Asia Pacific Mathematical Olympiad for Primary Schools

APMOPS PROBLEMS

from 2001 to 2012 with answer keys

The Chinese High School
Mathematics Learning And Research Centre

Singapore Mathematical Olympiad for Primary Schools 2001

First Round
2 hours
(150 marks)

Instructions to Participants

Attempt as many questions as you can.
Neither mathematical tables nor calculators may be used.
Write your answers in the answer boxes on the **separate answer sheet** provided.
Working may be shown in the space below each question.
Marks are awarded for correct answers only.

This question paper consists of 16 printed pages (including this page)

Number of correct answers for Q1 to Q10 : _______ Marks (**4**) : ________
Number of correct answers for Q11 to Q20 : ______ Marks (**5**) : ________
Number of correct answers for Q20 to Q30 : ______ Marks (**6**) : ________
Total Marks for **First Round** : ______

1. Find the value of

$$0.1 + 0.11 + 0.111 + \ldots + 0.1111111111 .$$

2. Find the missing number in the box.

$$5 \times \boxed{} \div 3 \times 4 - 299 = 2001$$

3. Find the missing number in the following number sequence.

1, 4, 10, 22, 46, ______, 190 , . . .

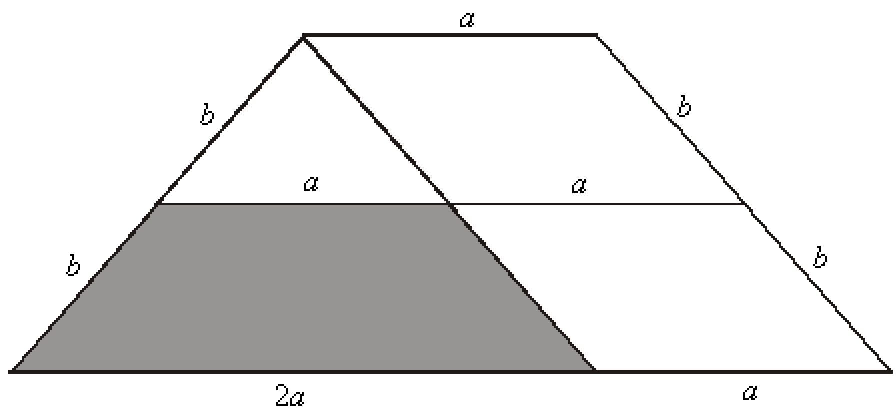
4. If numbers are arranged in 3 rows A, B and C according to the following table, which row will contain the number 1000 ?

A	1,	6,	7,	12,	13,	18,	19,	. . . .
B	2,	5,	8,	11,	14,	17,	20,	. . . .
C	3,	4,	9,	10,	15,	16,	21,	. . . .

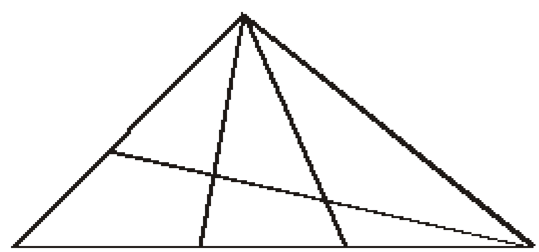
5. How many 5-digit numbers are multiples of 5 and 8 ?

6. John started from a point A, walked 10 m forwards and then turned 36° right. Again he walked 10 m forwards and then turned 36° right. He continued walking in this manner and finally returned to the starting point A. How many metres did he walk altogether ?

7. What fraction of the figure is shaded ?

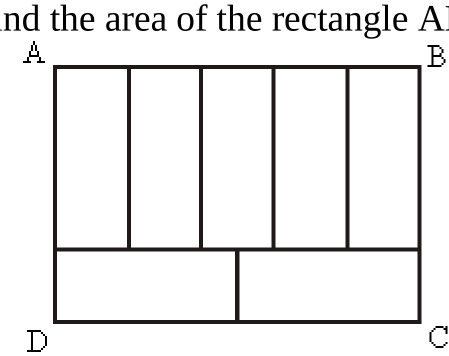


8. How many triangles are there in the figure ?



9. Between 12 o'clock and 1 o'clock, at what time will the hour hand and minute hand make an angle of 110° ?

10. The rectangle ABCD of perimeter 68 cm can be divided into 7 identical rectangles as shown in the diagram. Find the area of the rectangle ABCD.



11. Find the smallest number such that

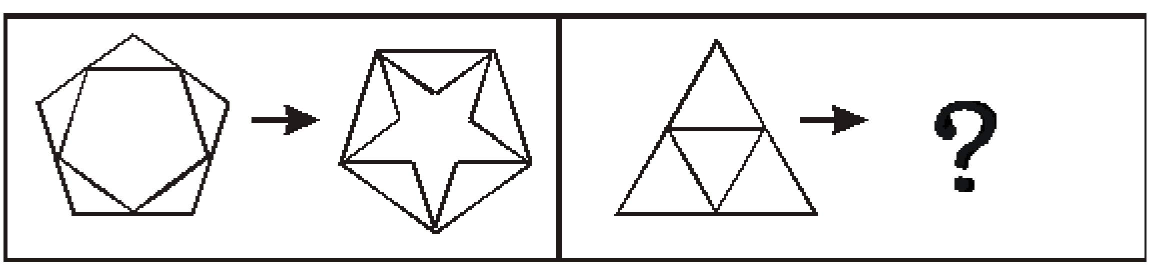
(i) it leaves a remainder 2 when divided by 3 ;
(ii) it leaves a remainder 3 when divided by 5 ;
(iii) it leaves a remainder 5 when divided by 7 .

12. The sum of two numbers is 168. The sum of $\frac{1}{8}$ of the smaller number and $\frac{3}{4}$ of the greater number is 76. Find the difference between the two numbers.

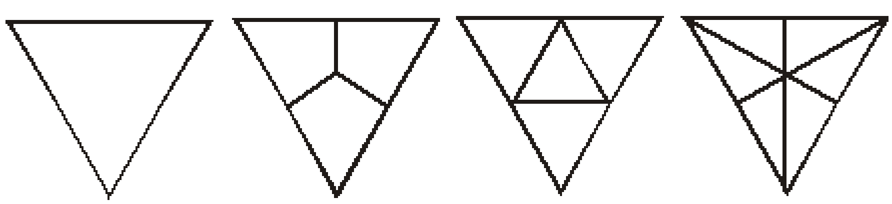
13. There are 325 pupils in a school choir at first. If the number of boys increases by 25 and the number of girls decreases by 5%, the number of pupils in the choir will become 341. How many boys are there in the choir at first ?

14. Mr Tan drove from Town A to Town B at a constant speed of $40\ km/h$. He then drove back from Town B to Town A at a constant speed of $70\ km/h$. The total time taken for the whole journey was 5.5 h. Find the distance between the two towns.

15.

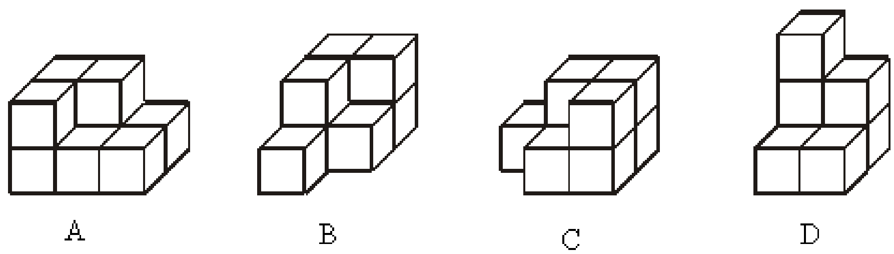


Which one of the following is the missing figure ?

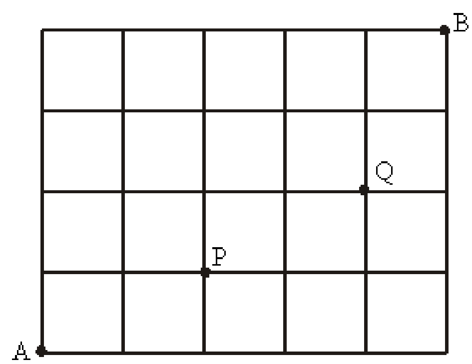


(A) (B) (C) (D)

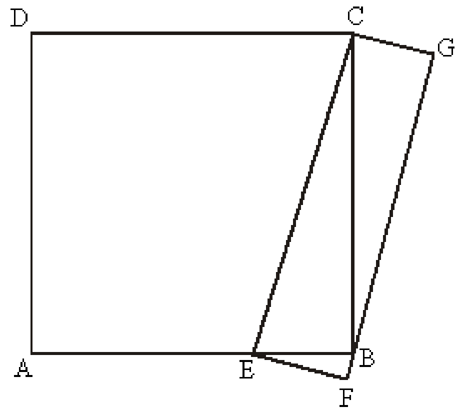
16. Which two of the following solid figures can be fitted together to form a cuboid ?



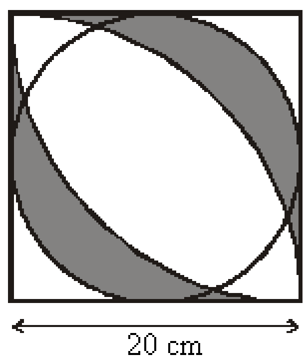
17. In how many different ways can you walk from A to B in the direction $\uparrow$ or $\rightarrow$, without passing through P and Q ?



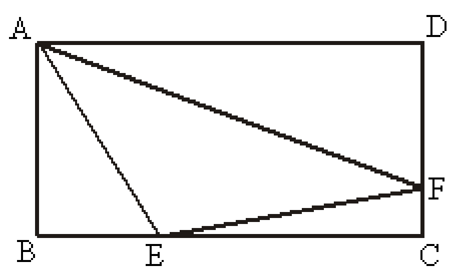
18. In the figure, ABCD is a square and EFGC is a rectangle. The area of the rectangle is $24\ cm^2$. Given that $AE = \frac{5}{8}AB$, find the length of one side of the square.



19. The diagram shows a circle and 2 quarter circles in a square. Find the area of the shaded region. (Take $\pi = 3.14$.)



20. The area of rectangle ABCD is $24\ cm^2$. The areas of triangles ABE and ADF are $4\ cm^2$ and $9\ cm^2$ respectively. Find the area of the triangle AEF.



21. A rectangular paper has a circular hole on it as shown. Draw a straight line to divide the paper into two parts of equal area..

22. What is the 2001th number in the following number sequence ?

$$\frac{1}{1}, \frac{2}{1}, \frac{1}{2}, \frac{3}{1}, \frac{2}{2}, \frac{1}{3}, \frac{4}{1}, \frac{3}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{1}, \frac{4}{2}, \frac{3}{3}, \ldots$$

23. There are 25 rows of seats in a hall, each row having 30 seats. If there are 680 people seated in the hall, at least how many rows have an equal number of people each ?

24. In the following columns, *A*, *B*, *C* and *X* are whole numbers. Find the value of *X*.

A	*A*	*A*	*A*	
B	*A*	*A*	*B*	
B	*B*	*A*	*C*	*A*
B	*B*	*B*	*C*	*B*
C	*C*	*C*	*C*	*C*
38	*36*	*34*	*28*	*X*

25. There were 9 cards numbered 1 to 9. Four people A, B, C and D each collected two of them.

A said : " The sum of my numbers is 6. "
B said : " The difference between my numbers is 5. "
C said : " The product of my numbers is 18. "
D said : " One of my numbers is twice the other. "

What is the number on the remaining card ?

26. Minghua poured out $\frac{1}{2}$ of the water in a container.

In the second pouring, he poured out $\frac{1}{3}$ of the remaining water ;

In the third pouring, he poured out $\frac{1}{4}$ of the remaining water ;

In the forth pouring, he poured out $\frac{1}{5}$ of the remaining water ;
and so on.

After how many times of pouring will the remaining water be exactly $\frac{1}{10}$ of the original amount of water ?

27. A bus was scheduled to travel from Town X to Town Y at constant speed $V \ km/h$. If the speed of the bus was increased by 20%, it could arrive at Town Y 1 hour ahead of schedule.

Instead, if the bus travelled the first 120 km at $V \ km/h$ and then the speed was increased by 25%, it could arrive at Town Y $\frac{4}{5}$ hours ahead of schedule. Find the distance between the two towns.

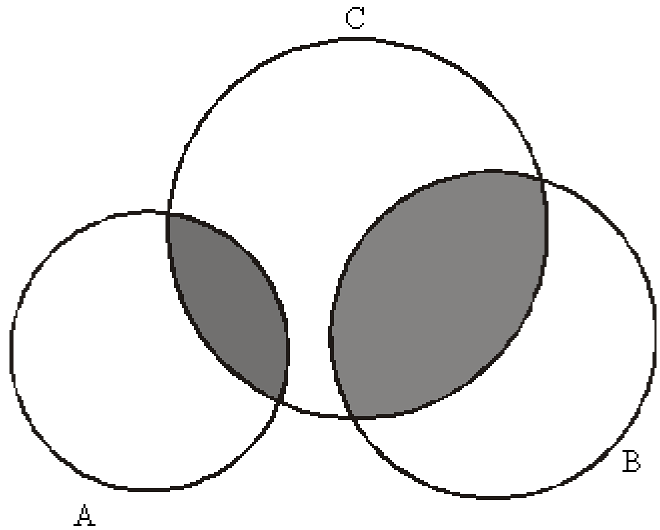
28. The diagram shows three circles A, B and C.

$\frac{1}{3}$ of the circle A is shaded,

$\frac{1}{2}$ of the circle B is shaded,

$\frac{1}{4}$ of the circle C is shaded.

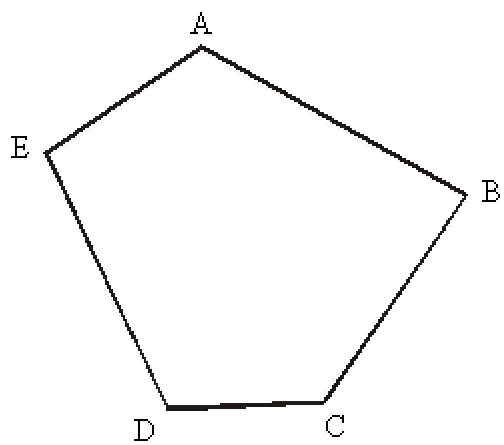
If the total area of A and B is equal to $\frac{2}{3}$ of the area of C, find the ratio of the area of A to the area of B.



29. Given that $m = \underbrace{999\,.\,.\,.\,.\,.\,99}_{2001\text{ digits}}$, $n = \underbrace{888\,.\,.\,.\,.\,.\,88}_{2001\text{ digits}}$,

find the sum of the digits in the value of $m \times n$.

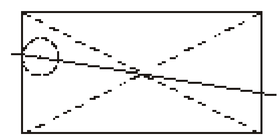
30. Each side of a pentagon ABCDE is coloured by one of the three colours : red, yellow or blue. In how many different ways can we colour the 5 sides of the pentagon such that any two adjacent sides have different colours ?



Singapore Mathematical Olympiad for Primary Schools 2001

First Round – Answers Sheet

	Answers	For markers' use only
1	1.0987654321	
2	345	
3	94	
4	Row C	
5	2250	
6	100 m	
7	$\frac{3}{8}$	
8	15	
9	12.20	
10	280 cm^2	
Questions 1 to 10 each carries 4 marks		
11	68	
12	8	
13	145	
14	140 km	

	Answers	For markers' use only
17	48	
18	8 cm	
19	129 cm^2	
20	9 cm^2	
Questions 11 to 20 each carries 5 marks		
21		The line drawn must pass through the centre of the circle and of the rectangle.
22	$\frac{16}{48}$	
23	4	
24	20	
25	9	
26	9	
27	360 km	
28	3:1	
29	18009	

15	A			30	30	
16	B and C			Questions 21 to 30 each carries 6 marks		

Mathematics Learning And Research Centre

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Singapore Mathematical Olympiad

for Primary Schools 2001

Invitation Round

2 hours

(60 marks)

Instructions to Participants

Attempt as many questions as you can.

