on top of the highest block. As the small ball falls down, it hits the top vertices of some wooden blocks below and rolls down the left or right side of a block with the same chance, until it falls into a bottle. How many different paths are there for the small ball to fall from the top of the highest block to a bottle?



Answer： ____________ paths

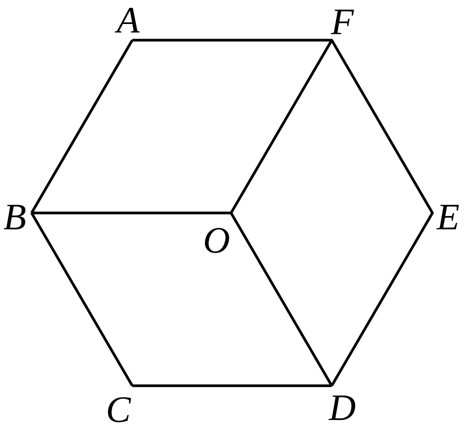
2009
Philippine
Elementary
Mathematics
International
Contest

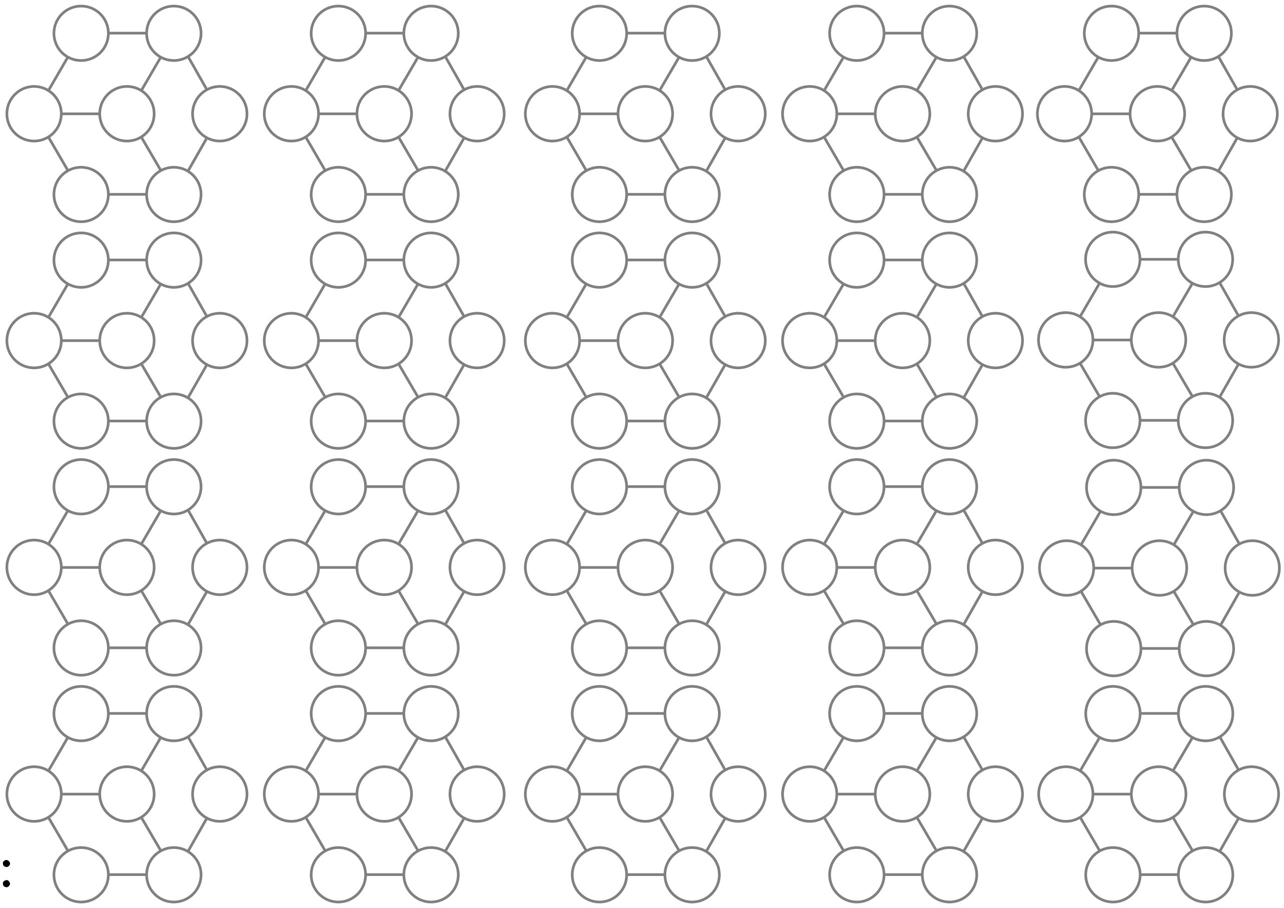
2009
Philippine
Elementary
Mathematics
International
Contest

TEAM CONTEST

Team：____________________________ Score：_______________

10. In the following figure, assign each of the numbers 1, 2, 3, 4, 5, 6, 7 to one of the six vertices of the regular hexagon *ABCDEF* and its center *O* so that sums of the numbers at the vertices of the rhombuses *ABOF*, *BCDO* and *DEFO* are equal. If solutions obtained by flipping or rotating the hexagon are regarded as identical, how many different solutions are there?





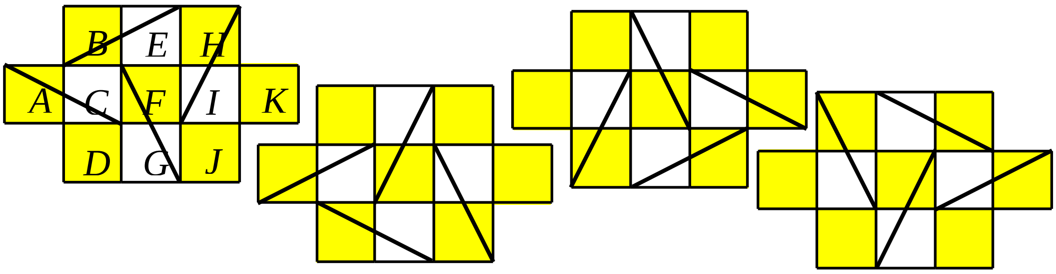
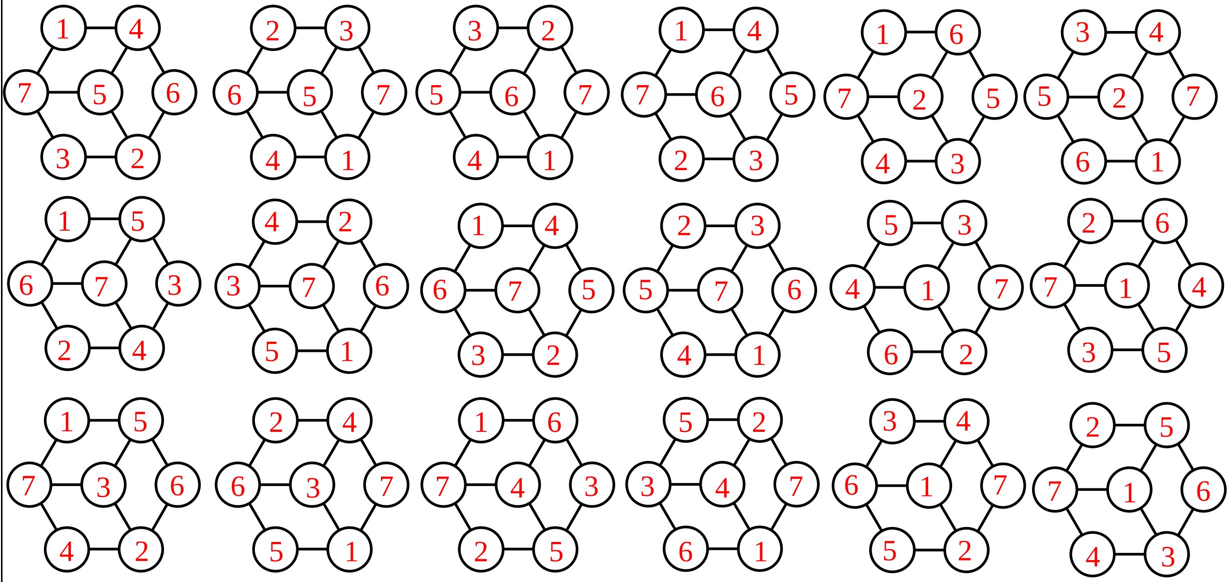
Answer：

2009 EMIC Answers

Individual

1.	4663	**2.**	24/4/1986	**3.**	23	**4.**	6:5	**5.**	5
6.	10	**7.**	1715	**8.**	495	**9.**	1444	**10.**	3:2
11.	15	**12.**	$\frac{33}{7}$	**13.**	24	**14.**	15	**15.**	240

Team

1.	12	**2.**	66
3.	251		
4.	4, 		
5.	5	**6.**	119
7.	4	**8.**	28.659
9.	64		
10.	18, 		

International Mathematics Competition,
25~29 July, 2010, Incheon, Korea,

Elementary Mathematics International Contest

Individual Contest

Time limit: 90 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Write down your name, your contestant number and your team's name on the answer sheet.
- Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.
- Answer all 15 problems. Each problem is worth 10 points and the total is 150 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.
- Diagrams shown may not be drawn to scale.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.

