

APMOPS 2011 Mock Test

Thời gian làm bài: 120 phút

Đề thi có 31 câu hỏi. Thí sinh chỉ viết câu trả lời cho mỗi câu hỏi, không cần giải thích, không cần trình bày lời giải. Gửi câu trả lời bằng đường email tới địa chỉ pvthuan@vnu.edu.vn

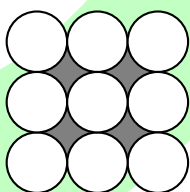
1 Compute the value of

$$\frac{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9}}{\frac{1}{10} + \frac{1}{12} + \frac{1}{14} + \frac{1}{16} + \frac{1}{18}}$$

2 A cuboid of dimension $7 \times 8 \times 10$ is painted on all the six faces. The cube is to be cut into identical cubes of side-length 1 cm. Find the total number of cubes that are NOT painted on any face.

3 Find the last digit of the number $4^{2011} + 7^{2011}$.

4 All the circles in the diagram below are congruent. The sum of their areas is 324π . Find the area of the shaded region.



5 Find the positive value of n such that all the following numbers are prime

$$5n - 7, 3n - 4, 7n + 3, 6n + 1, 9n + 5.$$

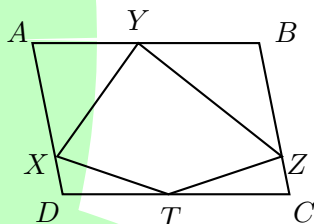
6 The beginning terms of a sequence are 2, 7, 16, 29, 46, 67, 92, 121, ... Find the 50th term of the sequence.

7 A square is to be cut into 226 squares of which 225 have area 1 cm^2 . What is the length of the side of the original square?

8 Find the number of three-digit numbers that give remainder 1 when divided by 3 and remainder 2 when divided by 4.

9 Given two triangles that are neither similar nor congruent, no two sides of the two triangles are on the same line. Let x be the number of point of intersections of the two triangles. What is the sum of all possible values of x ?

10 $ABCD$ is a parallelogram, $AB \parallel CD$, $AD \parallel BC$. Point P is inside the parallelogram. Two lines through P parallel to the sides of the parallelogram, intersecting its sides at X, Y, Z, T as shown in the diagram. If the area of $ABCD$ is 2012, find the area of $XYZT$.



11 Find the total number of positive integers less than 999 that are neither divisible by 3 nor by 5.

12 One or more of five siblings have eaten some cakes without their mother's approval. When questioned, they gave the following answers:

- the first child: "Only one of us ate the cake."
- the second child: "Exactly two of us ate the cake."
- the third child: "Exactly two of us did not eat the cake."
- the fourth child: "Only one of us did not eat the cake."
- the fifth child: "All of us ate the cake."

Their mother knew from past behaviour that the guilty children lied while the others told the truth. Find the number of children who ate the cakes.

13 There are both boys and girls in a class, fourteen of them are boys. If two pupils from the class are selected at random to represent the class at Hexagon Mathematics Contest, then the probability that both are boys is 0.26. How many girls are there in the class?

14 Given that

$$p = 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+\dots+2011},$$

find the largest whole number smaller than p .

15 Find the value of

$$1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + \dots + 21^3 - 22^3.$$

16 Let p be the greatest perfect square of the number $1 \times 2 \times 3 \times \dots \times 21 \times 22 \times 23$. What is the last digit of p ?

17 Find the total number of ways to arrange three yellow identical balls, two blue identical balls, and one red ball around a circle on a plane such that no two adjacent balls are of the same color.

18 Find the value of

$$\left(1 + \frac{4}{1 \times 5}\right) \left(1 + \frac{4}{2 \times 6}\right) \left(1 + \frac{4}{3 \times 7}\right) \dots \left(1 + \frac{4}{20 \times 24}\right).$$

19 Find the last digit of the number

$$1^{2011} + 2^{2011} + 3^{2011} + \dots + 100^{2011}.$$

20 A computer billboard is displaying the three words HEXAGON MATHLEY COMPETITIONS. A malfunction causes the initial letter of each of the three words to be shifted to the end of that word every minute. Thus, after one minute, the billboard reads EXAGONH ATHLEYM OMPETITIONSC, and after two minutes, it reads XAGONHE THLEYMA MPETITIONSCO. After how many minutes will the original three words reappear for the first time?