Neither mathematical tables nor calculators may be used.

Working *must* **be clearly shown in the space below each question.**

Marks are awarded for both method and answer.

Each question carries 10 marks.

This question paper consists of 7 printed pages (including this page)

Question	1	2	3	4	5	6
Marks						

Total Marks for Invitation Round **:** ___________

1. The value of the product

$$2 \times 4 \times 6 \times 8 \times 10 \times 12 \times 14 \times 16 \times 18 \times 20$$

ends with 2 consecutive zeros.

How many consecutive zeros with the value of each of the following products end with ?

(a) $2 \times 4 \times 6 \times 8 \times \ldots \times 100$,

(b) $2 \times 4 \times 6 \times 8 \times \ldots \times 1000$.

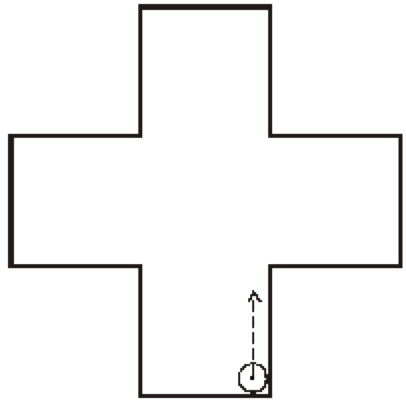
2. There are several red balls and white balls on the table.

If one red ball and one white ball are removed together each time until no red balls are left on the table, then the number of remaining white balls is 50.

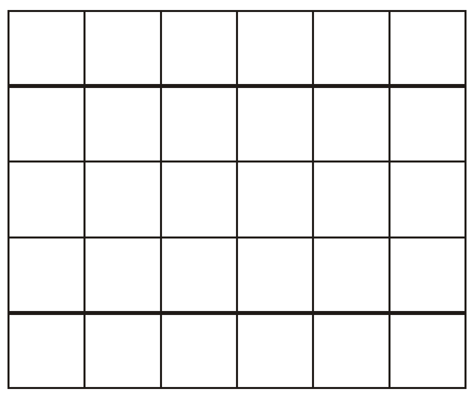
If one red ball and three white balls are removed together each time until no white balls are left on the table, then the number of remaining red balls is also 50.

Find the total number of red balls and white ball at first.

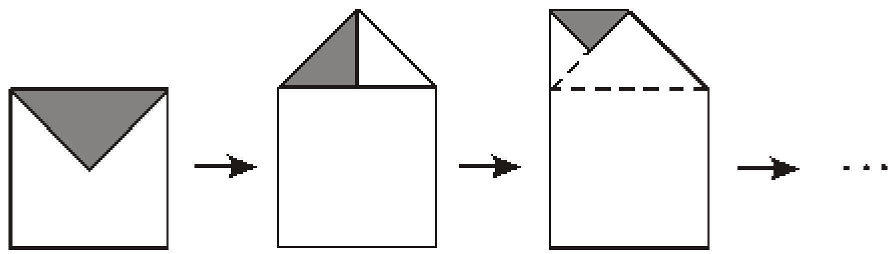
3. Each side of the figure is 10 cm long . A small circular disc of radius 1 cm is placed at one corner as shown. If the disc rolls along the sides of the figure and returns to the starting position, find the distance travelled by the centre of the disc .



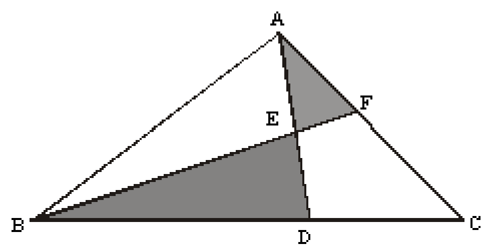
4. Draw two straight lines to divide the figure into four portions whose areas are in the ratio 1 : 2 : 3 : 4 .



5. The figure shows a shaded triangle attached to the square of side 2 cm. When the shaded triangle is unfolded, there is a smaller shaded triangle attached to it. When the smaller shaded triangle is unfolded, there is an even smaller triangle shaded triangle attached to it as shown. If there are infinitely many shaded triangle unfolded in this manner, find the total area of the figure unfolded.



6. In the figure on the right, the area of the ΔABC is $5\ cm^2$, $AE = ED$ and $BD = 2DC$. Find the total area of the shaded part.



THE END

Singapore Mathematical Olympiad for Primary Schools 2001
Invitation Round – Answers Sheet

Question 1:

Ans: a) 12 b) 124

Question 2:

Ans: 250

Question 3:

Ans: $(104+2\pi)$ cm

Question 4:

Question 5:

Ans: $6cm^2$

Question 6:
Ans: $2cm^2$

0900 h — 1100 h

The Chinese High School

Mathematics Learning And Research Centre

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Asia Pacific Mathematical Olympiad for Primary Schools 2002

First Round

2 hours

(150 marks)

Instructions to Participants

Attempt as many questions as you can.

Neither mathematical tables nor calculators may be used.

Write your answers in the answer boxes.

Marks are awarded for correct answers only.

This question paper consists of 4 printed pages (including this page)

Number of correct answers for Q1 to Q10 : ______ **Marks (** $\times 4$ **) :** ______

Number of correct answers for Q11 to Q20 : ______ **Marks (** $\times 5$ **) :** ______

Number of correct answers for Q20 to Q30 : ______ **Marks (** $\times 6$ **) :** ______

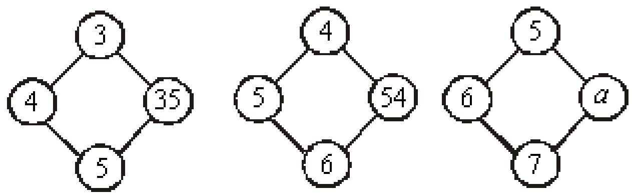
1. How many numbers are there in the following number sequence ?

1.11, 1.12, 1.13, ... , 9.98, 9.99.

2. What is the missing number in the following number sequence ?

$$\frac{1}{2},\ \frac{1}{12},\ \frac{1}{30},\ \frac{1}{56},\ \boxed{}\ ,\ \frac{1}{132}\ .$$

3. Observe the pattern and find the value of *a*.



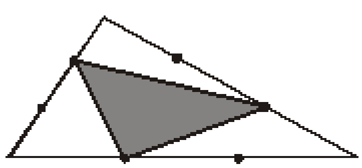
4. Find the value of

$$\frac{1}{2002} + \frac{2}{2002} + \frac{3}{2002} + \ . \ . \ . + \frac{2000}{2002} + \frac{2001}{2002}\ .$$

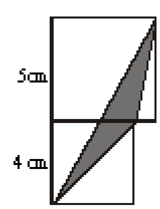
5. The average of 10 consecutive odd numbers is 100.

What is the greatest number among the 10 numbers ?

6. What fraction of the figure is shaded , when each side of the triangle is divided into 3 equal parts by the points?

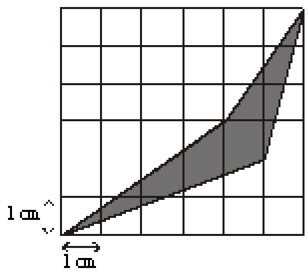


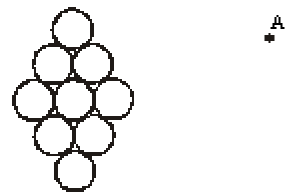
7. The figure is made up of two squares of sides 5 cm and 4 cm respectively. Find the shaded area.



8. Find the area of the shaded figure.

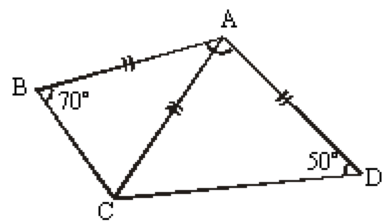
9. Draw a straight line through the point A to divide the 9 circles into two parts of equal areas.





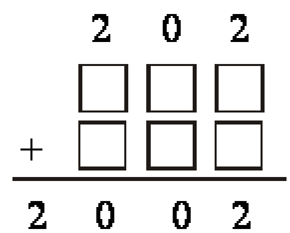
10 In the figure, $AB = AC = AD$, $\angle ABC = 70^0$ and $\angle ADC = 50^0$.

Find $\angle BAD$.



11. In the sum, each □ represents a non-zero digit.

What is the sum of all the 6 missing digits ?



12. The average of n whole numbers is 80. One of the numbers is 100. After removing the number 100, the average of the remaining numbers is 78. Find the value of n .

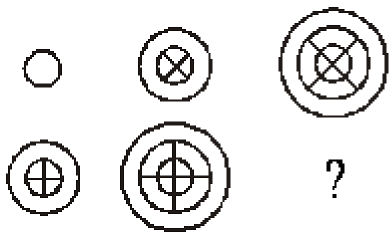
13. The list price of an article is $6000. If it is sold at half price, the profit is 25%. At what price must it be sold so that the profit will be 50% ?

14. $\frac{1}{7}$ of a group of pupils score A for Mathematics; $\frac{1}{3}$ of the pupils score B; $\frac{1}{2}$ of the pupils score C; and the rest score D.

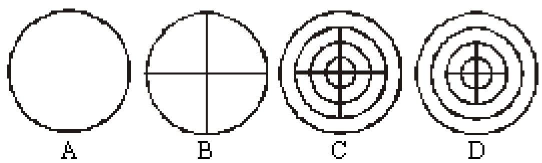
If a total of 100 pupils score A or B, how many pupils score D ?

15. At 8.00 a.m., car A leaves Town P and travels along an expressway. After some time, car B leaves Town P and travels along the same expressway. The two cars meet at 9.00 a.m. If the ratio of A's speed to B's speed is 4 : 5 , what time does B leave Town P ?

16.

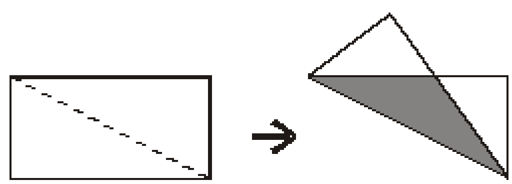


Which one of the following is the missing figure ?



17. A rectangle is folded along a diagonal as shown.

The area of the resulting figure is $\frac{5}{8}$ of the area of the original rectangle. If the area of the shaded triangle is $18\ cm^2$, find the area of the original rectangle.

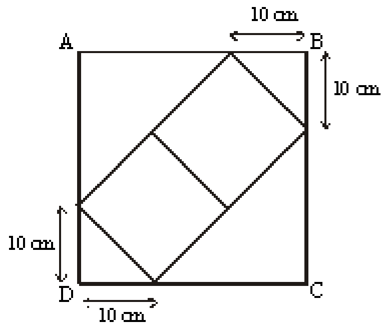


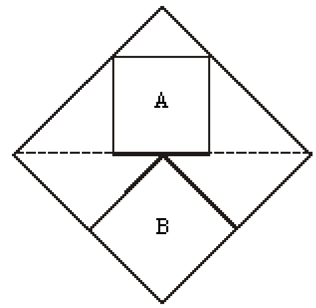
18. The square, ABCD is made up of 4 triangles and 2 smaller squares.

Find the total area of the square ABCD.

19. The diagram shows two squares A and B inside a bigger square.

Find the ratio of the area of A to the area of B.





20. There are 3 straight lines and 2 circles on the plane. They divide the plane into regions. Find the greatest possible number of regions.

21. The number 20022002 . . . 20022002 is formed by writing 2002 blocks of '2002'.

Find the remainder when the number is divided by 9.

22. Find the sum of the first 100 numbers in the following number sequence .

1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 0, 1, 1, 1, 2, 1, 3, 1, 4, 1, 5, . . .

23. In a number sequence : 1, 1, 2, 3, 5, 8, 13, 21, . . . , starting from the third number, each number is the sum of the two numbers that come just before it.

How many even numbers are there among the first 1000 numbers in the number sequence ?

24 10 years ago, the ratio of John's age to Peter's age was 5 : 2.

The ratio is 5 : 3 now. What will be the ratio 10 years later ?

25. David had \$100 more than Allen at first. After David's money had decreased by \$120 and

Allen's money had increased by \$200, Allen had 3 times as much money as David.

What was the total amount of money they had at first ?

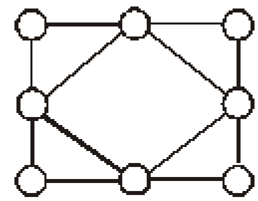
26. Two barrels X and Y contained different amounts of oil at first.

Some oil from X was poured to Y so that the amount of oil in Y was doubled. Then, some oil from Y was poured to X so that the amount of oil in X was doubled

After these two pourings, the barrels each contained 18 litres of oil. How many litres of oil were in X at first ?

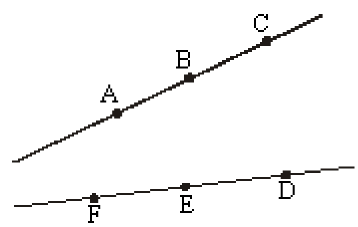
27. In the figure, each circle is to be coloured by one of the colours : red, yellow and blue.

In how many ways can we colour the 8 circles such that any two circles which are joined by a straight line have different colours ?



28. The points A, B, C, D, E and F are on the two straight lines as shown.

How many triangles can be formed with any 3 of the 6 points as vertices ?



29. Patrick had a sum of money.

On the first day, he spent $\frac{1}{4}$ of his money and donated $30 to charity.

On the second day, he spent $\frac{1}{3}$ of the money he still had and donated $20 to charity.

On the third day, he spent $\frac{1}{2}$ of the money he still had and donated $10 to charity.

At the end, he had $10 left. How much money did he have at first ?

30. Four football teams A, B, C and D are in the same group. Each team plays 3 matches, one with each of the other 3 teams.

The winner of each match scores 3 points; the loser scores 0 points; and if a match is a draw, each team scores 1 point.

After all the matches, the results are as follows :

(1) The total scores of 3 matches for the four teams are consecutive odd numbers.

(2) D has the highest total score.

(3) A has exactly 2 draws, one of which is the match with C.

Find the total score for each team..

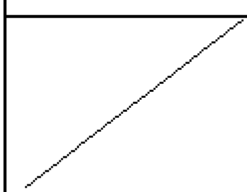
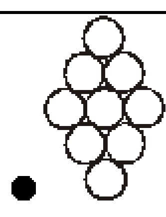
Name of Participant : ______________________________ Index No : ______ / ___

(Statutory Name)

Name of School : ____

Singapore Mathematical Olympiad for Primary Schools 2002

First Round – Answers Sheet

	Answers	For markers′ use only
1	889	
2	1/90	
3	77	
4	1000 ½	
5	109	
6	1/3	
7	8 cm²	
8	6 cm²	
 9		Line must pass through the centre of the middle circle.
10	120˚	

	Answers	For markers′ use only
16	C	
17	48 cm²	
18	900 cm²	
19	8 : 9	
20	21	
Questions 11 to 20 each carries 5 marks		
21	7	
22	365	
23	333	
24	10 : 7	
25	$360	

Questions 1 to 10 each carries 4 marks		
11	36	
12	11	
13	$3600	
14	5	
15	8.12a.m.	

26	22.5 l	
27	18	
28	18	
29	$160	
30	A : 5 B : 3 C : 1 D : 7	All correct – 6m 3 correct – 2m Others – 0m
Questions 21 to 30 each carries 6 marks		

The Chinese High School
Mathematics Learning And Research Centre

Asia Pacific Mathematical Olympiad for Primary Schools 2002

Invitation Round
2 hours
(60 marks)

Instructions to Participants

Attempt as many questions as you can.
Neither mathematical tables nor calculators may be used.
Working ***must*** be clearly shown in the space below each question.
Marks are awarded for both method and answer.
Each question carries 10 marks.