English Version

International Mathematics Competition,
25~29 July, 2010, Incheon, Korea,

Elementary Mathematics International Contest
Individual Contest

Time limit: 90 minutes 27th July 2010 Incheon, Korea

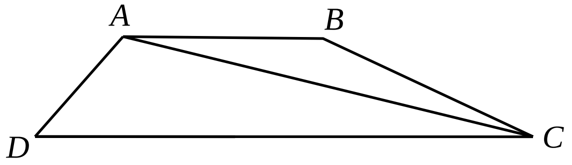
1. A computer billboard is displaying the three "words" : IMC 2010 INCHEON. A malfunction causes the initial "letter" of each of the three words to be shifted to the end of that word every minute. Thus after 1 minute, the billboard reads MCI 0102 NCHEONI, and after 2 minutes, it reads CIM 1020 CHEONIN. After how many minutes will the original three words reappear for the first time?

2. What is the sum of the digits of the number $10^{2010}-2010$?

3. By the notation d_n, we mean an *n*-digit number consisting of *n* times of the digit *d*. Thus 5_3=555 and $4_39_58_13_6$=444999998333333. If $2_w3_x5_y+3_y5_w2_x=$ $5_37_28_z5_17_3$ for some integers *w, x, y* and *z,* what is the value of *w* + *x* + *y* + *z*?

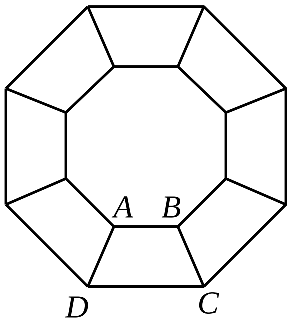
4. A man weighs 60 kg plus one-quarter of his weight. His wife weighs 64 kg plus one-fifth of her weight. What is the absolute difference between the weights of the man and his wife in kg?

5. In quadrilateral *ABCD*, *AB*=6 cm, *AD*=4 cm, *BC*=7 cm and *CD*=15 cm. If the length of *AC* is an integer number of cm, what is this number?



6. The speed of the current in the river is 1 km per hour. A man rows a boat at constant speed. He rows upstreams for 3 hours, and rows downstreams for 2 hours to return to his starting point. What is the distance, in km, between the starting point of the boat and the point at which the boat turns around?

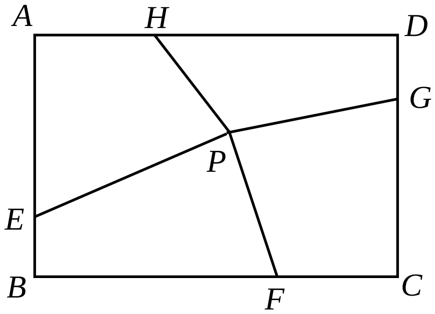
7. In the quadrilateral *ABCD*, *AB* is parallel to *DC* and *AD* = *BC*. If eight copies of this quadrilateral can be used to form a hollow regular octagon as shown in the diagram below, what is the measure of $\angle BAD$, in degree ?



8. Let $\overline{abc}$, $\overline{def}$ be two different 3-digit numbers. If the difference $\overline{abcdef} - \overline{defabc}$ is divisible by 2010, what is the largest possible sum of these two 3-digit numbers?

9. What is the average of all different 9-digits numbers where each consists of the digit 5 five times and the digit 4 four times?

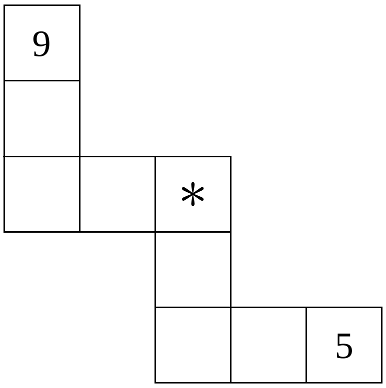
10. *ABCD* is a rectangle with *AB*=4 cm and *BC*=6 cm. *E*, *F*, *G* and *H* are points on the sides *AB*, *BC*, *CD* and *DA* respectively, such that *AE*=*CG*=3 cm and *BF*=*DH*=4 cm. If *P* is a point inside *ABCD* such that the area of the quadrilateral *AEPH* is 5 cm^2, what is the area the quadrilateral *PFCG*, in cm^2?



11. Narrow vegetable spring-rolls of length 8 cm are supposed to be made by rolling 8-cm bean sprouts inside 6 cm × 8 cm rice papers into cylinders. Instead, the workers are provided with 6 cm bean sprouts. So they roll the rice paper the other way and get wide cylinders of length 6 cm. For either kind of spring rolls, there is an overlap of 1 cm in order for the rice paper to stick. What is the ratio of the volume of the 8 cm spring roll to the volume of the 6 cm spring roll?

12. The largest of 23 consecutive odd numbers is 5 times the smallest. What is the average of these 23 numbers?

13. The digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 are to be written in the squares so that every row and every column with three numbers has a total of 13. Two numbers have already been entered. What is the number in the square marked $\ast$?



14. In a test given in four subjects, each of five students obtained a score of w, x, y or z in each individual subject, as shown in the table below. The total score of each student had been computed, as well as the class total for each subject except for one. What was the class total for Biology?

Students	Anna	Gail	Mary	Patty	Susie	Class Total
Algebra	w	z	w	z	y	416
Biology	w	x	y	y	z	?
Chemistry	x	y	y	w	x	428
Dictation	y	w	z	z	x	401
Individual Total	349	330	349	326	315	

15. What is the largest positive integer n which does not contain the digit 0, such that the sum of its digits is 15 and the sum of the digits of $2n$ is less than 20?

International Mathematics Competition,

25~29 July, 2010, Incheon, Korea,

Elementary Mathematics International Contest

TEAM CONTEST

Time：60 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on every page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each problem is worth 40 points and the total is 400 points. Each question is printed on a separate sheet of paper. Complete solutions of problems 1, 3, 4, 5, 6, 8 and 9 are required. Partial credits may be given. In case the spaces provided in each problem are not enough, you may continue you work at the back page of the paper. Only answers are required for Problem number 2, 7 and 10.
- The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each team member must solve at least one problem. Each will then have 35 minutes to write the solutions of the assigned problems independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator or calculating device or electronic devices are allowed.
- Answer must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

International Mathematics Competition,
25~29 July, 2010, Incheon, Korea,

Elementary Mathematics International Contest

TEAM CONTEST

Team：________________________ Score：_____________

1. Pat is building a number triangle so that the first row has only one number, and each subsequent row has two more numbers than the preceding one. Starting from 1, the odd numbers are used in order in the odd-numbered rows. Starting from 2, the even numbers are used in order in the even-numbered rows. Thus her triangle starts off as follows.

1

2 4 6

3 5 7 9 11

8 10 12 14 16 18 20

13 15 17 19 21 23 25 27 29

22 24 26 28 30 32 34 36 38 40 42

⋮

Determine the row number in which the number 2010 will appear in Pat’s number triangle.

Answer: ______________________________

Elementary Mathematics International Contest

TEAM CONTEST

Team：____________________ Score：____________

2. In a faulty calculator, only the keys 7, −, ×, ÷ and = work. If you press 7 after 7, you will get 77, and so on. As soon as an operation key is pressed, the preceding operation, if any, will be performed. When the = key is pressed, the final answer will appear. Find a sequence of key pressing which produces the final answer 34.