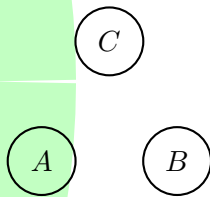
21 Thuan is building a number triangle so that the first row has only one number, and each subsequent row has two more numbers than the preceding one. Starting from 1, the odd numbers are used in order in the odd-numbered rows. Starting from 2, the even numbers are used in order in the even-numbered rows. Thus his triangle starts off as follows

				1						
			2	4	6					
		3	5	7	9	11				
	8	10	12	14	16	18	20			
	13	15	17	19	21	23	25	27	29	
22	24	26	28	30	32	34	36	38	40	42
					⋮					

Determine the row number in which the number 2011 will appear in Thuan's number triangle.

22 The Hexagon Publishing company invited representatives from a certain number of schools to its presentation where it planned to donate 3050 books to the schools so that each should receive an equal number of books. But it turned out that eleven of the schools invited did not come to the presentation. However, the publishing company then decided to split the books equally among the schools that attended the ceremony. How many books did each of the schools get at the presentation?

23 Three circles A, B, C of radius r are arranged in the plane such that their centers make an equilateral triangle of side-length $3r$, as shown in the figure. Find the total number of circles of radius $4r$ that are tangent to exactly two of the three circles.



24 On the blackboard is a 3×3 magic square. The sum of the three numbers in each row, each column and each diagonal is the same. As shown in the diagram below, all but three of the numbers are erased. What is the number represented by x in the cell at the upper left corner?

x	21	94
3		

25 If $x^2 - 9y^2 = 100$ and $x + 3y = 25$, find the value of $x - 3y$.

26 Suppose that the area of $\triangle ABC = 10 \text{ cm}^2$, D is on the side AB such that $AD = 2 \text{ cm}$, $DB = 3 \text{ cm}$ and E is on BC such that the area of $\triangle ABE$ is equal to area of quadrilateral $DBEF$. Find the area of $\triangle ABE$.

27 Compute the value of

$$\frac{2011^2}{2010^2 + 2012^2 - 2}$$

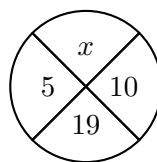
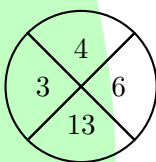
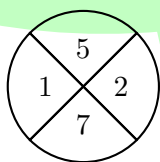
28 A farmer has ten baskets of eggs containing 12, 13, 14, 16, 18, 19, 22, 24, 29 and 34 eggs respectively. Some baskets have chicken eggs while other baskets have duck eggs. He sells one basket and finds that the number of remaining chicken eggs is three times the number of the remaining duck eggs. How many eggs were in the basket he sold?

29 The fraction

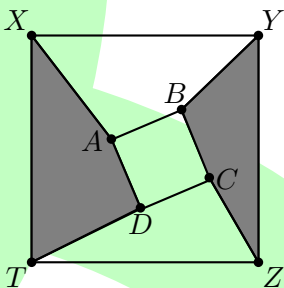
$$\frac{4}{\frac{\frac{4}{4+1}+1}{\frac{4}{4+1}+1} + 1 + \frac{4}{1+\frac{4}{1+\frac{4}{4+1}}}}$$

can be written in the form $\frac{p}{q}$ where p, q are relatively prime positive integers. Find the value of $4p + q$.

30 Find the value of x .



31 A square $ABCD$ is located inside in a larger square $XYZT$. If the sum of areas of quadrilateral $ABYX$ and $CDTZ$ is 360 cm^2 , find the area of the shaded region.



Answers

To have an idea of your performance, send your answer to pvthuanATvnu.edu.vn or contact website www.hexagon.edu.vn.

Credits

This set of problems are suggested by Phạm Văn Thuận, Nguyễn Tiến Lâm, Nguyễn Việt Hùng, Trần Quang Hùng.