This question paper consists of 7 printed pages (including this page)

Question	**1**	**2**	**3**	**4**	**5**	**6**
Marks						

Total Marks for **Invitation Round** : ___________

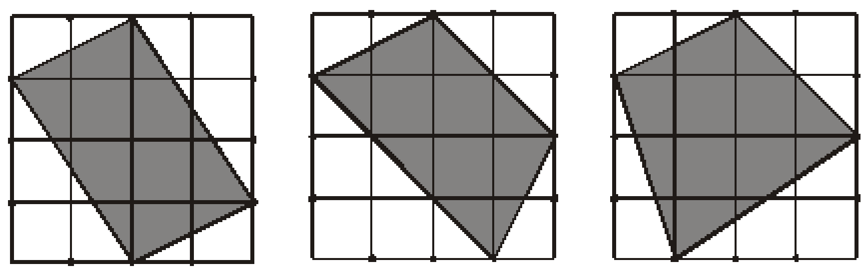
1. The following is an incomplete 9 by 9 multiplication table.

×	1	2	3	4	5	6	7	8	9
1				:			:		
2				:			:		
3				:			:		
4	. .	. .	. .	16			:		
5	. .	. .	. .	. .	. .	. .	35		
6									
7									
8									
9									

(a) Find out how many of the 81 products are odd numbers .

(b) If the multiplication table is extended up to 99 by 99, how many of the products are odd numbers ?

2. Find the area of each of the following shaded regions.



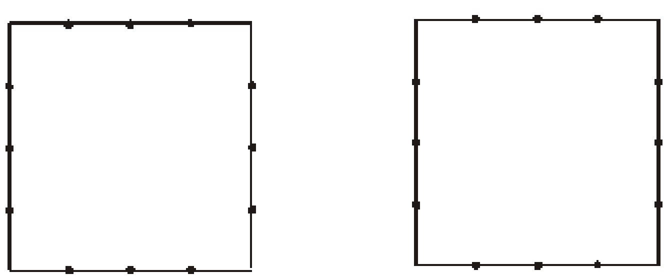
(A) (B) (C)

The shaded 4-sided figures above have been drawn with the four vertices at the dots, on each side of the square.

In the same manner,

(i) draw a 4-sided figure with the greatest possible area in (**D**),

(ii) draw a 4-sided figure with the smallest possible area in (**E**).



(D) (E)

3. Consider the following number sequence :

$$\frac{1}{2}\ ,\ \frac{3}{5}\ ,\ \frac{8}{13}\ ,\ \frac{21}{34}\ ,\ \ .\ \ .\ \ .\ \ ,\ \frac{2584}{4181}\ .$$

(i) Find the 5^{th} and 6^{th} numbers in the sequence.

(ii) How many numbers are there in the sequence ?

(iii) If this sequence continues, what is the number immediately after $\frac{2584}{4181}$?

4. There are two identical bottles A and B.

A contains $\frac{1}{2}$ bottle of pure honey.

B contains a full bottle of water.

First pour the water from B to fill up A and mix the content completely ;

then pour the mixture from A to fill up B and mix the content completely.

(i) What is the ratio of honey to water in B after the two pourings ?

(ii) If this process of pouring from A to B , and then from B to A, is repeated for another time, what will be the ratio of honey to water in B ?

(iii) If this process of pouring is repeated indefinitely, what will be the ratio of honey to water in B ?

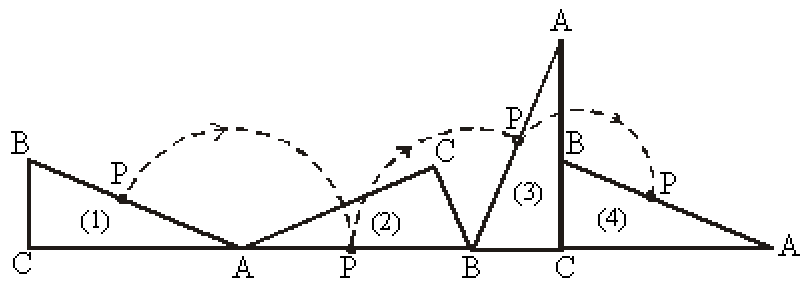
5. A right-angled triangle (1) is placed with one side lying along a straight line.

It is rotated about point A into position (2).

It is then rotated about point B into position (3).

Finally, it is rotated about point C into position (4).

Given that AP = BP = CP = 10 cm, find the total length of the path traced out by point P. (Take $\pi = 3.14$.)



6. Figure 1 shows a street network where *A, B, ..., I* are junctions. We observe that it takes <u>at most</u> 4 steps to travel from one junction to another junction.

e.g. From *A* to *I*, we may take the following 4 steps.

$$A \xrightarrow{(1)} B \xrightarrow{(2)} E \xrightarrow{(3)} H \xrightarrow{(4)} I$$

The street network is now converted to a one-way traffic system as shown in Figure 2. In this one-way traffic system, it takes <u>at most</u> 6 steps to travel from one junction to another junction.

e.g. From *A* to *I*, we may take the following 6 steps .

$$A \xrightarrow{(1)} D \xrightarrow{(2)} E \xrightarrow{(3)} B \xrightarrow{(4)} C \xrightarrow{(5)} F \xrightarrow{(6)} I$$

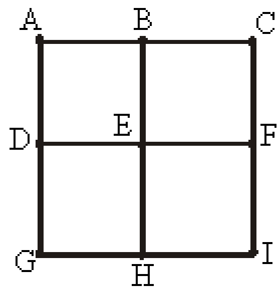


Figure 1

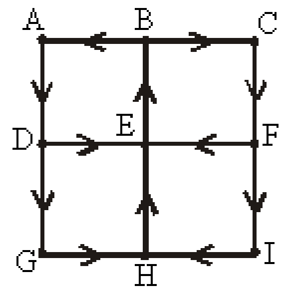


Figure 2

In Figure 3, design a one-way traffic system so that it takes **<u>at most</u>** 5 steps to travel between any two junctions.

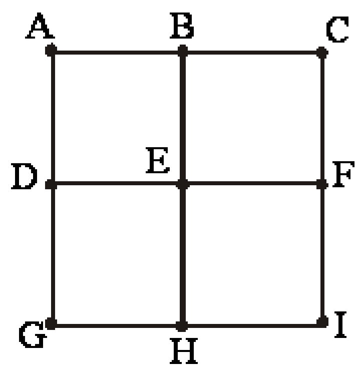


Figure 3

THE END

Singapore Mathematical Olympiad for Primary Schools 2002

Invitation Round – Answers Sheet

Question 1:

Ans: a) 25   b) 2500

Question 2:

Question 3:

Ans:

i) 5th number: $55/89$ 6th number: $144/233$

ii) $2584/4181$

iii) $6765/10946$

Question 4:

Ans:

i) 1 : 3

ii) 5 : 11

iii) 1 : 2

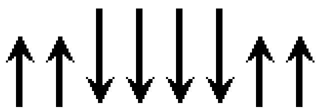
Question 5:

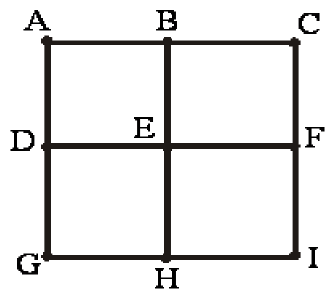
Ans: 62.8cm

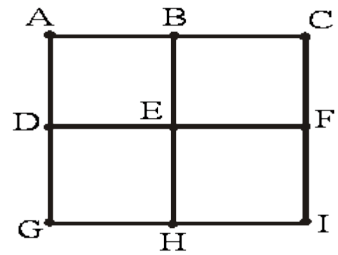
$\rightarrow \; \leftarrow \; \leftarrow \; \leftarrow \; \leftarrow \; \leftarrow \; \leftarrow \; \rightarrow$

$\rightarrow \; \rightarrow \; \rightarrow \; \rightarrow$

Question 6



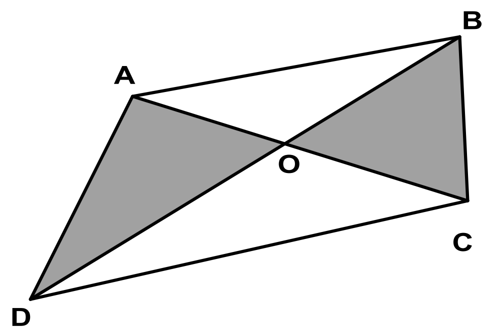

A
B
C
D
E
F
G
H
I


A
B
C
D
E
F
G
H
I

1. A student multiplies the month and the day in which he was born by 31 and 12 respectively. The sum of the two resulting products is 170.
Find the month and the date in which he was born.

2. Given that five whole numbers *a*, *b*, *c*, *d* and *e* are the ages of 5 people and that *a* is 2 times of *b*, 3 times of *c*, 4 times of *d* and 6 times of *e*, find the smallest possible value of $a+b+c+d+e$.

3. Lines *AC* and *BD* meet at point *O*.
Given that $OA = 40\ cm$, $OB = 50\ cm$, $OC = 60\ cm$ and $OD = 75\ cm$,
find the ratio of the area of triangle *AOD* to the area of triangle *BOC*.



4. 1000 kg of a chemical is stored in a container.
The chemical is made up of 99 % water and 1 % oil.
Some water is evaporated from the chemical until the water content is reduced to 96 %.
How much does the chemical weigh now?

5. A student arranges 385 identical squares to form a large rectangle without overlapping.
How many ways can he make the arrangement?

[Note: The arrangements as shown in figure (1) and figure (2) are considered the same arrangement.

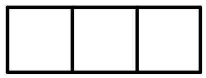


Figure (1)

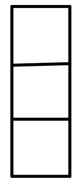


Figure (2)]

6. A bag contains identical sized balls of different colours :
10 red, 9 white, 7 yellow, 2 blue and 1 black.
Without looking into the bag, Peter takes out the balls one by one from it.
What is the least number of balls Peter must take out to ensure that at least 3 balls have the same colour?

7. Find the value of $\frac{1\times5\times18+2\times10\times36+3\times15\times54}{1\times3\times9+2\times6\times18+3\times9\times27}$.

8. If the base of a triangle is increased by 10% while its height decreased by 10%, find the area of the new triangle as a percentage of the original one.

9. A box of chocolate has gone missing from the refrigerator.

The suspects have been reduced to 4 children.

Only one of them is telling the truth.

John : " I did not take the chocolate."

Wendy : " John is lying."

Charles: " Wendy is lying."

Sally : " Wendy took the chocolate."

Who took the chocolate ?

10. How many digits are there before the hundredth 9 in the following number

9797797779777797777797777779…….?

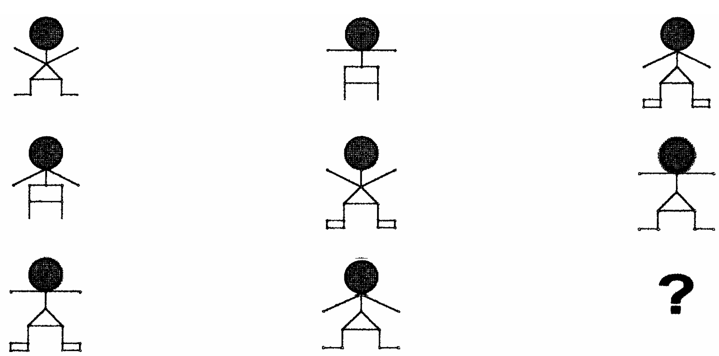
11. A particular month has 5 Tuesdays.
The first and the last day of the month are not Tuesday.
What day is the last day of the month?

12. In the following division, what is the sum of the first 2004 digits after the decimal point?

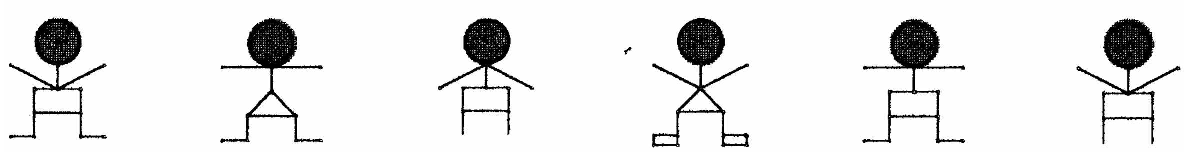
$$2004 \div 7 = 286.285714285714......$$

13. A three digit number 5ab is written 99 times as 5ab5ab5ab……..5ab.
The resultant number is a multiple of 91.
What is the three digit number?

14.

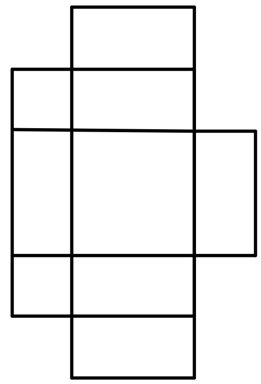


Which one of the following is the missing figure?



A B C D E F

15. How many rectangles are there in the following diagram?



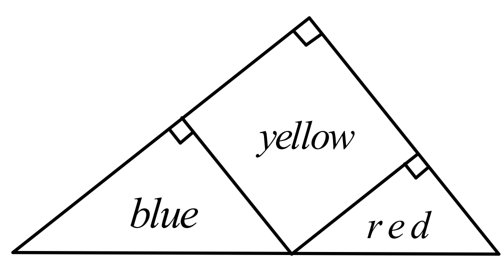
16. Placed on a table is a mathematics problem,

$$89 + 16 + 69 + 6\Delta + \square 8 + 88$$

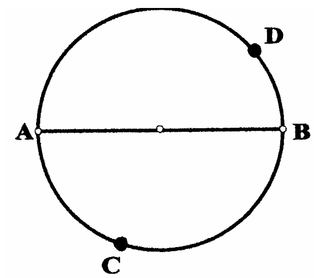
where each of the symbols Δ and $\square$ represents a digit.
Two students *A* and *B* sit on the opposite sides of the table facing each other.
They read the problem from their directions and both get the same answer.
What is their answer?

17. Find the value of $\frac{1}{4\times 9}+\frac{1}{9\times 14}+\frac{1}{14\times 19}+...+\frac{1}{1999\times 2004}$.

18. The diagram shows a right-angled triangle formed from three different coloured papers.
The red and blue coloured papers are right-angled triangles with the longest sides measuring 3 cm and 5 cm respectively.
The yellow paper is a square.
Find the total area of the red and blue coloured papers.



19. The diagram shows a circular track with **AB** as its diameter.
Betty starts walking from point **A** and David starts from point **B**.
They walk toward each other along the circular track.
They meet at point **C** which is 80 m from **A** the first time.
Then, they meet at point **D** which is 60 m from **B** the second time.
What is the circumference of the circular track?



20. A triangle, figure (1), is folded along the dotted line to obtain a figure as shown in figure (2).
The area of the triangle is 1.5 times that of the resulting figure.
Given that the total area of the three shaded regions is 1 unit^2,
find the area of the original triangle.

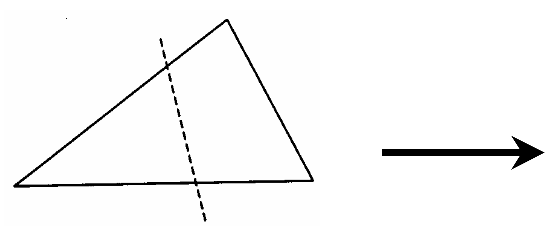


Figure (1)

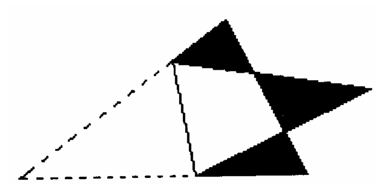
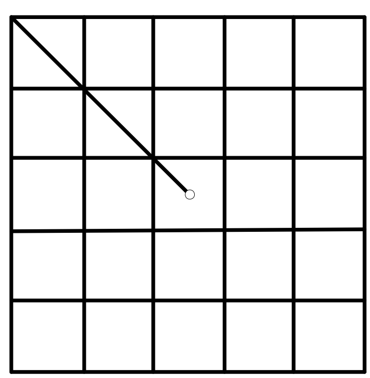


Figure (2)

21. What is the missing number in the following number sequence?

2, 2, 3, 5, 14, $\boxed{}$, 965.