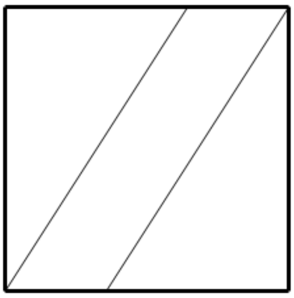
Answer: ____________________

Elementary Mathematics International Contest

TEAM CONTEST

Team：________________ Score：________

3. A square is divided into three parts of equal area by two parallel lines drawn from opposite vertices, as shown in the diagram below. Determine the area of the square, in cm^2, if the distance between the two parallel lines is 1 cm?



Answer: ________________ cm^2

International Mathematics Competition,

25~29 July, 2010, Incheon, Korea,

Elementary Mathematics International Contest

TEAM CONTEST

Team：______________________ Score：____________

4. John and Mary live in the same building which has ten apartments on each floor. The apartments are numbered consecutively, with 1 to 10 on the first floor, 11 to 20 on the second floor, 21 to 30 on the third floor, and so on. The number of Mary's apartment is equal to John's floor number, and the sum of their apartment numbers is 239. Determine the number of John's apartment.

Answer: ______________________

Elementary Mathematics International Contest

TEAM CONTEST

Team：____________________ Score：____________

5. Three couples went shopping in a mall. The following facts were known.
   (1) Each person spent a whole number of dollars.
   (2) The three wives spent \$2408 among them.
   (3) Lady **A** spent \$400 plus half of what Lady **B** spent.
   (4) Lady **C** spent \$204 more than Lady **A** .
   (5) Mr. **X** spent four times as much as his wife.
   (6) Mr. **Y** spent \$8 more than his wife.
   (7) Mr. **Z** spent one and a half times as much as his wife.
   (8) The three couples spent altogether \$8040.
   Determine the three husband-wife pairs.

Answer:
Mr. X - Lady ____________
Mr. Y - Lady ____________
Mr. Z - Lady ____________

International Mathematics Competition,

25~29 July, 2010, Incheon, Korea,

Elementary Mathematics International Contest

TEAM CONTEST

Team：__________________________ Score：______________

6. A nine-digit number contains each of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 exactly once, and every two adjacent digits of this nine-digit number form a two-digit number which is the product of two one-digit numbers. Determine this nine-digit number.

Answer: ______________________________

International Mathematics Competition,

25~29 July, 2010, Incheon, Korea,

Elementary Mathematics International Contest

TEAM CONTEST

Team：________________ Score：________

7. Sixteen students, labelled A to P, are writing a five-day examination. On each day, they write in four rooms, with four of them in a room. No two students are to be in the same room for more than one day. The published schedule, as shown in the diagram below, contains smudges, and unreadable entries are replaced by Xs. Replace each X by the correct letter.

Room	Day 1				Day 2				Day 3				Day 4				Day 5			
1	A	B	C	D	X	G	I	P	X	X	X	M	X	H	I	X	X	G	X	X
2	E	F	G	H	X	X	X	N	D	F	X	O	X	E	J	X	B	X	J	O
3	I	J	K	L	C	E	L	X	X	H	L	P	A	X	K	X	A	X	X	M
4	M	N	O	P	D	X	K	X	X	X	K	X	B	X	X	X	C	F	X	X

Answer:

Room	Day 1				Day 2				Day 3				Day 4				Day 5			
1	A	B	C	D		G	I	P				M		H	I			G		
2	E	F	G	H				N	D	F		O		E	J		B		J	O
3	I	J	K	L	C	E	L			H	L	P	A		K		A			M
4	M	N	O	P	D		K				K		B				C	F		

International Mathematics Competition,

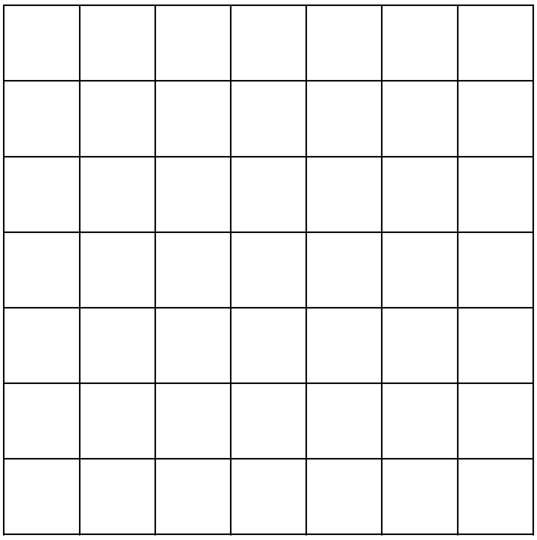
25~29 July, 2010, Incheon, Korea,

Elementary Mathematics International Contest

TEAM CONTEST

Team：________________________ Score：____________

8. A 1×4 alien spaceship is going to land on a 7×7 airfield, occupying 4 of the 49 squares in a row or a column. Mines are placed in some of the squares, and if the alien space ship lands on a square with a mine, it will blow up. Determine the smallest number of mines required to guarantee that the alien spaceship will be blown up, wherever it lands on this airfield. Show where the mines should be placed.



Answer: ______________________________

International Mathematics Competition,
25~29 July, 2010, Incheon, Korea,

Elementary Mathematics International Contest

TEAM CONTEST

Team：________________ Score：________

9. All but one of the numbers from 1 to 21 are to be filled into the squares of a 4×5 table, one number in each square, such that the sum of all the numbers in each row is equal to a number, and the sum of all the numbers in each column is equal to another number. Find all possible values of the number which is deleted, and find a way of filling in the table for each number that was deleted.

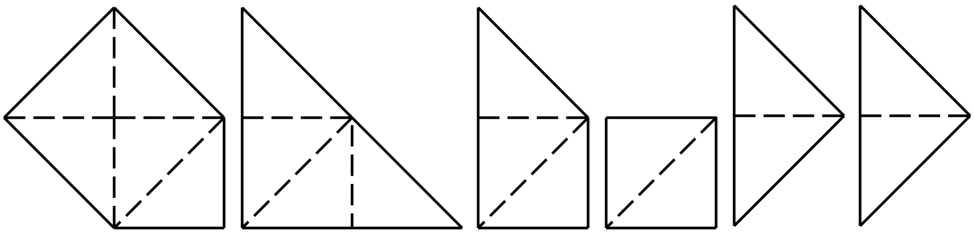
Answer: ________________

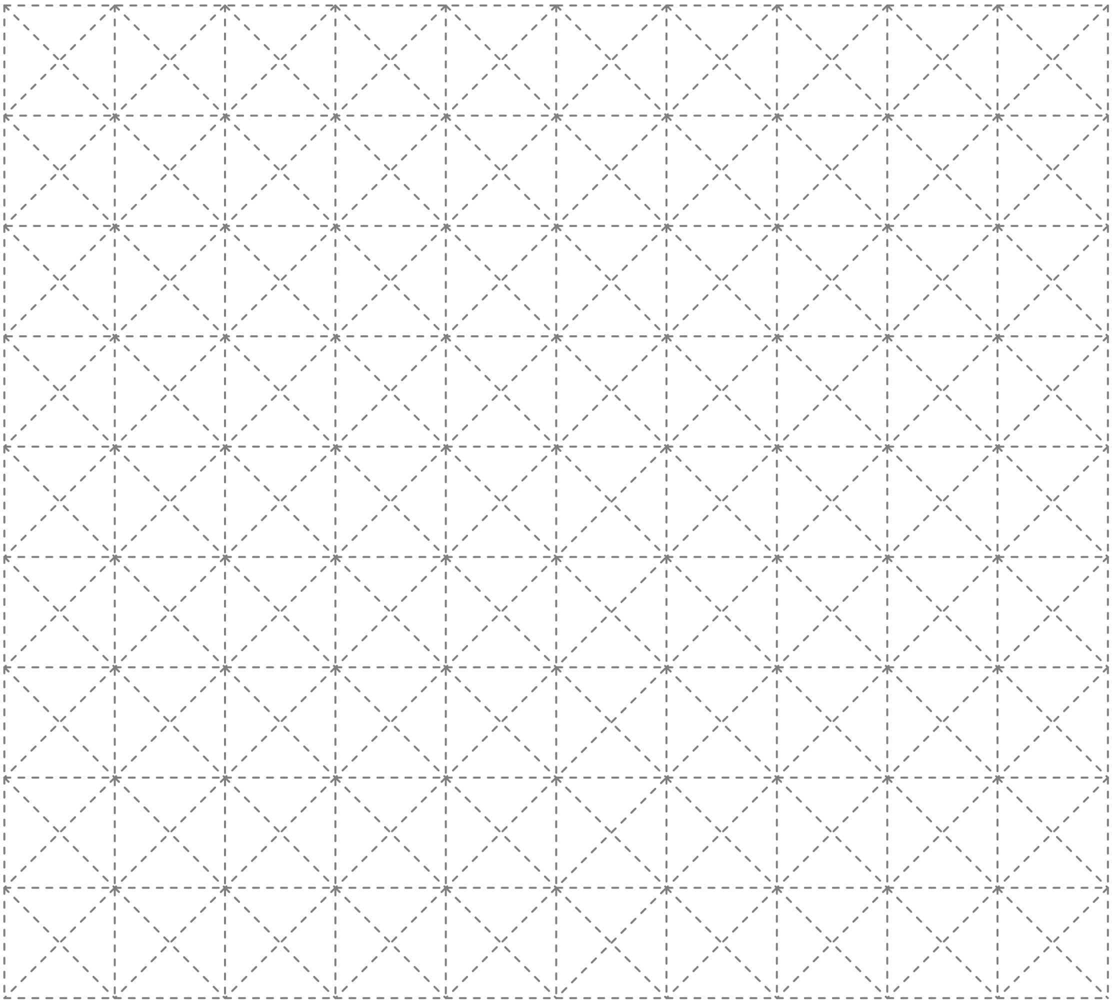
Elementary Mathematics International Contest