22. The figure shown in the diagram below is made up of 25 identical squares.
A line is drawn from one corner of the figure to its centre.
On the answer sheet provided, show how to add in 4 more non-parallel lines so as to divide the figure into 5 equal areas.



23. There are three bowls on a table, each containing different types of fruits.
To the right of the green bowl is the banana.
To the left of the banana is the orange.
To the right of the star-fruit is the green bowl.
To the left of the white bowl is the blue bowl.
What is the colour of the bowl containing the orange?

[Note: The "right" or "left" here do not necessarily refer to the immediate right nor immediate left.]

24. At 7.00 am, a vessel contained 4000 cm^3 of water.
Water was removed from the vessel at a constant rate of 5 cm^3 per minute.
At 8.00 am, 80 cm^3 of water was added.
A further 80 cm^3 was added at the end of each hour after that.
Find the time when the vessel was empty for the first time.

25. A car travels from town **P** to town **Q** at a constant speed.
When it increases its speed by 20%,
the journey from **P** to **Q** takes 1 hour less than its usual time.
When it travels at its usual speed for 100 km before increasing its speed by 30%,
the journey also takes 1 hour less than usual.
Find the distance between the two towns.

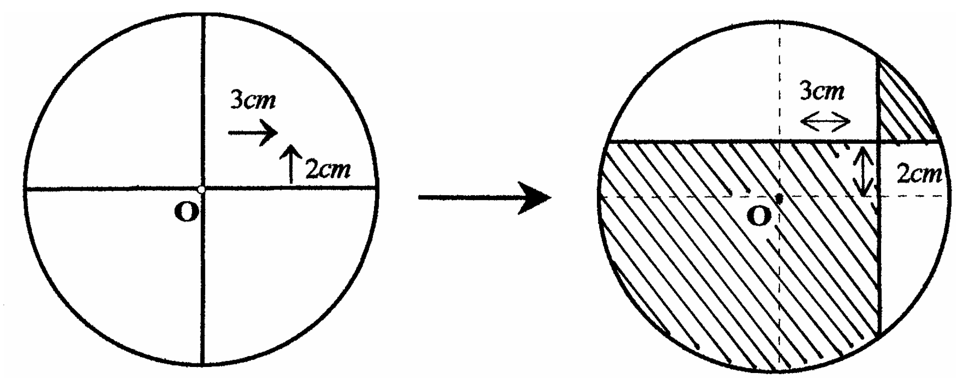
26. A piece of pasture grows at a constant rate everyday.

200 sheep will eat up the grass in 100 days.
150 sheep will eat up the grass in 150 days.

How many days does it take for 100 sheep to eat up the grass?

27. The digits 3, 4, 5 and 7 can form twenty four different four digit numbers. Find the average of these twenty four numbers.

28. The vertical diameter of a circle is shifted to the right by 3 cm and the horizontal diameter is shifted up by 2 cm as shown in the diagram below.



Find the difference between the shaded and the unshaded areas.

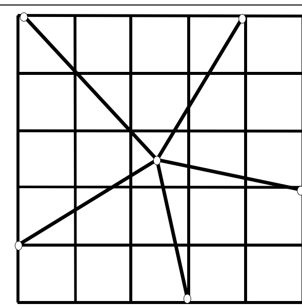
29. The Sentosa High School's telephone number is an eight digit number.
The sum of the two numbers formed from the first three digits and the last five digits respectively is 66558.
The sum of the two numbers formed from the first five digits and the last three digits is 65577.
Find the telephone number of The Sentosa High School.

30. A confectionery shop sells three types of cakes.
Each piece of chocolate and cheese cake costs \$5 and \$3 respectively.
The mini-durian cakes are sold at 3 pieces a dollar.
Mr Ng bought 100 pieces of cakes for \$100.
How many chocolate, cheese and durian cakes did he buy?
Write down all the possible answers.

THE END

Singapore Mathematical Olympiad for Primary Schools 2004

First Round – Answers Sheet

Question	Answers	For markers' use Only		Question	Answers	For markers' use Only
	Questions 1 to 10 each carries 4 marks					
1	**9th February**			18	**7.5 cm ^{2}**	
2	**27**			19	**360 m**	
3	**1 : 1**			20	**3 unit ^{2}**	
4	**250kg**				**Q11 to Q20 total Sub-Score**	
5	**4**				**Questions 21 to 30 each carries 6 marks**	
6	**10**			21	**69**	
7	$3\frac{1}{3}$			22		
8	**99%**					
9	**John**					
10	**5049**					
	Q1 to Q10 total Sub-Score			23	**Green**	
	Questions 11 to 20 each carries 5 marks			24	**00.52 am**	
11	**Wednesday**			25	**360 km**	
12	**9018**			26	**300 days**	
13	**546**			27	**5277.25**	
14	**F**			28	**24 cm^2**	
15	**30**			29	**64665912**	
16	**421**			30	**Chocolate : 4, 8, 12** **Cheese : 18, 11, 4** **Mini-durian : 78, 81, 84**	2 marks for each set of correct answer e.g. (4,18,78)
17	$\frac{25}{501}$				**Q21 to Q30 total Sub-Score**	

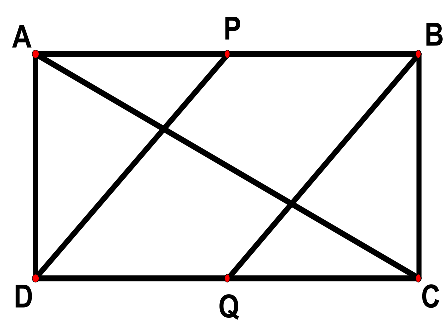
1 There is a triangle and a circle on a plane. Find the greatest number of regions that the plane can be divided into by the triangle and the circle.

---

2 The diagram shows a rectangle **ABCD**.

P and **Q** are the midpoints of **AB** and **CD** respectively.

What fraction of the rectangle is shaded?



3 There is a glass of water and a glass of wine. A small amount of water is poured from the glass containing water into the glass containing wine. Then an equal amount of the wine - water mixture is poured back into the glass containing water.

Which one of the following statements is correct?

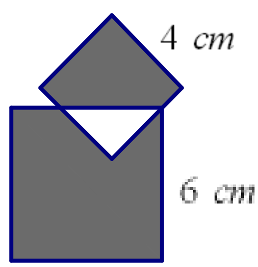
(A) There is now more water in the wine than there is wine in the water.

(B) There is now less water in the wine than there is wine in the water.

(C) There is now an equal amount of water in the wine as there is wine in the water.

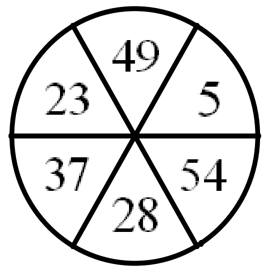
(D) It is uncertain whether there is more or less water in the wine than wine in the water.

4 Two squares, with lengths **4** cm and **6** cm respectively, are partially overlapped as shown in the diagram below. What is the difference between shaded area **A** and shaded area **B**?



5 The diagram shows a dartboard.

What is the least number of throws needed in order to get a score of exactly 100?



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6 Find the value of $1-\frac{5}{6}+\frac{7}{12}-\frac{9}{20}+\frac{11}{30}-\frac{13}{42}+\frac{15}{56}-\frac{17}{72}+\frac{19}{90}$.

7 Julia and Timothy each has a sum of money.

Julia's amount is $\frac{3}{5}$ that of Timothy's.

If Timothy were to give Julia $168, then his remaining amount will be $\frac{7}{9}$ that of Julia's.

How much does Julia have originally?

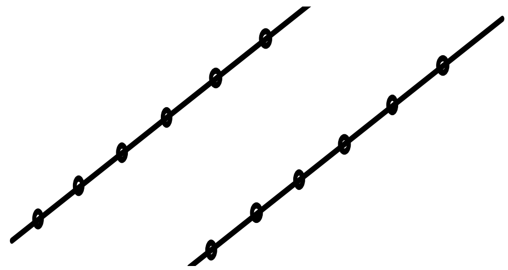
---

8 When a number (the dividend) is divided by another number (the divisor), the quotient is 4 and the remainder is 8. Given that the sum of the dividend, the divisor, the quotient and the remainder is 415, find the dividend.

[*Note* : When 17 is divided by 3, the quotient is 5 and the remainder is 2 .]

9 Six different points are marked on each of two parallel lines.

Find the number of different triangles which may be formed using 3 of the 12 points.



10 How long, in hours, after 12 noon, will it take for the hour hand of a clock to lie directly over its minute hand for the first time?

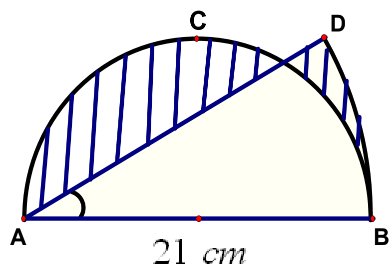
11 The lengths of two rectangles are 11 m and 19 m respectively.
Given that their total area is 321 m^2, find the difference in their areas.
[*Note* : Both their widths are whole numbers.)

12 Lala speaks the truth only on Monday, Wednesday, Friday and Sunday.
Nana speaks the truth only on Monday, Tuesday, Wednesday and Thursday.
Find the day when both said “Yesterday I lied”.

13 The diagram shows a semicircle **ACB** with diameter **AB** = 21 cm.

Angle **DAB** = 30º and arc **DB** is part of another circle with centre **A**.

Find the perimeter of the shaded region, using π as $\frac{22}{7}$.



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14 Find the value of $20042005 \times 20052004 - 20042004 \times 20052005$.

15 In how many ways can $\frac{7}{12}$ be written as a sum of two fractions in lowest term given that the denominators of the two fractions are different and are each not more than 12?

---

16 There are 72 students in Grade 6.
Each of them paid an equal amount of money for their mathematics text books.
The total amount collected is $\$\square 3 5. 0 \square$, where two of the digits indicated with $\square$ cannot be recognized.
How much money did **each** student pay for the books?

17 Numbers such as **1001, 23432, 897798, 3456543** are known as palindromes.
If **all** of the digits **2**, **7**, **0** and **4** are used and each digit cannot be used more than twice, find the number of different palindromes that can be formed.

---

18 There are three classes **A**, **B** and **C**.
Class **A** has 2 more students than class **B**.
Class **B** has 1 more student than class **C**.
The product of the numbers of students in the three classes is **99360**.
How many students are there in class **A**?

19 Find the value of

$$1 + 2 + 2^2 + 2^3 \ldots\ldots + 2^{11}.$$

[*Note* : $2^2 = 2 \times 2$, $2^3 = 2 \times 2 \times 2$]

---

20 Three boys **Abel**, **Ben** and **Chris** participated in a 100 m race.

When Abel crossed the 100 m mark, Ben was at 90 m.

When Ben crossed the 100 m mark, Chris was at 90 m.

By how many metres did Abel beat Chris?

[*Note* : They were running at constant speeds throughout the race.]

21 There are 5 students. Each time, two students are weighed, giving a total of 10 readings, in kilograms, as listed below :

103, 115, 116, 117, 118, 124, 125, 130, 137, 139.

What is the weight of the **third heaviest** student?

---

22 One pan can fry 2 pieces of meat at one time. Every piece of meat needs two minutes to be cooked (one minute for each side). Using only one pan, find the least possible time required to cook

(i) **2000**,

(ii) **2005** pieces of meats.

23 The distance between **A** and **B** is 7 km. At the beginning, both **Gregory** and **Catherine** are at **A**. Gregory walked from **A** to **B** at a constant speed. After Gregory had walked for 1 km, Catherine discovered that he had left his belongings at **A.** She immediately ran after him at a constant speed of 4 km/h. After handing over the belongings to Gregory, she turned back and started running towards **A** at the same speed. Given that both Catherine and Gregory reached **A** and **B** respectively at the same time, what was Gregory's speed?

---

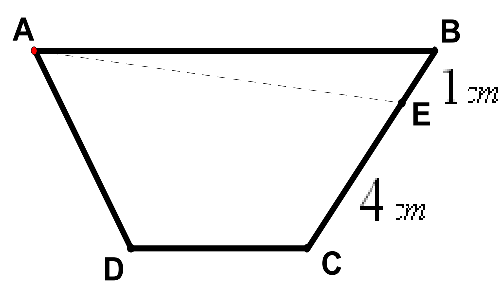
24 The diagram shows a trapezium **ABCD**.

E is a point on **BC** such that BE = 1 cm and **EC** = 4 cm.

AE divides **ABCD** into two parts.

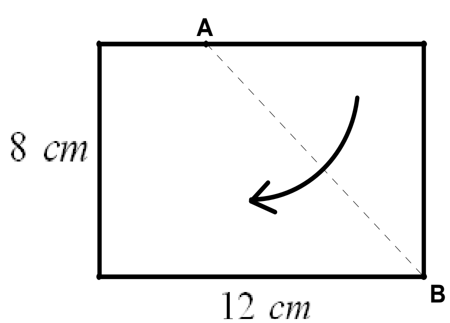
The areas of the two parts are in the ratio 1 : 6.

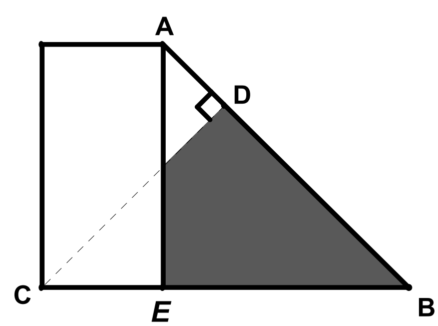
Find the ratio of the lengths **AB** : **DC.**



25 David's father picks him up from school every evening at 6 pm. One day, David was dismissed early at 5 pm. He walked home taking the same route that his father usually drives. When he met his father along the way he boarded the car and returned home 50 minutes earlier than usual. Given that his father drove at a constant speed and planned to reach the school at 6 pm sharp, how long, in minutes, had he walked before he was picked up by his father?

26 A piece of rectangular paper measuring 12 cm by 8 cm is folded along the dotted line **AB** to form the figure on the right. Given that **CD** is perpendicular to **AB**, find the area of the shaded region.





27 Four girls need to cross a dark narrow tunnel and they have only a torch among them.

Girl **A** can cross the tunnel in 1 minute.
Girl **B** can cross the tunnel in 2 minutes.
Girl **C** can cross the tunnel in 5 minutes.
Girl **D** can cross the tunnel in 10 minutes.

Given that they need a torch to cross at all times and that the tunnel can only allow two girls to go through at any given time, find the least possible time for the four girls to get across the tunnel.

[*Note* : The time taken by the slower girl is taken to be the time of each crossing.]

---

28 Which of the following is **true**?

(A) $\frac{10}{13} > \frac{11}{14}$,

(B) $\frac{4567}{6789} > \frac{3456}{5678}$,

(C) $\frac{12}{19} > \frac{20}{31}$,

(D) $\frac{111}{1111} > \frac{1111}{11111}$.

29 Mrs Wong is preparing gifts for the coming party.

She buys 540 notepads, 720 pens and 900 pencils.

The total cost of each type of stationery is equal.

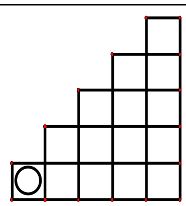
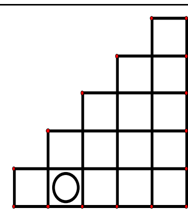
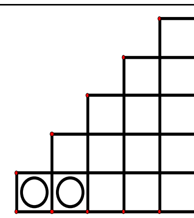
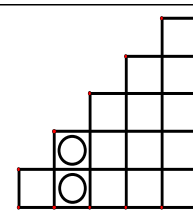
She divides the notepads equally in red gift boxes, pens equally in yellow gift boxes and pencils equally in blue gift boxes.