TEAM CONTEST

Team： ________________ Score： ________

10. Each of the six pieces shown in the diagram below consists of two to five isosceles right triangles of the same size. A square is to be constructed, without overlap, using *n* of the six pieces. For each possible value of *n*, give a construction.





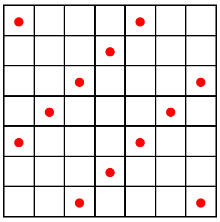
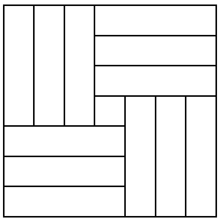
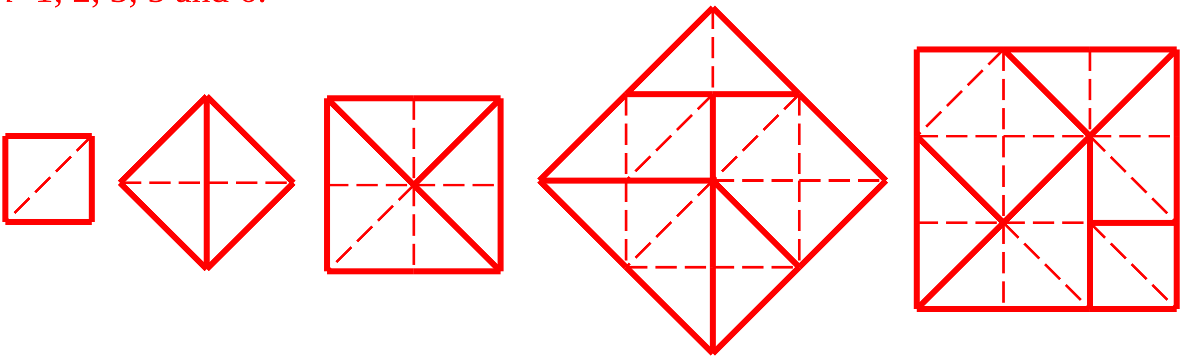
Answer: ________________

2010 EMIC Answers

Individual

1.	84	**2.**	18079	**3.**	15	**4.**	0	**5.**	12
6.	12	**7.**	$112\frac{1}{2}$°	**8.**	1328	**9.**	506172839	**10.**	8
11.	100:147	**12.**	33	**13.**	4	**14.**	424	**15.**	5511111

Team

<table>
<tr><td>1.</td><td>46</td><td>2.</td><td>777 ÷ 7 − 77 = 34</td></tr>
<tr><td>3.</td><td>13</td><td>4.</td><td>217</td></tr>
<tr><td>5.</td><td>Mr. X — Lady C,
Mr. Y — Lady A
Mr. Z — Lady B</td><td>6.</td><td>728163549</td></tr>
<tr><td>7.</td><td colspan="3">
<table>
<tr><td>Room</td><td colspan="4">Day 1</td><td colspan="4">Day 2</td><td colspan="4">Day 3</td><td colspan="4">Day 4</td><td colspan="4">Day 5</td></tr>
<tr><td>1</td><td>A</td><td>B</td><td>C</td><td>D</td><td>B</td><td>G</td><td>I</td><td>P</td><td>C</td><td>G</td><td>J</td><td>M</td><td>C</td><td>H</td><td>I</td><td>N</td><td>D</td><td>G</td><td>L</td><td>N</td></tr>
<tr><td>2</td><td>E</td><td>F</td><td>G</td><td>H</td><td>A</td><td>F</td><td>J</td><td>N</td><td>D</td><td>F</td><td>I</td><td>O</td><td>D</td><td>E</td><td>J</td><td>P</td><td>B</td><td>H</td><td>J</td><td>O</td></tr>
<tr><td>3</td><td>I</td><td>J</td><td>K</td><td>L</td><td>C</td><td>E</td><td>L</td><td>O</td><td>A</td><td>H</td><td>L</td><td>P</td><td>A</td><td>G</td><td>K</td><td>O</td><td>A</td><td>E</td><td>I</td><td>M</td></tr>
<tr><td>4</td><td>M</td><td>N</td><td>O</td><td>P</td><td>D</td><td>H</td><td>K</td><td>M</td><td>B</td><td>E</td><td>K</td><td>N</td><td>B</td><td>F</td><td>L</td><td>M</td><td>C</td><td>F</td><td>K</td><td>P</td></tr>
</table>
</td></tr>
<tr><td>8.</td><td>12,  </td><td>9.</td><td>11,
<table>
<tr><td>21</td><td>4</td><td>5</td><td>15</td><td>10</td></tr>
<tr><td>20</td><td>3</td><td>6</td><td>14</td><td>12</td></tr>
<tr><td>1</td><td>18</td><td>16</td><td>7</td><td>13</td></tr>
<tr><td>2</td><td>19</td><td>17</td><td>8</td><td>9</td></tr>
</table>
</td></tr>
<tr><td>10.</td><td colspan="3"><i>n</i>=1, 2, 3, 5 and 6:
</td></tr>
</table>

Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

Elementary Mathematics International Contest

Individual Contest

Time limit: 90 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Write down your name, your contestant number and your team's name on the answer sheet.
- Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.
- Answer all 15 problems. Each problem is worth 10 points and the total is 150 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.
- Diagrams shown may not be drawn to scale.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.

English Version

Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

Elementary Mathematics International Contest

Individual Contest

Time limit: 90 minutes 20th July 2011 Bali, Indonesia

1. For any two numbers *a* and *b*, $a * b$ means $a + b - \frac{2011}{2}$.
   Calculate: $1 * 2 * 3 * \cdots * 2010 * 2011$.

2. Suppose 11 coconuts have the same cost as 14 pineapples, 22 mango have the same cost as 21 pineapples, 10 mango have the same cost as 3 bananas, and 5 oranges have the same cost as 2 bananas. How many coconuts have the same cost as 13 oranges?

3. A girl calculates $\frac{1+2}{3} + \frac{4+5}{6} + \cdots + \frac{2011+2012}{2013}$ and a boy calculates $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{671}$. What is the sum of their answers?

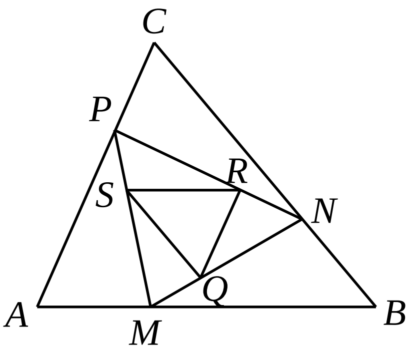
4. What is the first time between 4:00 and 5:00 that the hour hand and the minute hand are exactly 10° apart?

5. Two squirrels, Tim and Kim, are dividing a pile of hazelnuts. Tim starts by taking 5 hazelnuts. Thereafter, they take alternate turns, each time taking 1 more hazelnut than the other in the preceding turn. If the number of hazelnuts to be taken is larger than what remains in the pile, then all remaining hazelnuts are taken. At the end, Tim has taken 101 hazelnuts. What is the exact number of hazelnuts at the beginning?

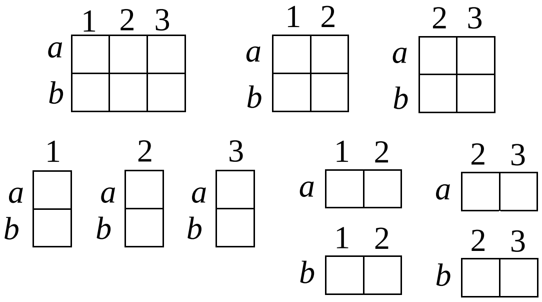
6. In how many ways can we pay a bill of \$500 by a combination of \$10, \$20 and \$50 notes?

7. The least common multiple of the numbers 16, 50 and *A* is 1200. How many positive integers *A* have this property?

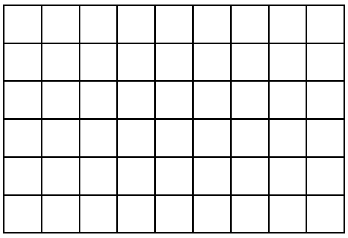
8. In the figure below, $\frac{AM}{MB}=\frac{BN}{NC}=\frac{CP}{PA}=\frac{1}{2}$ and $\frac{MQ}{QN}=\frac{NR}{RP}=\frac{PS}{SM}=\frac{1}{2}$. If the area of $\triangle ABC$ is 360 cm^2, what is the area of $\triangle QRS$, in cm^2?



9. In a 2×3 table, there are 10 rectangles which consist of an even number of unit squares.

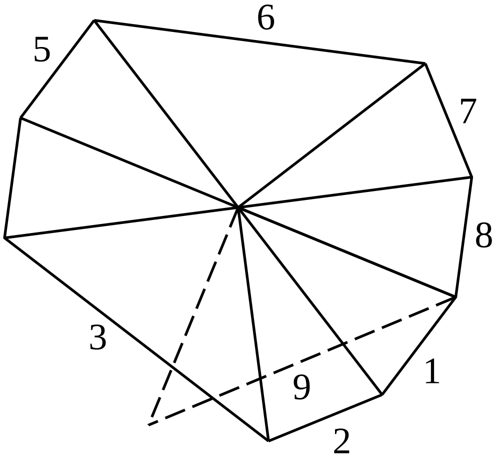


How many rectangles are there in a 6×9 table which consist of an even number of unit squares?



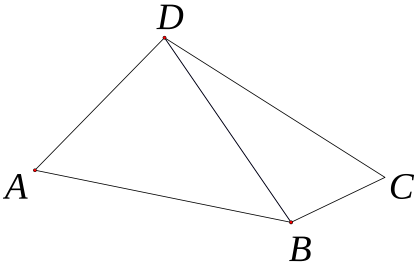
10. Find the smallest positive common multiple of 4 and 6 such that each digit is either 4 or 6, there is at least one 4 and there is at least one 6.

11. We have two kinds of isosceles triangles each with two sides of length 1. The acute triangle has a 30° angle between the two equal sides, and the right triangle has a right angle between the two equal sides. We place a sequence of isosceles triangles around a point according to the following rules. The *n*-th isosceles triangle is a right isosceles triangle if *n* is a multiple of 3, and an acute isosceles triangle if it is not. Moreover, the *n*-th and (*n*+1)-st isosceles triangles share a common side, as shown in the diagram below. What is the smallest value of *n*>1such that the *n*-th isosceles triangle coincides with the 1-st one?



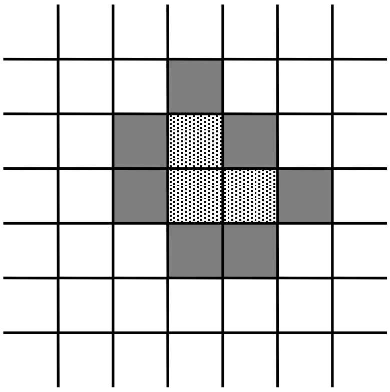
12. When the digits of a two-digit number are reversed, the new number is at least 3 times as large as the original number. How many such two-digit numbers are there?

13. In the quadrilateral *ABCD*, *AB*=*CD*, $\angle BCD$=57°, and $\angle ADB + \angle CBD$ = 180°. Find the value of $\angle BAD$.

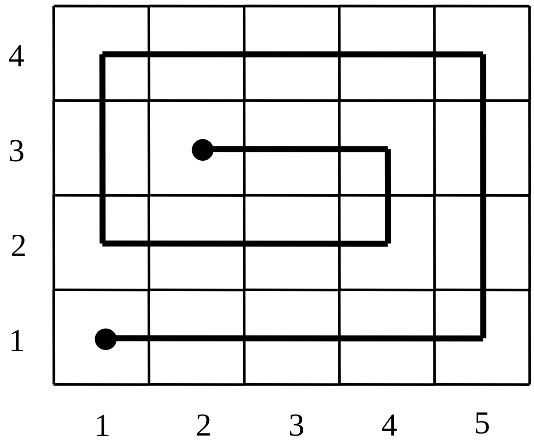


14. Squares on an infinite chessboard are being painted. As shown in the diagram below, three squares (lightly shaded) are initially painted. In the first step, we paint all squares (darkly shaded) which share at least one edge with squares already painted. The same rule applies in all subsequent steps. Find the number of painted squares after one hundred steps.

First step



15. The rows of a 2011×4024 chessboard are numbered from 1 to 2011 from bottom to top, and the columns from 1 to 4024 from left to right. A snail starts crawling from the cell on row 1 and column 1 along row 1. Whenever it is about to crawl off the chessboard or onto a cell which it has already visited, it will make a left turn and then crawl forwards in a straight line. Thus it follows a spiraling path until it has visited every cell. Find the sum of the row number and the column number of the cell where the path ends. (The answer is 3+2=5 for a 4×5 table.)



Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

Elementary Mathematics International Contest

TEAM CONTEST

Time：60 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on every page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. Each problem is worth 40 points and complete solutions of problem 2, 4, 6, 8 and 10 are required for full credits. Partial credits may be awarded. In case the spaces provided in each problem are not enough, you may continue your work at the back page of the paper. Only answers are required for problem number 1, 3, 5, 7 and 9.
- The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must attempt at least one problem. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator or calculating device or electronic devices are allowed.
- Answer must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Elementary Mathematics International Contest

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team：________________ Score：________

1. There are 18 bags of candies. The first bag contains 1 piece. The second bag contains 4 pieces. In general, the k-th bag contains k^2 pieces. The bags are to be divided into three piles, each consisting of 6 bags, such that the total number of pieces inside the bags in each pile is the same. Find one way of doing so.

Answer:

1st pile：________________

2nd pile：________________

3rd pile：________________

Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

Elementary Mathematics International Contest

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team : ______________________ Score : ______________

2. There are eight positive integers in a row. Starting from the third, each is the sum of the preceding two numbers. If the eighth number is 2011, what is the largest possible value of the first one?

Answer: ______________________________

Elementary Mathematics International Contest

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team： ______________________________ Score： ________________

3． O is the centre of a circle. A light beam starts from a point A_0 on the circle, hits a point A_1 on the circle and then reflects to hit another point A_2 on the circle, where $\angle A_0A_1O = \angle A_2A_1O$. Then it reflects to hit another point A_3, and so on. If A_{95} is the first point to coincide with A_0, how many different choices of the point A_1 can there be?

Answer: ______________________________ different choices

Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

Elementary Mathematics International Contest

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team：________________ Score：________________

4. The capacities of a large pipe and four identical small pipes, in m^3 per hour, are positive integers. The large pipe has a capacity of 6 m^3 per hour more than a small pipe. The four small pipes together can fill a pool 2 hours faster than the large pipe. What is the maximum volume of the pool, in m^3?