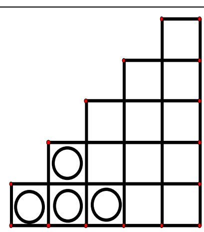
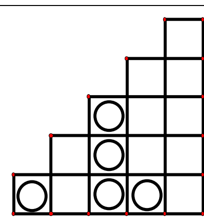
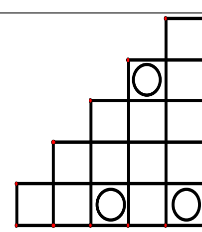
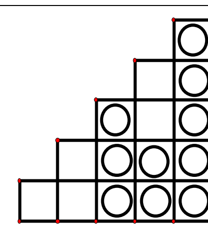
She wishes that the cost of each gift box is equal and as low as possible.

Find the **number of notepads** she needs to put in **each red gift box**.

---

30 Find the value of x in figure **H**.

A	B	C	D
			
1	2	3	4

E	F	G	H
			
11	43	150	x

END OF PAPER

Singapore Mathematical Olympiad for Primary Schools 2005
First Round – Answers Sheet

Question	Answers	For markers' use Only	Question	Answers	For markers' use Only
	Questions 1 to 10		17	36	
	each carries 4 marks		18	48	
1	8		19	4095	
2	$\frac{1}{4}$		20	19 m	
3	(c)			Q11 to Q20	
4	20 cm^2			total Sub-Score	
5	3			Questions 21 to 30	
6	$\frac{3}{5}$			each carries 6 marks	
7	$336		21	64 kg	
8	324		22	(i) 2000 mins	Both must
9	180				be correct to
10	$1\frac{1}{11}$ hours			(ii) 2005 mins	score 6 marks
	Q1 to Q10		23	3 km/h	
	total Sub-Score		24	5 : 2	
	Questions 11 to 20		25	35 mins	
	each carries 5 marks		26	28 cm^2	
11	211 m^2		27	17 mins	
12	Friday		28	(B)	
13	65 cm		29	3	
14	10000		30	x = 666	
15	3 ways			Q21 to Q30	
16	$8.82			total Sub-Score	

1 **Paul** and **Samuel** are playing the same computer game on their own computers. At the start of the game, there are a certain number of targets on the screen. The targets will disappear from the screen one by one at a constant rate. **Paul** can shoot down the targets twice as fast as **Samuel**. Given that at the end of the game, **Paul** shot down 54 targets while **Samuel** shot down 36 targets. Find the number of targets at the start of the game.

[Omission of essential working will result in loss of marks]

2 How many whole numbers from **1** to **1000** can be expressed as the difference of the squares of two whole numbers? [*Note* : 0 is a whole number.]

[Omission of essential working will result in loss of marks]

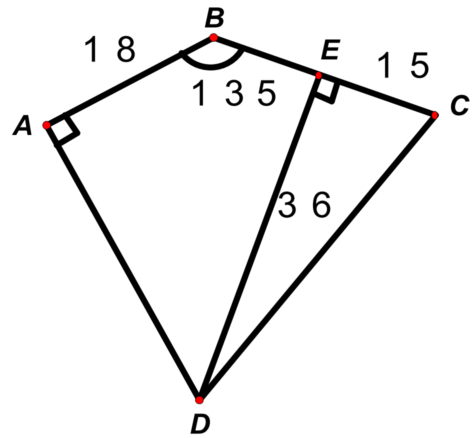
3 The following number is made up of all the digits of the whole numbers **1** to **2005**.

12345678910111213141516……………20042005

Find the number of zeros in this number.

[Omission of essential working will result in loss of marks]

4 **ABCD** is a quadrilateral (4 sided figure). ∠**ABC** = 135º, ∠**BAD** = ∠**CED** = 90º, **AB** = 18 cm, **CE** = 15 cm and **DE** = 36 cm. Find the area of the quadrilateral **ABCD**.



[Omission of essential working will result in loss of marks]

5 Five classes **A**, **B**, **C**, **D** and **E** took part in an international chess competition.

Each class sent in 2 participants.

The rules of the competition were :

(a) Participants do not compete with each other for more than 1 game,

(b) Participants from the same class cannot compete with each other.

After a few rounds of competitions, it turned out that all had completed different number of games, except for a participant from class **A**.

How many games had the **2 participants** from class **A** completed?

[Omission of essential working will result in loss of marks]

6 **Allen, Benedict and Carl** started at the same instant from the same point using the same route trying to overtake a fourth cyclist **Donald** traveling at a constant speed ahead of them. **Allen** and **Benedict** each took 10 hours and 2 hours respectively to overtake **Donald**.

Given that **Allen**, **Benedict** and **Carl** each cycled at the constant speed of 4 km/h, 5 km/h and 10 km/h respectively throughout the journey, find the time, in hours, that **Carl** took to overtake **Donald**.

[Omission of essential working will result in loss of marks]

END OF PAPER

Name of Participant : ______________________________ (Statutory Name)

Index No : ________ / _____ Name of School : ______________________________

Date Of Birth : ______________________________ (DD/MM/YY)

19 April 2008 **0900 h — 1100 h**

Hwa Chong Institution
Mathematics Learning And Research Centre

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Singapore Mathematical Olympiad for Primary Schools 2008

First Round
2 hours
(150 marks)

Instructions to Participants :

Attempt as many questions as you can.
Neither mathematical tables nor calculators may be used.
Shade your answers on the OMR form.
Working may be done in the space below each question.
Marks are awarded for correct answers only.

This question paper consists of 16 printed pages (including this page)

Number of correct answers for Q1 to Q10 : ____________ *Marks (x 4) :* ____________
Number of correct answers for Q11 to Q20 : ____________ *Marks (x 5) :* ____________
Number of correct answers for Q21 to Q30 : ____________ *Marks (x 6) :* ____________

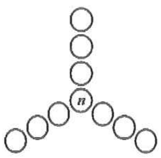
1 What is the difference between the sum of the first 2008 even numbers and the sum of the first 2008 odd numbers?

2 The sides of a triangle have lengths that are consecutive whole numbers and its perimeter is greater than 2008 cm. If the least possible perimeter of the triangle is x cm, find the value of x.

3 Find the value of 2008 x 20072007 – 2007 x 20082007.

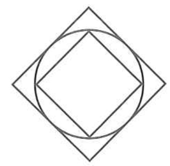
4 When a rectangular sheet of paper with length 8 cm is folded exactly into half, the ratio of its length to its width remains unchanged. If the square of the width of the original piece of paper = x cm^2, find the value of x.

5 The numbers 1 to 10 are arranged in the circles in such a way that the sum of the four numbers on each line is 21. What is the value if n?



6 Find the value of (56789 + 67895 + 78956 + 89567 + 95678) ÷ 5

7 The diagram shows a circle whose circumference touches the sides and the vertices of a large and a small square respectively. If the area of the small square is 9 cm^2 and the area of the large square is x cm^2, find the value of x.



8 One hundred numbers are placed along the circumference of a circle. When any five adjacent numbers are added, the total is always 40. Find the difference between the largest and the smallest of these numbers.

9 In triangle PQR, PQ = 6 cm, PR = 4 cm and QR = 6 cm. If sides PQ and PR are tripled while QR remains unchanged, then

(1) the area is tripled.

(2) the area increases by 9 times.

(3) the altitude is tripled.

(4) the area decreases to 0 cm^2.

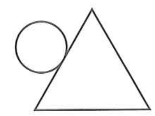
(5) none of the above.

10 Find the last 5 digits of the sum

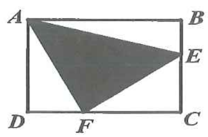
$1 + 22 + 333 + 4444 + 55555 + 666666 + 7777777 + 88888888 + 999999999$.

11 If an arc of 80° on circle *A* has the same length as an arc of 60° on circle *B*, and that the ratio of the area of circle *A* to the area of circle *B* is a : b, find the smallest value of a + b.

12 A circle of circumference 1 m rolls around the equilateral triangle of perimeter 3 m. How many turns does the circle make as it rolls around the triangle once without slipping?



13 The diagram shows a rectangle *ABCD* with area 32 cm^2. Given that area of triangle *ADF* = 2 cm^2, area of triangle *ABE* = 8 cm^2 and area of the shaded region = x cm^2, find the value of x.



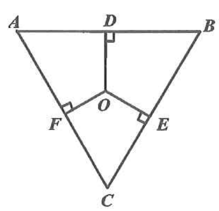
14 There are two containers A and B. Each of them contains 9 white marbles, 9 black marbles and 9 red marbles. If 10 marbles are removed from A and placed into B, how many marbles must be returned from B to A to make sure that there are at least 8 marbles of each colour in A?

15 9^{10} is a 10-digit number. If **A** is the sum of all digits of 9^{10}, **B** is the sum of all digits of **A** and **C** is the sum of all digits of **B**, find the value of **C**.

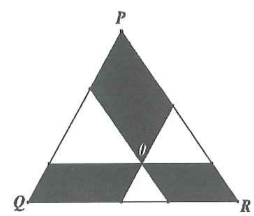
16 A car travels from point A to B at a constant speed of V km/h. If the car increases its speed by 20%, it will reach B one hour earlier. If the car increases its speed by 25% after traveling at V km/h for 120 km, it will reach B forty eight minutes earlier. If the distance between the two towns is x km, find the value of x.

17 After all the faces of a rectangular block are painted green, the block is cut into unit cubes each of volume 1 cm^3. It is found that 7 of the unit cubes have none of their faces painted green. How many of the unit cubes have exactly two faces painted green?

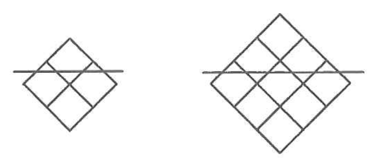
18 The diagram shows an equilateral triangle ABC with OD, OE and OF perpendicular to AB, BC and CA respectively. If $OD + OE + OF = 28$ cm and the height of triangle $ABC = x$ cm, find the value of x.



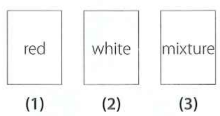
19 The diagram shows a triangle PQR. Three lines parallel to the sides of the triangle are drawn through a point O. Given that the areas of the three shaded triangles are 32 cm^2, 48 cm^2 and 96 cm^2 respectively, and the area of the triangle $PQR = x$ cm^2, find the value of x.



20 As shown in the diagram, a straight line can cut across at most 3 squares in a 2 by 2 square and at most 5 squares in a 3 by 3 square. What is the greatest number of squares that can be cut across by a straight line in a 2008 by 2008 square?

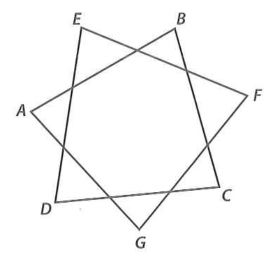


21 There are three containers. One contains red marbles, another white marbles and the third one a mixture of red and white marbles. Given that all of them are labelled wrongly and you are allowed to open only one of them to take out only one marble in order to state correctly where all the labels ought to go, which container should you open?



22 Given that $\underbrace{200820082008\ldots2008}_{n \text{ of } 2008}623$, find the smallest value of n such that the number is divisble by 11.

23 Given that $\angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFG + \angle FGA + \angle GAB = x^0$, find the value of x.



24 Peter and Jane are to take turns to subtract perfect squares from a given whole number and the one who reaches zero first is the winner. If the whole number is 29, and Peter is the first player, what perfect number must he subtract in order for him to definitely win.

[*Note: 4, 9 and 16 are examples of perfect squares.*]

25 The inhabitants of an island are either gentlemen or liars. A gentleman always tells the truth and a liar always lies. *A*, *B* and *C* are three of the inhabitants. A sailor who landed on the island asked *A*: "Are you a gentleman or a liar?" *A* answered but the sailor could not hear clearly what he said. He then asked *B*, "What did *A* say?" *B* replied, "*A* said that he is a liar." At that instant, *C* immediately shouted "*B* is lying!"

I It is impossible to tell whether *A* is a gentlemen or a liar.
II *B* is a gentlemen and *C* is a liar.
III *B* is a liar and *C* is a gentlemen.

(1) Only I is true.
(2) Only II is true.
(3) Only III is true.
(4) Only I and II are true.
(5) Only I and III are true.

26 The product of n whole numbers 1 x 2 x 3 x 4 x 5 x … x (n – 1) x n has twenty eight consecutive zeros. Find the largest value of n.

27 Find the largest number n such that there is only one whole number k that satisfies

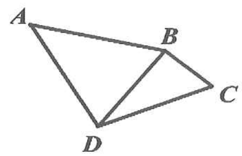
$$\frac{8}{21} < \frac{n}{n+k} < \frac{5}{13}$$

[Note: A < C < B means that value of C is between A and B, example 4 < 9 < 16]

28 How many ways are there to distribute 28 identical marbles into 3 different boxes such that no box is empty?

29 If Peter walks up an up-going escalator at the rate of 1 step per second, he is able to reach the top in 10 steps. If he increases his rate to 2 steps per second, he can reach the top in 16 steps. Find the number of steps of the escalator.

30 The diagram shows a quadrilateral ABCD. If AB = CD, angle ADB + angle CBD = 180°, angle BCD = 55° and angle BAD = x°, find the value of x.



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Singapore Mathematical Olympiad for Primary Schools 2008 First Round – Answers Keys					Total Marks 150
Question	**Answers**	**For markers' use only**	**Question**	**Answers**	**For markers' use only**
	Questions 1 to 10 Each carries 4 marks		16	360	
1	2008		17	36	
2	2010		18	28	
3	2007		19	288	
4	32		20	4015	
5	4			Questions 21 to 30 Each carries 6 marks	
6	77777		21	3	
7	18		22	3	
8	0		23	540	
9	4		24	9	
10	93685		25	5	
	Questions 11 to 20 Each carries 5 marks		26	124	
11	25		27	80	
12	4		28	351	
13	15		29	40	
14	27		30	55	
15	9				

FULL SOLUTIONS TO SELECTED QUESTIONS (FIRST ROUND)

1 2008
Hint : Consider the difference between the sum of the first 4 even numbers and the sum of the first 4 odd numbers.

2 $(a) + (a + 1) + (a + 2) > 2008 \Rightarrow 3a + 3 > 2008 \Rightarrow a > 668\frac{1}{3}$

The three sides are 669, 670,671, the least possible perimeter = 2010.

3 Let $x = 2007$
$2008 \times 20072007 - 2007 \times 20082007$
$= (x + 1) \times (10001x) - x (1000 (x + 1) + x)$
$= x$
$= 2007$

4 *Let length* $= 8 = 2L$, *width* $= w$
$\frac{length}{width} = \frac{2L}{w} = \frac{w}{L} \Rightarrow w^2 = 2L^2 = 2(4)^2 = 32$

5 4

6 $(56789 + 67895 + 78956 + 89567 + 95678) \div 5$
$= (350000 + 35000 + 3500 + 350 + 35) \div 5$
$= 77777$

7 Rotate the inner smaller square about the centre by 90 degrees, you will notice that the size of the small square is exactly half that of the large square.
Therefore area of the large square is 18 cm^2 .

8 $n_1 + n_2 + n_3 + n_4 + n_5 = 40$
$n_2 + n_3 + n_4 + n_5 + n_6 = 40$
$\Rightarrow n_1 = n_6 = n_{11} = \ldots = n_{886}$
$n_{885} + n_{886} + n_{887} + n_{888} + n_1 = 40$
$n_{886} + n_{887} + n_{888} + n_1 + n_2 = 40$
$\therefore n_1 = n_2$
They are all 8s. Therefore answer is 0.

9 Option (4).
In the new triangle, PQ = PR + QR, that is R lies on the line PQ.
The height from R is zero, therefore area of the triangle is zero.