Answer: ________________ m^3

Indonesia International Mathematics Competition 2011

Ministry of National Education
Republic of Indonesia

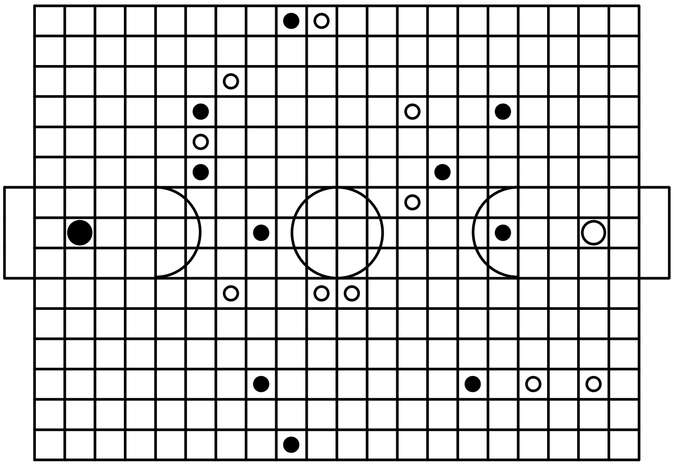
Elementary Mathematics International Contest

TEAM CONTEST

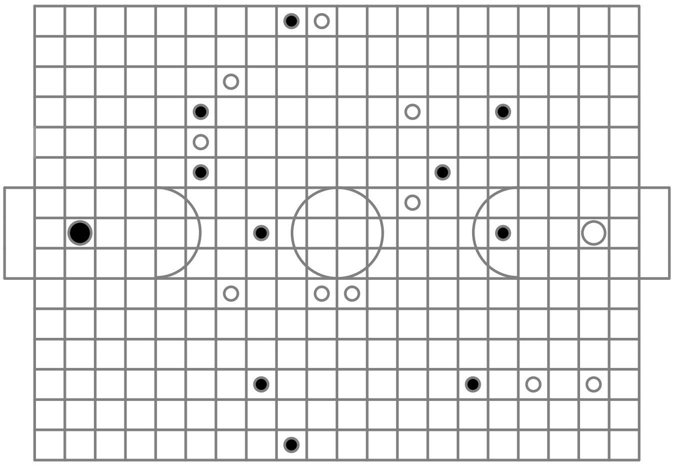
20th July 2011 Bali, Indonesia

Team：______________________ Score：______________

5. The boys in Key Stage II, wearing white, are playing a soccer match against the boys in Key Stage III, wearing black. At one point, the position of the players on the field are as shown in the diagram below. The ball may be passed from one team member, in any of the eight directions along a row, a column or a diagonal, to the first team member in line. The ball may not pass through an opposing team member. The goalkeeper of Stage II, standing in front of his goal on the right, has the ball. Pass the ball so that each member of the white team touches the ball once, and the last team member shoots the ball into the black team's net.



Answer:



Elementary Mathematics International Contest

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team：_______________ Score：_______________

6. A palindrome is a positive integer which is the same when its digits are read in reverse order. In the addition 2882+9339=12221, all three numbers are palindromes. How many pairs of four-digit palindromes are there such that their sum is a five-digit palindrome? The pair (9339, 2882) is not considered different from the pair (2882, 9339).

Answer: _______________ pairs

Ministry of National Education
Republic of Indonesia

Indonesia International Mathematics Competition 2011

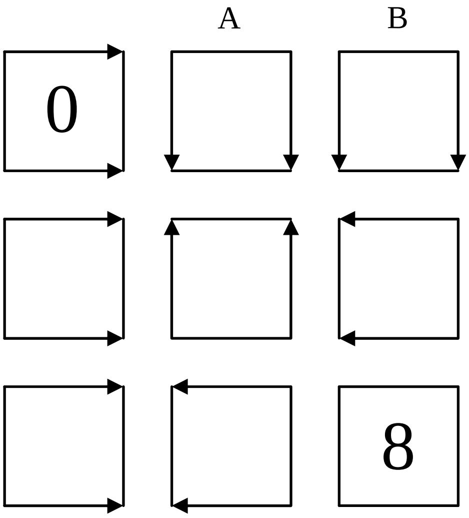
Elementary Mathematics International Contest

TEAM CONTEST

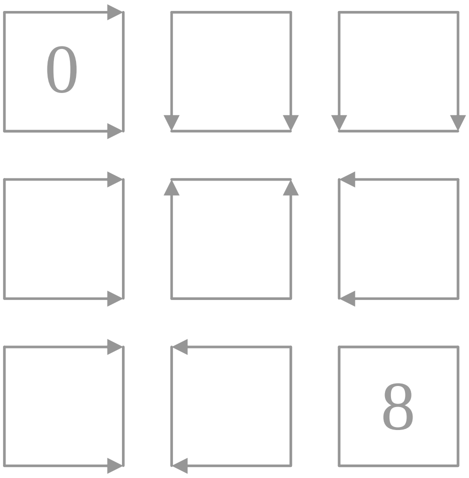
20th July 2011 Bali, Indonesia

Team : ______________________ Score : ______________

7. Place each of 1, 2, 3, 4, 5, 6 and 7 into a different vacant box in the diagram below, so that the arrows of the box containing 0 point to the box containing 1. For instance, 1 is in box A or B. Similarly, the arrows of the box containing 1 point to the box containing 2, and so on.



Answer:



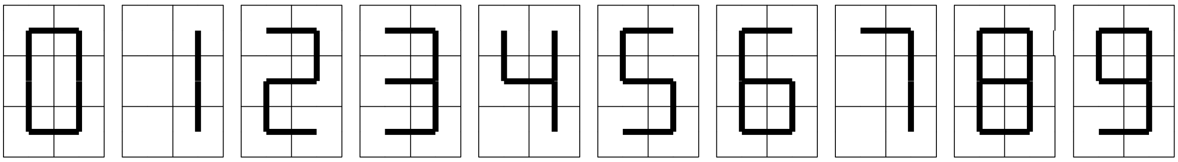
Elementary Mathematics International Contest

TEAM CONTEST

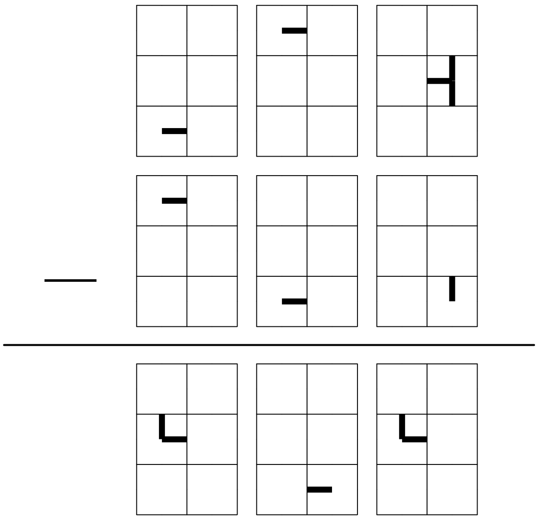
20th July 2011 Bali, Indonesia

Team：__________________________ Score：_______________

8. On calculators, the ten digits are displayed as shown in the diagram below, each consisting of six panels in a 3×2 configuration.



A calculator with a two-dimensional display was showing the subtraction of a three-digit number from another three-digit number, but the screen was malfunctioning so that only one panel of each digit was visible.
What is the maximum value of the three-digit difference?



Answer: ____________________

Elementary Mathematics International Contest

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team : ______________________________ Score : ________________

9. Six villages are evenly spaced along a country road. It takes one hour to ride on a bicycle from one village to the next. Mail delivery is once a day. There are six packets of letters, one for each village. The mailman's introductions are as follows:
   (1) Ask the Post Office van to drop you off at the village on the first packet and deliver it.
   (2) Ride the bicycle non-stop to the village on the second packet and deliver it.
   (3) Repeat the last step until all packets have been delivered.
   (4) Phone the Post Office van to pick you up.

   The mailman is paid 20000 rupiahs an hour on the bicycle. Taking advantage that the Post Office has no instructions on how the packets are to be ordered, what is the maximum amount of money he can earn in a day?

Answer: ______________________________ rupiahs

Elementary Mathematics International Contest

TEAM CONTEST

20th July 2011 Bali, Indonesia

Team：______________________________ Score：________________

10. How many different ways can 90 be expressed as the sum of at least two consecutive positive integers?

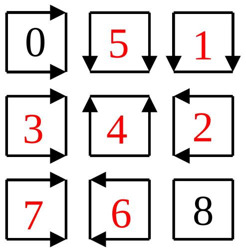
Answer: ______________________________

2011 EMIC Answers

Individual

1.	2011	**2.**	13	**3.**	1342	**4.**	4 : 20	**5.**	205
6.	146	**7.**	15	**8.**	40	**9.**	645	**10.**	4464
11.	23	**12.**	6	**13.**	57°	**14.**	20503	**15.**	4025

Team

1.	The first pile consists of 324, 169, 121, 64, 16 and 9, the second pile consists of 256, 225, 144, 49, 25 and 4 and the third pile consists of 289, 196, 100, 81, 36 and 1.		
2.	240	**3.**	72
4.	72	**5.**	1–2–3–8–10–4–7–9–6–11–5or 1–2–3–8–10–11–6–9–7–4–5
6.	36	7.	
8.	529	**9.**	340000
10.	5		

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

Individual Contest

Time limit: 90 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Write down your name, your contestant number and your team's name on the answer sheet.
- Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.
- Answer all 15 problems. Each problem is worth 10 points and the total is 150 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.
- Diagrams shown may not be drawn to scale.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.

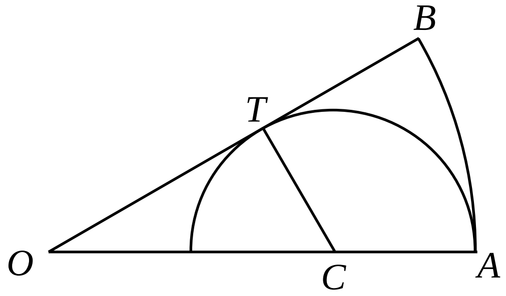
English Version

1. In how many ways can 20 identical pencils be distributed among three girls so that each gets at least 1 pencil?

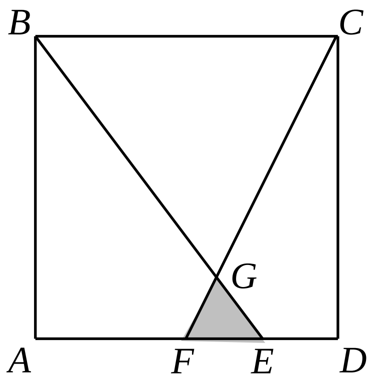
2. On a circular highway, one has to pay toll charges at three places. In clockwise order, they are a bridge which costs \$1 to cross, a tunnel which costs \$3 to pass through, and the dam of a reservoir which costs \$5 to go on top. Starting on the highway between the dam and the bridge, a car goes clockwise and pays toll-charges until the total bill amounts to \$130. How much does it have to pay at the next place if he continues?

3. When a two-digit number is increased by 4, the sum of its digits is equal to half of the sum of the digits of the original number. How many possible values are there for such a two-digit number?

4. In the diagram below, *OAB* is a circular sector with *OA* = *OB* and $\angle AOB = 30°$. A semicircle passing through *A* is drawn with centre *C* on *OA*, touching *OB* at some point *T*. What is the ratio of the area of the semicircle to the area of the circular sector *OAB*?

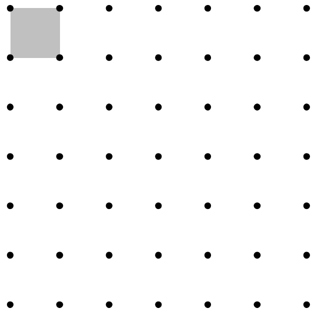


5. *ABCD* is a square with total area 36 cm^2. *F* is the midpoint of *AD* and *E* is the midpoint of *FD*. *BE* and *CF* intersect at *G*. What is the area, in cm^2, of triangle *EFG*?



6. In a village, friendship among girls is mutual. Each girl has either exactly one friend or exactly two friends among themselves. One morning, all girls with two friends wear red hats and the other girls all wear blue hats. It turns out that any two friends wear hats of different colours. In the afternoon, 10 girls change their red hats into blue hats and 12 girls change their blue hats into red hats. Now it turns out that any two friends wear hats of the same colour. How many girls are there in the village? (A girl can only change her hat once.)

7. The diagram below shows a 7 × 7 grid in which the area of each unit cell (one of which is shaded) is 1 cm^2. Four congruent squares are drawn on this grid. The vertices of each square are chosen among the 49 dots, and two squares may not have any point in common. What is the maximum area, in cm^2, of one of these four squares?



8. The sum of 1006 different positive integers is 1019057. If none of them is greater than 2012, what is the minimum number of these integers which must be odd?

9. The desks in the TAIMC contest room are arranged in a 6 × 6 configuration. Two contestants are neighbours if they occupy adjacent seats along a row, a column or a diagonal. Thus a contestant in a seat at a corner of the room has 3 neighbours, a contestant in a seat on an edge of the room has 5 neighbours, and a contestant in a seat in the interior of the room has 8 neighbours. After the contest, a contestant gets a prize if at most one neighbour has a score greater than or equal to the score of the contestant. What is maximum number of prize-winners?

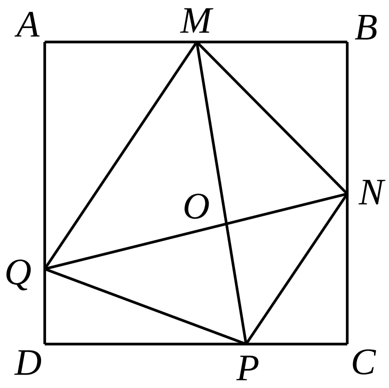
10. The sum of two positive integers is 7 times their difference. The product of the same two numbers is 36 times their difference. What is the larger one of these two numbers?

11. In a competition, every student from school A and from school B is a gold medalist, a silver medalist or a bronze medalist. The number of gold medalist from each school is the same. The ratio of the percentage of students who are gold medalist from school A to that from school B is 5:6. The ratio of the number of silver medalists from school A to that from school B is 9:2. The percentage of students who are silver medalists from both school is 20%. If 50% of the students from school A are bronze medalists, what percentage of the students from school B are gold medalists?

12. We start with the fraction $\frac{5}{6}$. In each move, we can either increase the numerator by 6 or increases the denominator by 5, but not both. What is the minimum number of moves to make the value of the fraction equal to $\frac{5}{6}$ again?

13. Five consecutive two-digit numbers are such that 37 is a divisor of the sum of three of them, and 71 is also a divisor of the sum of three of them. What is the largest of these five numbers?

14. $ABCD$ is a square. M is the midpoint of AB and N is the midpoint of BC. P is a point on CD such that $CP = 4$ cm and $PD = 8$ cm, Q is a point on DA such that $DQ = 3$ cm. O is the point of intersection of MP and NQ. Compare the areas of the two triangles in each of the pairs (QOM, QAM), (MON, MBN), (NOP, NCP) and (POQ, PDQ). In cm^2, what is the maximum value of these four differences?



15. Right before Carol was born, the age of Eric is equal to the sum of the ages of Alice, Ben and Debra, and the average age of the four was 19. In 2010, the age of Debra was 8 more than the sum of the ages of Ben and Carol, and the average age of the five was 35.2. In 2012, the average age of Ben, Carol, Debra and Eric is 39.5. What is the age of Ben in 2012?

Taiwan International Mathematics Competition 2012 (TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

TEAM CONTEST

Time：60 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name in the space indicated on every page.
- There are 10 problems in the Team Contest, arranged in increasing order of difficulty. Each question is printed on a separate sheet of paper. Each problem is worth 40 points and complete solutions of problem 2, 4, 6, 8 and 10 are required for full credits. Partial credits may be awarded. In case the spaces provided in each problem are not enough, you may continue your work at the back page of the paper. Only answers are required for problem number 1, 3, 5, 7 and 9.
- The four team members are allowed 10 minutes to discuss and distribute the first 8 problems among themselves. Each student must attempt at least one problem. Each will then have 35 minutes to write the solutions of their allotted problem independently with no further discussion or exchange of problems. The four team members are allowed 15 minutes to solve the last 2 problems together.
- No calculator or calculating device or electronic devices are allowed.
- Answer must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

For Juries Use Only

No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**	Sign by Jury
Score												
Score												

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

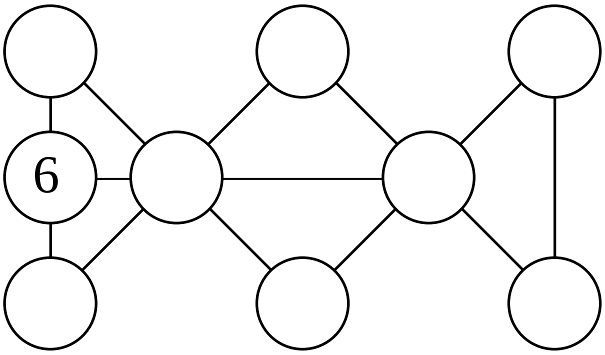
Elementary Mathematics International Contest