10 1,000,000,000 (= 1 + 999,999,999)
88, 888,910 (= 22 + 88,888,888)
7,778,110 (= 333 + 7,777,777)
671,110 (= 4,444 + 666,666)
55,555
1097<u>393685</u> (Total)

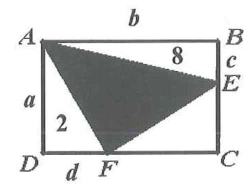
11 $\frac{80}{360} \times 2 \times \pi \times r_A = \frac{60}{360} \times 2 \times \pi \times r_B \Rightarrow \frac{r_A}{r_B} = \frac{60}{80} = \frac{3}{4}$

$\frac{a}{b} = \frac{\pi r_A^2}{\pi r_A^2} = \frac{9}{16}$

$\therefore a + b = 25$

12 Four turns. One -third turn at each of the vertices plus one turn for each side of the triangle.

13



Area of triangle CEF $= \frac{1}{2}(a - c)(b - d)$

$= \frac{1}{2}(ab - ad - bc + cd)$

$= \frac{1}{2}(32 - 4 - 16 + \frac{4}{a} \times \frac{16}{b})$

$= 7$

Area of shaded region $= 32 - 2 - 8 - 7 = 15\text{ cm}^2$

14 27

15 The sum of all the digits of a multiple of 9 is also a multiple of 9. Therefore **A** must be equal or less than 9 x 10 = 90, **B** must be equal or less than 8 + 9 = 17, **C** must be 9.

16 360

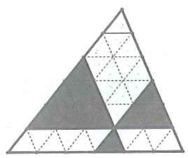
17 36

18 $\frac{1}{2} \times b \times h = \frac{1}{2} \times b \times OD + \frac{1}{2} \times b \times OE + \frac{1}{2} \times b \times OF$

$= \frac{1}{2} \times b \times (OD + OE + OF)$

Therefore $h = (OD + OE + OF) = 28$.

19



Therefore area of the triangle $PQR = 288\text{ cm}^2$.

20 For a 1 by 1 square, maximum number of squares that can be crossed
$= 1 \times 2 - 1 = 1$

For a 2 by 2 square, maximum number of squares that can be crossed
$= 2 \times 2 - 1 = 3$

For a 3 by 3 square, maximum number of squares that can be crossed
$= 3 \times 2 - 1 = 5$

For a 2008 by 2008 square, maximum number of squares that can be crossed
$= 2008 \times 2 - 1 = 4015$

21 The container labeled "mixture", option (3).
If a red marble is selected from the "mixture" container, since it is wrongly labelled, this container must contain only red marbles. The one labelled "white" must contain a mixture of red and white marbles while the one labelled "red" must contain only white marbles.
The same argument applies if a white marble is selected from the container labelled "mixture".

22 $n = 3$
Sum of even digits $= 8n + 2$
Sum of odd digits $= 2n + 9$, Difference $= 6n - 7$,
smallest $n = 3$ for the difference to be divisible by 11.

23 $7 \times 180^0 - 2 \times 360^0 = 540^0$

24 9

25 **Option (5)**.
If A is a gentleman, A would tell the truth and say "I am a gentleman".
If A is a liar, A would lie and say " I am a gentleman".
Regardless of what A is, he could not have said "I am a liar."
Hence B is lying and C spoke the truth.
Lastly, it is impossible to tell what A is.

26 When 124 is divided by 5, we get quotient 24, remainder 4.
When 124 is divided by 25, we get quotient 4, remainder 24.
The largest value of n is 124.

27 Can simplify the inequality into $64n < 40k < 65n$.
The interval $64n$ to $65n$ contains $(n - 1)$ whole numbers.
If $n - 1 \geq 2 \times 40 = 80$, it will contain at least two multiples of 40.
Therefore if $n = 80$, we will get largest value of n for which there is only one multiple of 40.
Therefore $64 \times 80 < 40k < 65 \times 80$ which implies $128 \times 40 < 129 \times 40 < 130 \times 40$.
Hence $k = 129$.

28 Consider

1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1=28

If we mark two of the positive signs, we can divide the left hand side into 3 parts. For example

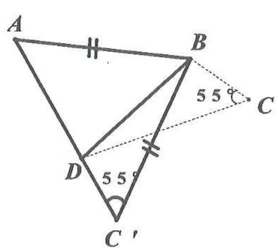
1+1+1+1⁎1+1+1+1+1+1+1⁎1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1=28

means that there are 4 , 7 and 17 marbles in boxes A, B and C respectively.

Now there are 27 ways to choose the first positive sign to mark.
Then we are left with 26 ways to choose the second positive sign to mark.
Since order is not important, there are $(27 \times 26) \div 2 = 351$ ways

29 Let the rate of the escalator be r steps per second..
In the 1st scenario, Peter covers **10 steps** in 10 seconds while the escalator covers **10*r* steps**.
In the 2nd scenario, Peter covers **16 steps** in 8 seconds while the escalator covers **8*r* steps**.
Number of steps of the escalator $= 10 + 10r = 16 + 8r$, we get $r = 3$.
Therefore the number of steps of the escalator $= 10 + 10(3) = 40$.

30 Reflecting triangle *BCD* in the line perpendicular to *BD* will give rise to the figure below. It is clear that triangle *ABC'* is isosceles and hence $\angle BAD$ is 55°.



1 Given that the product of two whole numbers $m \times n$ is a prime number, and the value of m is smaller than n, find the value of m.

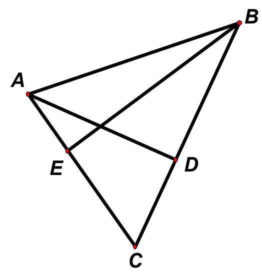
2 Given that $(2009 \times n - 2009) \div (2008 \times 2009 - 2006 \times 2007) = 0$, find the value of n.

3 Find the missing number x in the following number sequence.

2, 9, $-$18, $-$11, x, 29, $-$58, $-$51,…

4 Jane has 9 boxes with 9 accompanying keys. Each box can only be opened by its accompanying key.
If the 9 keys have been mixed up, find the maximum number of attempts Jane must make before she can open all the boxes

5 The diagram shows a triangle *ABC* with *AC* = 18 cm and *BC* = 24 cm. *D* lies on *BC* such that *AD* is perpendicular to *BC*. *E* lies on *AC* such that *BE* is perpendicular to *AC*.
Given that *BE* = 20 cm and *AD* = x cm, find the value of x.



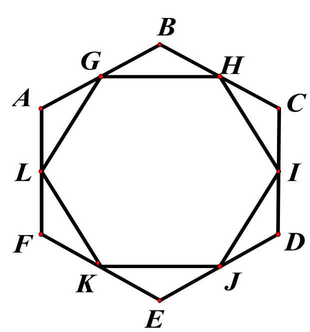
6 A language school has 100 pupils in which 69% of the pupils study French, 79% study German, 89% study Japanese and 99% study English. Given that at least x % of the students study all four languages, find the value of x.

7 Find the value of x.

89	35	9	1
35	x	7	1
9	7	5	1
1	1	1	3

8 Given that $9 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 + n_8 + n_9$ where $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$ and n_9 are consecutive numbers, find the value of the product $n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6 \times n_7 \times n_8 \times n_9$.

9 The diagram shows a regular 6-sided figure ***ABCDEF***. *G,H, I, J, K* and *L* are mid-points of *AB, BC*, *CD, DE, EF* and *FA* respectively. Given that the area of ***ABCDEF*** is 100 cm^2 and the area of ***GHIJKL*** is x cm^2, find the value of x.

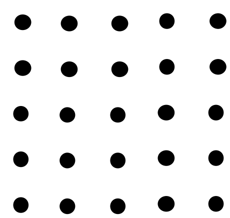


10 Three pupils A, B and C are asked to write down the height of a child, the circumference of a circle, the volume of a cup and the weight of a ball. Their responses are tabulated below :

Pupil	Height of the child (cm)	Circumference of the circle (cm)	Volume of the cup (cm^3)	Weight of the ball (g)
A	90	22	250	510
B	70	21	245	510
C	80	22	250	520

If each pupil has only two correct responses, and the height of the child is x cm, find the value of x.

11 The diagram shows a square grid comprising 25 dots. A circle is attached to the grid.
Find the largest possible number of dots the circle can pass through.



12 Jane and Peter competed in a 100 m race. When Peter crossed the finishing line, Jane just crossed the 90 m mark. If Peter were to start 10 m behind the starting line, the distance between them when one of them crosses the finishing line is x m. Find the value of x.

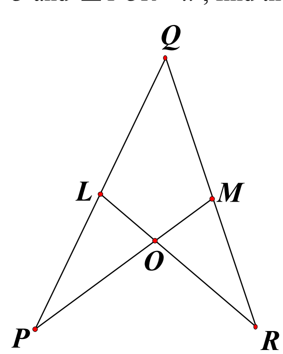
13 Given that $(1+2+3+4+5+4+3+2+1)\times(123454321)=x^2$, find the value of x.

14 A $5\times5\times5$ cube is to be assembled using only $1\times1\times1$ cuboid(s) and $1\times1\times2$ cuboid(s).
Find the maximum number of $1\times1\times2$ cuboid(s) required to build this $5\times5\times5$ cube.

15 Given that $(n_1)^2+(2n_2)^2+(3n_3)^2+(4n_4)^2+(5n_5)^2+(6n_6)^2+(7n_7)^2+(8n_8)^2+(9n_9)^2=285$,
find the value of $n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9$ if n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8 and n_9 are non-zero whole numbers.

16 A circle and a square have the same perimeter. Which of the following statement is true?
(1) Their areas are the same.
(2) The area of the circle is four times the area of the square.
(3) The area of the circle is greater than that of the square.
(4) The area of the circle is smaller than that of the square.
(5) None of the above.

17 As shown in the diagram, the points L and M lie on PQ and QR respectively. O is the point of intersection of the lines LR and PM. Given that $MP = MQ$, $LQ = LR$, $PL = PO$ and $\angle POR = x°$, find the value of x.

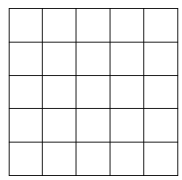


18 Given that the value of the sum $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ lies between $\frac{28}{29}$ and 1, find the smallest possible value of $a+b+c$ where *a*, *b* and *c* are whole numbers.

19 Jane has nine 1 cm long sticks, six 2 cm long sticks and three 4 cm long sticks. Given that Jane has to use all the sticks to make a single rectangle, how many rectangles with different dimensions can she make?
(1) Three **(2)** Two **(3)** One **(4)** Zero **(5)** None of the above.

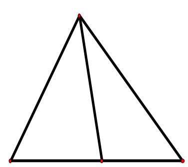
20 Peter wants to cut a 63 cm long string into smaller segments so that one or more of the segments add up to whole numbers in centimetres from 1 to 63. Find the least number of cuts he must make.

21 The diagram shows a 5 by 5 square comprising twenty five unit squares. Find the **least number** of unit squares to be shaded so that any 3 by 3 square has **exactly four** unit squares shaded.



22 Peter and Jane were each given a candle. Jane's candle was 3 cm shorter than Peter's and each candle burned at a different rate. Peter and Jane lit their candles at 7 pm and 9 pm respectively. Both candles burned down to the same height at 10 pm. Jane's candle burned out after another 4 hours and Peter's candle burned out after another 6 hours. Given that the height of Peter's candle at the beginning was x cm, find the value of x.

23 Three straight lines can form a maximum of one triangle.
Four straight lines can form a maximum of two non-overlapping triangles as shown below.
Five straight lines can form a maximum of five non-overlapping triangles.
Six straight lines can form a maximum of x non-overlapping triangles.
Find the value of x.

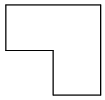
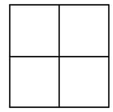


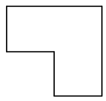
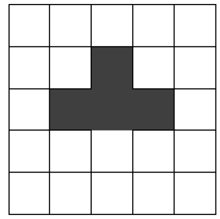
24 Given that $N=\underbrace{2\times2\times2\times\ldots\times2}_{2009}\times\underbrace{5\times5\times5\times\ldots\times5}_{2000}$, find the number of digits in *N*.

25 Jane and Peter are queueing up in a single line to buy food at the canteen. There are x persons behind Jane and there are y persons in front of Peter. Jane is z persons in front of Peter.
The number of people in the queue is _______ persons.

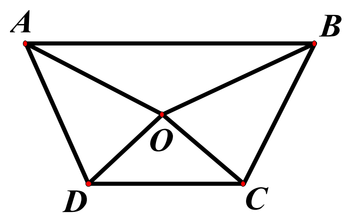
(1) $-x+y+z-1$ **(2)** $x+y-z+1$ **(3)** $-x+y+z$

(4) $-x+y+z+2$ **(5)** $x+y-z$

26 There are 4 ways to select  from .

Find the number of ways to select  from .

27 The diagram shows a trapezium *ABCD*. The length of *AB* is $2\frac{1}{2}$ times that of *CD* and the areas of triangles *OAB* and *OCD* are 20 cm^2 and 14 cm^2 respectively.
Given that the area of the trapezium is x cm^2, find the value of x.



28 Given that $\dfrac{1}{10+\dfrac{1}{9+\dfrac{1}{9}}}+\dfrac{1}{a+\dfrac{1}{b+\dfrac{1}{b+\dfrac{1}{b}}}}=1$ where a and b are whole numbers, find the value of $a+b$.

29 A shop sells dark and white chocolates in three different types of packaging as shown in the table.

	Number of Dark Chocolate	***Number of White Chocolate***
Package *A*	9	3
Package *B*	9	6
Package *C*	6	0

Mr Tan bought a total of 36 packages which consisted of 288 pieces of dark chocolates and 105 pieces of white chocolates.
How many packages of type ***A*** did he buy?

30 There are buses travelling to and fro between Station *A* and Station *B*. The buses leave the stations at regular interval and a bus will meet another bus coming in the opposite direction every 6 minutes.
Peter starts cycling from *A* towards *B* at the same time Jane starts cycling from *B* towards *A*.
Peter and Jane will meet a bus coming in the opposite direction every 7 and 8 minutes respectively.
After 56 minutes of cycling on the road, they meet each other.
Find the time taken by a bus to travel from *A* to *B*.

End of Paper

Singapore Mathematical Olympiad for Primary Schools 2009 First Round – Answers Keys					Total Marks 150
Question	**Answers**		**Question**	**Answers**	
	Questions 1 to 10 Each carries 4 marks		**16**	3	
1	1		**17**	108	
2	1		**18**	12	
3	22		**19**	4	
4	45		**20**	5	
5	15				
6	36			**Questions 21 to 30 Each carries 6 marks**	
7	19		**21**	7	
8	0		**22**	18	
9	75		**23**	7	
10	80		**24**	2003	
			25	2	
			26	30	
			27	77	
	Questions 11 to 20 Each carries 5 marks		**28**	10	
11	8		**29**	13	
12	1		**30**	68	
13	55555				
14	62				
15	9				

Singapore-Asia Pacific Mathematical Olympiad for Primary Schools (APMOPS 2010)

1 Giới thiệu về kỳ thi

Kỳ thi APMOPS được tổ chức lần đầu tiên tại Việt Nam năm 2009, thu hút sự tham gia của hơn 300 thí sinh (130 em ở Hà Nội và 207 em ở Tp HCM). Thí sinh Việt Nam dành được 16 HCV, 14 HCB và 31 HCĐ. Kế hoạch mời tham dự và tài trợ 10 thí sinh Việt Nam sang Singapore để tham dự Vòng 2 của kỳ thi đã bị hủy bỏ do dịch cúm A. Vòng 1 của kỳ thi năm nay (2010) sẽ diễn ra vào ngày 24 tháng 4.

Bài thi APMOPS thường có 30 bài toán ở vòng 1, thể hiện sự đa dạng về nội dung toán học. Các bài toán thường được sắp từ trình tự dễ đến khó, thuộc các phân môn như: số học (number theory), hình học (geometry), tổ hợp (combinatorics), logic, ... http://www.hexagon.edu.vn/smo/smoj2008.pdf

Dưới đây chúng tôi trích một số bài toán (và lời giải hoặc gợi ý) đã được sử dụng trong các bài giảng tập huấn của thầy giáo Phạm Văn Thuận cho một số học sinh ở trường Hanoi Amsterdam, Giảng Võ, Đoàn Thị Điểm, Lê Quý Đôn Hanoi năm 2009 và năm 2010. Các bài toán này hoặc do chúng tôi đề nghị, hoặc chọn từ các đề thi cũ của kỳ thi APMOPS, hoặc từ các nguồn tài liệu do các bạn nước ngoài gửi. Các thầy cô giáo, các bậc phụ huynh có thể sử dụng tài liệu này để hướng dẫn các em học sinh chuẩn bị cho kỳ thi này. Tài liệu này có thể có nhiều sai sót, hoặc không đầy đủ.

2 Bài toán

1 A student multiplies the month and the day in which he was born by 31 and 12 respectively. The sum of the two resulting products (tích) is 170. Find the month and the date in which he was born.

2 Given that the product of two whole numbers $m \times n$ is a prime number, and the value of m is smaller than n, find the value of m.

3 Given that $(2009 \times n - 2009) \div (2008 \times 2009 - 2006 \times 2007) = 0$, find the value of n.

4 Find the missing number x in the following number sequence (dãy số).

$$2,\ 9,\ -18,\ -11,\ x,\ 29,\ -58,\ -51,\ldots.$$

5 Jane has 9 boxes with 9 accompanying keys. Each box can only be opened by its accompanying key. If the 9 keys have been mixed up, find the maximum number of attempts Jane must make before she can open all the boxes.

6 A triangle ABC with $AC = 18$ cm and $BC = 24$ cm. D lies on BC such that AD is perpendicular to BC. E lies on AC such that BE is perpendicular (vuông góc) to AC. Given that $BE = 20$ cm and $AD = x$ cm, find the value of x.

7 A language school has 100 pupils in which 69% of the pupils study French, 79% study German, 89% study Japanese and 99% study English. Given that at least x % of the students study all four languages, find the value of x.

8 Given that $9 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 + n_8 + n_9$, where $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$, and n_9 are consecutive numbers, find the value of the product $n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6 \times n_7 \times n_8 \times n_9$.

9 Given a regular 6-sided figure $ABCDEF$, that is, all of its sides are equal. G, H, I, J, K and L are mid-points (trung điểm) of AB, BC, CD, DE, EF and FA respectively. Given that the area of $ABCDEF$ is 100 cm^2 and the area of $GHIJKL$ is x cm^2, find the value of x.

10 Write the following numbers in descending order (thứ tự giảm dần).

$$\frac{1005}{2002}, \frac{1007}{2006}, \frac{1009}{2010}, \frac{1011}{2014}.$$

11 Ten players took part in a round-robin tournament (i.e. every player must play against every other player exactly once). There were no draws in this tournament. Suppose that the first player won x_1 games, the second player won x_2 games, the third player won x_3 games and so on. Find the value of

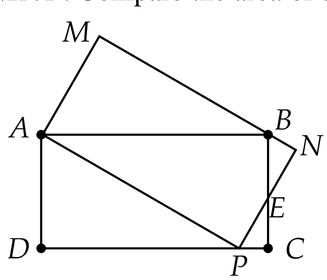
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}.$$

12 What is the difference (hiệu) between the sum of the first 2008 even numbers and the sum of the first 2008 odd numbers?

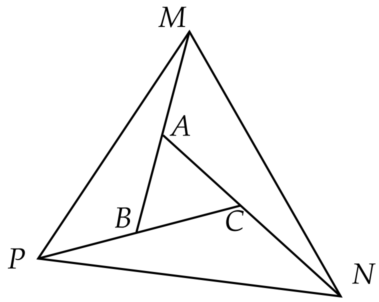
13 The sides of a triangle have lengths that are consecutive whole numbers (các số nguyên liên tiếp) and its perimeter (chu vi) is greater than 2008 cm. If the least possible perimeter of the triangle is x cm, find the value of x.

14 Albert wrote a least possible number on the board that gives remainders 1, 2, 3, 4, 5 upon division by 2, 3, 4, 5, 6 respectively and the written number is divisible by 7. Find the number Albert wrote on the board.

15 The diagram shows two identical rectangular pieces of papers overlapping each other, $ABCD$ and $AMNP$. Compare the area of the region that is common to both rectangles and the



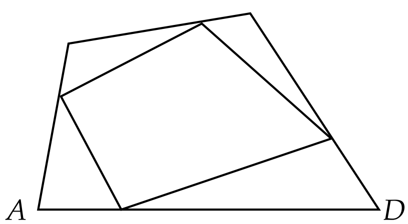
16 A triangle ABC has area 30 cm^2. Another triangle MNP is produced by extending the sides AB, AC, CB such that A, B, C are the midpoints of the sides MB, PC, and NA respectively, as shown in the diagram. Compute the area of triangle MNP.



17 Given a quadrilateral $ABCD$ with area 120 cm^2, points M, N, P, Q are chosen on sides AB, BC, CD, DA such that

$$\frac{MA}{MB} = \frac{PC}{PD} = 2, \ \frac{NB}{NC} = \frac{QD}{QA} = 3.$$

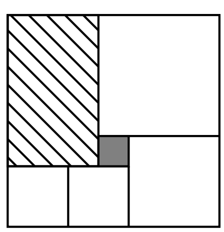
Compute the area of $MNPQ$.



18 Compare the two numbers A, B, where $A = \frac{39}{40}$ and

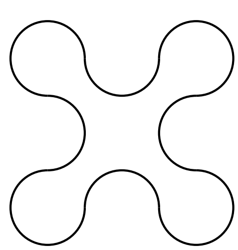
$$B = \frac{1}{21} + \frac{1}{22} + \cdots + \frac{1}{80}.$$

19 A square is formed by five squares and one rectangle (hình chữ nhật). Given that the area of the shaded square is 4 cm^2, compute the area of the rectangle.



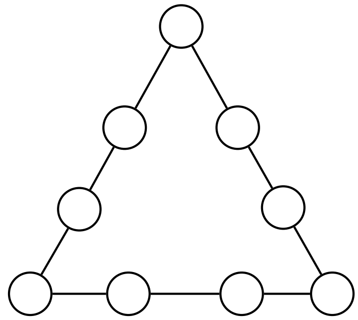
20 For each positive two-digit number (số có hai chữ số), Jack subtracts the units digit from the tens digit; for example, the number 34 gives $3 - 4 = -1$. What is the sum (tổng) of all results?

21 The figure below is made up of 20 quarter-circles that have radius 1 cm. Find the area of the figure.



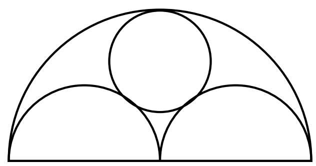
22 In a sequence of positive integers, each term is larger than the previous term. Also, after the first two terms, each term is the sum of the previous two terms. The eighth term (số hạng) of the sequence is 390. What is the ninth term?

23 Each of the numbers from $\{1,2,3,4,5,6,7,8,9\}$ is to be placed in the circles so that the sum of each line of four numbers is 17.



24 The numbers $1, 2, 3, ..., 20$ are written on a blackboard. It is allowed to erase any two numbers a, b and write the new number $a + b - 1$. What number will be on the blackboard after 19 such operations?

25 Two semicircles (nửa đường tròn) of radius 3 are inscribed in a semicircle of radius 6. A circle of radius R is tangent to all three semicircles, as shown. Find R.



26 You are given a set of 10 positive integers. Summing nine of them in ten possible ways we get only nine different sums: $86, 87, 88, 89, 90, 91, 93, 94, 95$. Find those numbers

27 Let natural numbers be assigned to the letters of the alphabet as follows: $A = 1, B = 2, C = 3, \ldots, Z = 26$. The value of a word is defined to be the product of the numbers assigned the the letters in the word. For example, the value of MATH is $13 \times 1 \times 20 \times 8 = 2080$. Find a word whose value is 285.

28 An 80 m rope is suspended at its two ends from the tops of two 50 m flagpoles. If the lowest point to which the mid-point of the rope can be pulled is 36 m from the ground, find the distance, in metres, between the flagpoles.

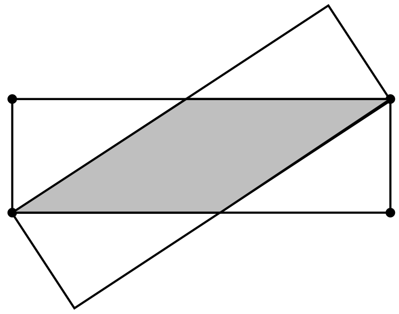
29 Suppose that A, B, C are positive integers such that

$$\frac{24}{5} = A + \frac{1}{B + \frac{1}{C+1}}.$$

Find the value of $A + 2B + 3C$.

30 If $x^2 + xy + x = 14$ and $y^2 + xy + y = 28$, find the possible values of $x + y$.

31 Two congruent rectangles each measuring 3 cm $\times$ 7 cm are placed as in the figure. Find the area of the overlap.



32 Find the least positive integer k such that

$$(k+1)+(k+2)+\cdots+(k+19)$$

is a perfect square.

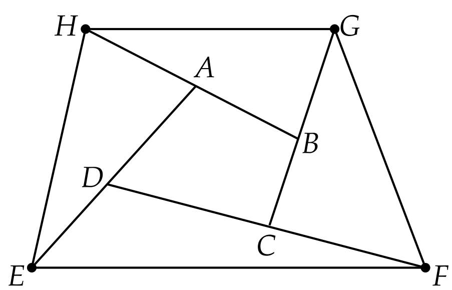
33 A six-digit number begins with 1. If this digit is moved from the extreme left to the extreme right without changing the order of the other digits, the new number is three times the original. Find the sum of the digits in either number.

34 Find the number of positive integers between 200 and 2000 that are multiples (bội số) of 6 or 7 but not both.

35 Find the sum

$$\frac{25}{72}+\frac{25}{90}+\frac{25}{110}+\frac{25}{132}+\cdots+\frac{25}{9900}.$$

36 The convex quadrilateral $ABCD$ has area 1, and AB is produced to E, BC to F, CD to G and DA to H, such that $AB = BE, BC = CF, CD = DG$ and $DA = AH$. Find the area of the quadrilateral $EFGH$.



37 The corners of a square of side 100 cm are cut off so that a regular octagon (hình bát giác đều) remains. Find the length of each side of the resulting octagon.

38 When 5 new classrooms were built for Wingerribee School, the average class size was reduced by 6. When another 5 classrooms were built, the average class size reduced by another 4. If the number of students remained the same throughout the changes, how many students were there at the school?

39 The infinite sequence

$$12345678910111213141516171819202122232425\ldots$$

is obtained by writing the positive integers in order. What is the 210^{th} digit in this sequence?

40 Alice and Bob play the following game with a pile of 2009 beans. A move consists of removing one, two or three beans from the pile. The players move alternately, beginning with Alice. The person who takes the last bean in the pile is the winner. Which player has a winning strategy for this game and what is the strategy?

41 For how many integers n between 1 and 2010 is the improper fraction $\frac{n^2+4}{n+5}$ NOT in lowest terms?

42 Find the number of digits 1s of number n,

$$n = 9 + 99 + 999 + \cdots + \underbrace{9999\cdots99999}_{\text{2010 digits}}.$$

43 In the figure, the seven rectangles are congruent and form a larger rectangle whose area is 336 cm^2. What is the perimeter of the large rectangle?

44 Determine the number of integers between 100 and 999, inclusive, that contains exactly two digits that are the same.

45 Two buildings A and B are twenty feet apart. A ladder thirty feet long has its lower end at the base of building A and its upper end against building B. Another ladder forty feet long has its lower end at the base of building B and its upper end against building A. How high above the ground is the point where the two ladders intersect?

46 A regular pentagon is a five-sided figure that has all of its angles equal and all of its side lengths equal. In the diagram, $TREND$ is a regular pentagon, PEA is an equilateral traingle, and $OPEN$ is a square. Determine the size of $\angle EAR$.

47 Let p, q be positive integers such that $\frac{72}{487} < \frac{p}{q} < \frac{18}{121}$. Find the smallest possible value of q.

48 Someone forms an integer by writing the integers from 1to 82 in ascending order, i.e.,

12345678910111213...808182.

Find the sum of the digits of this integer.

49 How many digits are there before the hundredth 9 in the following number?

$$97977977797777977777977777797777777 9\cdots?$$

50 Find the value of

$$\frac{1}{4\times 9} + \frac{1}{9\times 14} + \frac{1}{14\times 19} + \cdots + \frac{1}{2005\times 2010}.$$

51 What is the missing number in the following number sequence?

$$2,\ 2,\ 3,\ 5,\ 14,\ \ldots,\ 965.$$

52 A confectionery shop sells three types of cakes. Each piece of chocolate and cheese cake costs \$ 5 and \$ 3 respectively. The mini-durian cakes are sold at 3 pieces a dollar. Mr Ngu bought 100 pieces of cakes for \$ 100. How many chocolate, cheese and durian cakes did he buy? Write down all the possible answers.

53 The Sentosa High School telephone number is an eight digit number. The sum of the two numbers formed from the first three digits and the last five digits respectively is 66558. The sum of the two numbers formed from the first five digits and the last three digits is 65577. Find the telephone number of the Sentosa High School.

54 There are 50 sticks of lengths 1 cm, 2 cm, 3 cm, 4 cm, ..., 50 cm. Is it possible to arrange the sticks to make a square, a rectangle?

55 Find the least natural three-digit number whose sum of digits is 20.

56 Given four digits $0, 1, 2, 3$, how many four digit numbers can be formed using the four numbers?

57 One person forms an integer by writing the integers from 1 to 2010 in ascending order, i.e.

$$123456789101112131415161718192021\ldots2010.$$

How many digits are there in the integer.

58 A bag contains identical sized balls of different colours: 10 red, 9 white, 7 yellow, 2 blue and 1 black. Without looking into the bag, Peter takes out the balls one by one from it. What is the least number of balls Peter must take out to ensure that at least 3 balls have the same colour?

59 Three identical cylinders weigh as much as five spheres. Three spheres weigh as much as twelve cubes. How many cylinders weigh as much as 60 cubes?

60 Two complete cycles of a pattern look like this

$$AABBBCCCCCAABBBCCCCC\ldots$$

Given that the pattern continues, what is the 103^{rd} letter?

61 Set A has five consecutive positive odd integers. The sum of the greatest integer and twice the least integer is 47. Find the least integer.

62 Which fraction is exactly half-way between $\frac{2}{5}$ and $\frac{4}{5}$?

63 Let n be the number of sides in a regular polygon where $3 \leq n \leq 10$. What is the value of n that result in a regular polygon where the common degree measure of the interior angles is non-integral?

64 What is the 200^{th} term of the increasing sequence of positive integers formed by omitting only the perfect squares?

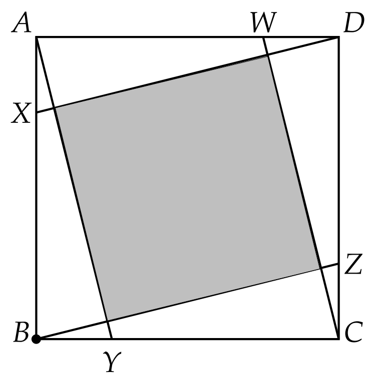
65 If w, x, y, z are consecutive positive integers such that $w^3 + x^3 + y^3 = z^3$, find the least value of z.

66 The mean of three numbers is $\frac{5}{9}$. The difference between the largest and smallest number is $\frac{1}{2}$. Given that $\frac{1}{2}$ is one of the three numbers, find the smallest number.

67 Five couples were at a party. Each person shakes hands exactly one with everyone else except his/her spouse. So how many handshakes were exchanged?

68 What is the positive difference between the sum of the first 20 positive multiples of 5 and the sum of the first 20 positive, even integers?

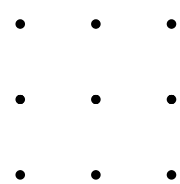
69 The sides of unit square $ABCD$ have trisection points X, Y, Z and W as shown. If $AX : XB = BY : YC = CZ : ZD = DW : WA = 3 : 1$, what is the area of the shaded region?