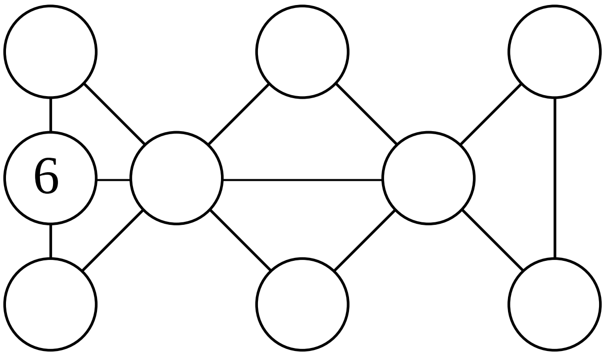
TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：____________________ Score：______________

1. Each of the nine circles in the diagram below contains a different positive integer. These integers are consecutive and the sum of numbers in all the circles on each of the seven lines is 23. The number in the circle at the top right corner is less than the number in the circle at the bottom right corner. Eight of the numbers have been erased. Restore them.





Answer: ____________________

Taiwan International Mathematics Competition 2012 (TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：____________________ Score：______________

2. A clay tablet consists of a table of numbers, part of which is shown in the diagram below on the left. The first column consists of consecutive numbers starting from 0. In the first row, each subsequent number is obtained from the preceding one by adding 1. In the second row, each subsequent number is obtained from the preceding one by adding 2. In the third row, each subsequent number is obtained from the preceding one by adding 3, and so on. The tablet falls down and breaks up into pieces, which are swept away except for the two shown in the diagram below on the right in magnified forms, each with a smudged square. What is the sum of the two numbers on these two squares?

0	1	2	3	4	5	
1	3	5	7	9	11	
2	5	8	11	14	17	
3	7	11	15	19	23	
4	9	14	19	24	29	
5	11	17	23	29	35	

?	2012	2023

2012
2683
?

Answer: ____________________

Taiwan International Mathematics Competition 2012 (TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：____________________ Score：______________

3. In a row of numbers, each is either 2012 or 1. The first number is 2012. There is exactly one 1 between the first 2012 and the second 2012. There are exactly two 1s between the second 2012 and the third 2012. There are exactly three 1s between the third 2012 and the fourth 2012, and so on. What is the sum of the first 2012 numbers in the row?

Answer: ________________________

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team : ______________________ Score : ______________

4. In a test, one-third of the questions were answered incorrectly by Andrea and 7 questions were answered incorrectly by Barbara. One fifth of the questions were answered incorrectly by both of them. What was the maximum number of questions which were answered correctly by both of them?

Answer: ______________________________

Taiwan International Mathematics Competition 2012 (TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：__________________________ Score：_______________

5. Five different positive integers are multiplied two at a time, yielding ten products. The smallest product is 28, the largest product is 240 and 128 is also one of the products. What is the sum of these five numbers?

Answer: ___________________________________

Taiwan International Mathematics Competition 2012 (TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

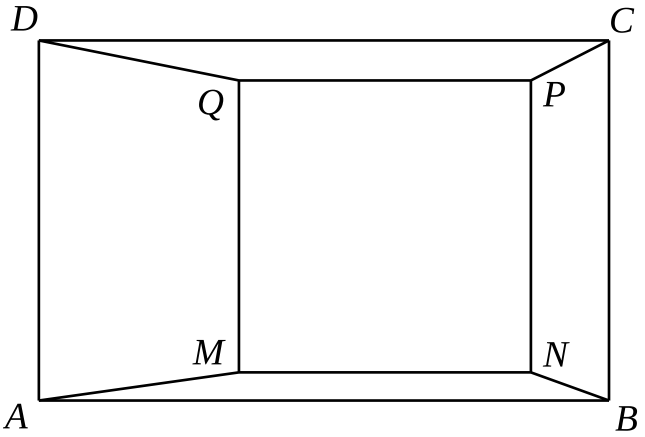
Elementary Mathematics International Contest

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：____________________ Score：______________

6. The diagram below shows a square *MNPQ* inside a rectangle *ABCD* where $AB - BC = 7$ cm. The sides of the rectangle parallel to the sides of the square. If the total area of *ABNM* and *CDQP* is 123 cm^2 and the total area of *ADQM* and *BCPN* is 312 cm^2, what is the area of *MNPQ* in cm^2?



Answer: ____________________ cm^2

Taiwan International Mathematics Competition 2012 (TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：__________________________ Score：_______________

7. Two companies have the same number of employees. The first company hires new employees so that its workforce is 11 times its original size. The second company lays off 11 employees. After the change, the number of employees in the first company is a multiple of the number of employees in the second company. What is the maximum number of employees in each company before the change?

Answer: ___________________________________

Taiwan International Mathematics Competition 2012 (TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

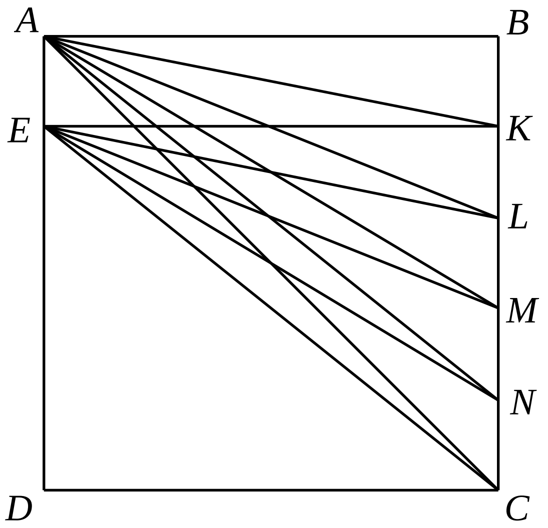
TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：________________ Score：________

8. *ABCD* is a square. *K*, *L*, *M* and *N* are points on *BC* such that $BK = KL = LM = MN = NC$. *E* is the point on *AD* such that $AE = BK$. In degrees, what is the measure of

$$\angle AKE + \angle ALE + \angle AME + \angle ANE + \angle ACE\,?$$



Answer: ________________ °

Taiwan International Mathematics Competition 2012 (TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

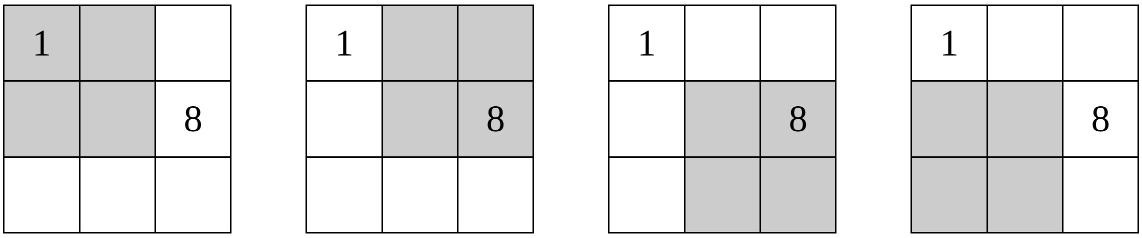
Elementary Mathematics International Contest

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：________________ Score：________

9. The numbers 1 and 8 have been put into two squares of a 3×3 table, as shown in the diagram below. The remaining seven squares are to be filled with the numbers 2, 3, 4, 5, 6, 7 and 9, using each exactly once, such that the sum of the numbers is the same in any of the four 2×2 subtables shaded in the diagram below. Find all possible solutions.



1		
		8

1		
		8

1		
		8

1		
		8

1		
		8

1		
		8

Answer: ________________

Taiwan International
Mathematics Competition 2012
(TAIMC 2012)
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

TEAM CONTEST

25th July 2012 Taipei, Taiwan

Team：______________________ Score：______________

10. At the beginning of each month, an adult red ant gives birth to three baby black ants. An adult black ant eats one baby black ant, gives birth to three baby red ants, and then dies (Also, it is known that there are always enough baby black ants to be eaten.) During the month, baby ants become adult ants, and the cycle continues. If there are 9000000 red ants and 1000000 black ants on Christmas day, what was the difference between the number of red ants and the number of black ants on Christmas day a year ago?

Answer: ______________________________