70 We have a box with red, blue and green marbles. At least 17 marbles must be selected to make sure at least one of them is green. At least 18 marbles must be selected without replacement to be sure that at least 1 of them is red. And at least 20 marbles must be selected without replacement to be sure all three colors appear among the marbles selected. So how many marbles are there in the box?

71 The three-digit integer N yields a perfect square when divided by 5. When divided by 4, the result is a perfect cube. What is the value of N?

72 How many different sets of three points in this 3 by 3 grid of equally spaced points can be connected to form an isosceles triangle (having two sides of the same length)?



73 Given the list of integer

$$1234567898765432123456\cdots,$$

find the 1000^{st} integer in the list.

74 A person write the letters from the words $LOVEMATH$ in the following way

$$LOVEMATHLOVEMATHLOVEMATH\cdots.$$

i) Which letter is in the 2010^{th} place?

ii) Assume that there are 50 letters M in a certain sequence. How many letters E are there in the sequence?

iii) If the letters are to be coloured blue, red, purple, yellow, blue, red, purple, yellow, ... What colour is the letter in the 2010^{th} place?

3 Hướng dẫn, Lời giải

1 A student multiplies the month and the day in which he was born by 31 and 12 respectively. The sum of the two resulting products (tích) is 170. Find the month and the date in which he was born.

Solution. Let d, m be the day, moth the student was born. Notice that $d, m \in \mathbb{Z}$ and $1 \leq d \leq 31$, $1 \leq m \leq 12$. We have the equation

$$31m + 12d = 170.$$

Since both 170 and $12d$ are divisible by 2, we must choose m such that $31m$ is divisible by 2.

If $m = 2$, then $d = \frac{170-62}{12} = 9$. Hence, $m = 2, d = 9$.

If $m = 4$, then $d = \frac{170-124}{12}$, which is not an integer.

If $m \geq 6$, then $31m > 17$, which invalidates the equality in the given equation.

Answer: The student was born on Feb 9$^{\text{th}}$. □

2 Given that the product of two whole numbers $m \times n$ is a prime number, and the value of m is smaller than n, find the value of m.

Solution. Since a prime number only has 1 and the number itself as factors (ước số), we have $m = 1$. □

3 Given that $(2009 \times n - 2009) \div (2008 \times 2009 - 2006 \times 2007) = 0$, find the value of n.

Solution. $n = 1$. □

4 Find the missing number x in the following number sequence (dãy số).

$$2,\ 9,\ -18,\ -11,\ x,\ 29,\ -58,\ -51, \ldots.$$

Solution. Notice that $2 + 7 = 9$, $9 \times (-2) = -18$, $-18 + 7 = -11$, and $-11 \times (-2) = 22$. Then $x = 22$. □

5 Jane has 9 boxes with 9 accompanying keys. Each box can only be opened by its accompanying key. If the 9 keys have been mixed up, find the maximum number of attempts Jane must make before she can open all the boxes.

Solution. In the worst case, he needs 9 attempts for the first boxes, 8 attempts for the second box, 7 attempts for the third box, ... and 1 attempt for the last box. Hence, the maximum number of attempts Jane must make is

$$1 + 2 + \cdots + 8 + 9 = \frac{9(9+1)}{2} = 45.$$

□

6 A triangle ABC with $AC = 18$ cm and $BC = 24$ cm. D lies on BC such that AD is perpendicular to BC. E lies on AC such that BE is perpendicular (vuông góc) to AC. Given that $BE = 20$ cm and $AD = x$ cm, find the value of x.

Solution. The area of triangle ABC is computed by multiplying its altitude by its corresponding base divided by 2. Hence, we have

$$\frac{24 \times x}{2} = \frac{20 \times 18}{2}.$$

Solving for x gives $x = 15$. □

7 A language school has 100 pupils in which 69% of the pupils study French, 79% study German, 89% study Japanese and 99% study English. Given that at least x % of the students study all four languages, find the value of x.

Hint. The idea of Venn diagram works perfectly for this problem.

□

8 Given that $9 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 + n_8 + n_9$, where $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$, and n_9 are consecutive numbers, find the value of the product $n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6 \times n_7 \times n_8 \times n_9$.

Solution. Let $n_1 = k - 4$, then $n_2 = k - 3$, $n_3 = k - 2$, etc. Summing up the terms gives

$$k - 4 + k - 3 + (k - 2) + (k - 1) + (k) + (k + 1) + (k + 2) + (k + 3) + (k + 4) = 9,$$

which yields $9k = 9$, or $k = 1$. Hence, one of the nine numbers is zero, which implies that the product is zero. That is,

$$n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6 \times n_7 \times n_8 \times n_9 = 0.$$

□

9 Given a regular 6-sided figure $ABCDEF$, that is, all of its sides are equal. G, H, I, J, K and L are mid-points (trung điểm) of AB, BC, CD, DE, EF and FA respectively. Given that the area of $ABCDEF$ is 100 cm^2 and the area of $GHIJKL$ is x cm^2, find the value of x.

10 Write the following numbers in descending order (thứ tự giảm dần).

$$\frac{1005}{2002}, \frac{1007}{2006}, \frac{1009}{2010}, \frac{1011}{2014}.$$

Solution. Notice that

$$\frac{1005}{2002} = \frac{1001}{2002} + \frac{4}{2002} = \frac{1}{2} + \frac{2}{1001}.$$

Similarly,

$$\frac{1007}{2006} = \frac{1}{2} + \frac{2}{1003}, \frac{1009}{2010} = \frac{1}{2} + \frac{2}{1005}, \frac{1011}{2014} = \frac{1}{2} + \frac{2}{2007}.$$

Since $\frac{2}{1001} > \frac{2}{1003} > \frac{2}{1005} > \frac{2}{1007}$, we have

$$\frac{1005}{2002} > \frac{1007}{2006} > \frac{1009}{2010} > \frac{1011}{2014}.$$

□

11 Ten players took part in a round-robin tournament (i.e. every player must play against every other player exactly once). There were no draws in this tournament. Suppose that the first player won x_1 games, the second player won x_2 games, the third player won x_3 games and so on. Find the value of

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}.$$

Solution. A total number of $\frac{9\times 10}{2} = 45$ games were played among the 10 players. Since there is only one winner for each game, there are altogether 45 wins in the games. Hence,

$$x_1 + x_2 + \cdots + x_{10} = 45.$$

□

12 What is the difference (hiệu) between the sum of the first 2008 even numbers and the sum of the first 2008 odd numbers?

Solution. Answer: 2008. Let S(even)$= 2+4+6+\cdots+2008$, and S(odd)$= 1+3+5+\cdots+2007$. Hence,

$$S(\text{even}) - S(\text{odd}) = 2-1+4-3+6-5+8-7+\cdots+2008-2007 = 2008.$$

One trick in this sort of problem is that you can try for sums that involve smaller number of numbers, say, first four (having the same parity with 2008) even numbers and first four odd numbers to see if any regular pattern appears.

$$2+4+6+8-(1+3+5+7) = 1+1+1+1 = 4.$$

□

13 The sides of a triangle have lengths that are consecutive whole numbers (các số nguyên liên tiếp) and its perimeter (chu vi) is greater than 2008 cm. If the least possible perimeter of the triangle is x cm, find the value of x.

Solution. Let a be the shortest side of the triangle. Then, the other two sides are $a+1$, $a+2$. We have

$$a + a + 1 + a + 2 > 2008,$$

which implies $3a + 3 > 2008$ or $a > 668\frac{1}{3}$. The three sides are $669, 670, 671$, the least possible perimeter is 2010. □

14 Albert wrote a least possible number on the board that gives remainders 1, 2, 3, 4, 5 upon division by 2, 3, 4, 5, 6 respectively and the written number is divisible by 7. Find the number Albert wrote on the board.

Solution. Let N be the number that Albert wrote on the board. Since N gives remainders (số dư) $1, 2, 3, 4, 5$ when divided by $2, 3, 4, 5, 6$, we have $N+1$ is divisible by $2, 3, 4, 5, 6$. Since $N+1$ are divisible by both 3 and 4, we have $N+1$ is divisible by 12, then $N+1$ is also divisible by 2 and 6. Furthermore, $N+1$ is divisible by 5, then $N+1$ is divisible by 60. Hence, $N+1 = \{60, 120, 180, ...\}$ which means that

$$N = \{59,\ 119,\ 179,\ ...\}.$$

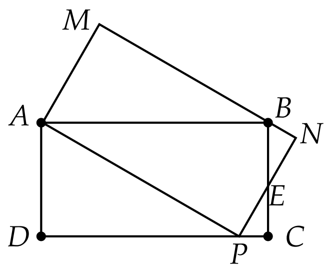
If $N = 59$, then 59 is not divisible by 7.

If $N = 119$, then $119 : 7 = 17$.

Then, the least number is 119.

□

15 The diagram shows two identical rectangular pieces of papers overlapping each other, $ABCD$ and $AMNP$. Compare the area of the region that is common to both rectangles and the

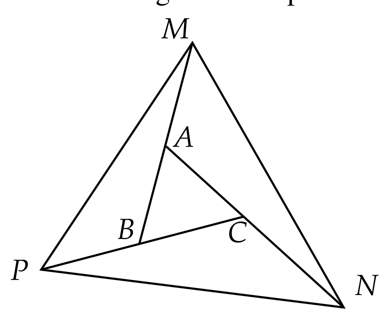


Solution. Draw rectangles $BHKE$, and $EKPPC$ as shown in the diagramm. Denote by $(.)$ the area of a polygon. Since $AHPD$ is a rectangle with diagonal AP, we have $(AHP) = (APD)$. We also have $(KEP) = (ECP)$ since $KECP$ is a rectangle with diagonal PE. Hence,

$$(ABEP) = (AHP) + (KEP) + (HBEK) = (APD) + (ECP) + HBEK).$$

It follows that the area of the shaded region is greater than the other region of the rectangle. □

16 A triangle ABC has area 30 cm^2. Another triangle MNP is produced by extending the sides AB, AC, CB such that A, B, C are the midpoints of the sides MB, PC, and NA respectively, as shown in the diagram. Compute the area of triangle MNP.



Solution. Denote by $(.)$ the area of polygon. We have $(CAB) = (CAM) = 30$ cm^2, $(MAC) = (MCN) = 30$ cm^2. Hence $(MAN) = 30 + 30 = 60$ cm^2. Since $(MBC) = 30 + 30 = 60$ cm^2, we have $(MPB) = (MBC) = 60$ cm^2. Since $(PAC) = 30 \times 2 = 60$ cm^2, we have $(PCN) = (PAC) = 60$ cm^2. Hence,

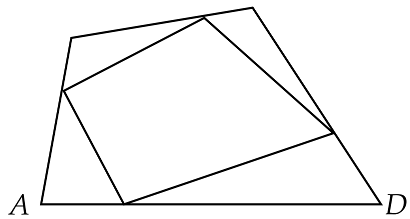
$$(MNP) = 60 \times 3 + 30 = 210\,\text{cm}^2.$$

□

17 Given a quadrilateral $ABCD$ with area 120 cm^2, points M, N, P, Q are chosen on sides AB, BC, CD, DA such that

$$\frac{MA}{MB} = \frac{PC}{PD} = 2, \; \frac{NB}{NC} = \frac{QD}{QA} = 3.$$

Compute the area of $MNPQ$.



Solution. Let $(.)$ denote the area of the polygon. We have $(BMN) = \frac{1}{3}(ABN)$, $(ABN) = \frac{3}{4}(ABC)$. Then, $(BMN) = \frac{1}{4}(ABC)$. Similarly, we have $(DPQ) = \frac{1}{3}(CDQ)$. Since $(CDQ) = \frac{3}{4}(ACD)$, we have $(DPQ) = \frac{1}{4}(ACD)$. Hence,

$$(BMN) + (DPQ) = \frac{1}{4}[(ABC) + (ACD)].$$

Since $(ABC) + (ACD) = (ABCD) = 120 \text{ cm}^2$, we have $(BMN) + (DPQ) = 120 : 4 = 30 \text{ cm}^2$. Similarly, we have $(AMQ) + (CNP) = \frac{1}{6}(ABCD) = \frac{1}{6} \times 120 = 20 \text{ cm}^2$. Hence, $(MNPQ) = 120 - (30 + 20) = 70 \text{ cm}^2$. □

18 Compare the two numbers A, B, where $A = \frac{39}{40}$ and

$$B = \frac{1}{21} + \frac{1}{22} + \cdots + \frac{1}{80}.$$

Solution. Rewriting B as

$$B = \left(\frac{1}{21} + \frac{1}{22} + \cdots + \frac{1}{50}\right) + \left(\frac{1}{51} + \frac{1}{52} + \cdots + \frac{1}{80}\right).$$

Notice that the sum $\frac{1}{21} + \frac{1}{22} + \cdots + \frac{1}{50}$ has $50 - 21 + 1$ summands, hence

$$\frac{1}{21} + \frac{1}{22} + \cdots + \frac{1}{50} > \frac{30}{50} = \frac{3}{5}.$$

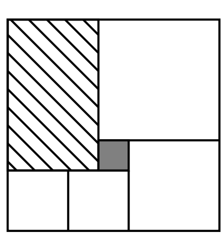
The second sum $\frac{1}{51} + \frac{1}{52} + \cdots + \frac{1}{80} > \frac{30}{80} = \frac{3}{8}$. Hence,

$$B > \frac{3}{5} + \frac{3}{8} = \frac{39}{40}.$$

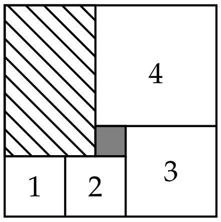
Then, $A < B$.

□

19 A square is formed by five squares and one rectangle (hình chữ nhật). Given that the area of the shaded square is 4 cm^2, compute the area of the rectangle.



Solution. The side length of the shaded square is 2 since its area is 4. Label the other four squares as shown.



Let a be the side length of square 1, then the side length of square 2 is a and that of square 3 is $a+2$, square 4 has side-length $a+4$. Hence we have

$$a+a+a+2 = a+2+a+4.$$

Solving this equation gives $a = 4$. Hence the length of the rectangle is $4+4+2 = 10$ and the breadth of the rectangle is $4+4-2 = 6$. The area of the rectangle is $10 \times 6 = 60$. $\square$

20 For each positive two-digit number (số có hai chữ số), Jack subtracts the units digit from the tens digit; for example, the number 34 gives $3-4 = -1$. What is the sum (tổng) of all results?

Solution. Let n be the two-digit number of the form $\overline{ab}$, $1 \le a \le 9$, $0 \le b \le 9$. We may classify these two-digit numbers into the following sets: $P = \{12, 13, 14, ..., 23, 24, 25, ..., 34, 35, ..., ..., 56, 57, ..., 67, 68, ..., 89$ $Q = \{21, 31, 41, ..., 32, 42, ..., 43, 53, ..., 98, 99\}$, R consits of all palindromes of the form aa, and S contains all numbers ab, where $a > 0$ and $b = 0$.

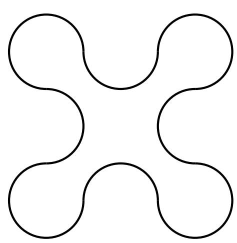
The result $a-b$ from each of the numbers in set P can be matched with the result $b-a$ from each corresponding number in the set Q, giving a total of $(a-b)+(b-a) = 0$. For each of the numbers in the set R, the result is $a-a$ which is 0. Finally, the sum of all results is

$$1+2+3+\cdots+9 = 45.$$

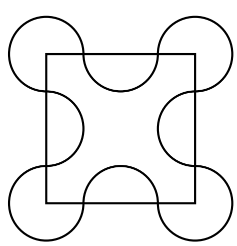
Answer: 45.

$\square$

21 The figure below is made up of 20 quarter-circles that have radius 1 cm. Find the area of the figure.



Solution. Notice that the area of the figure is equal to the sum of area of the square and one circle that is formed by four quarters. The area of the square is $8^2 = 64$ cm^2, and the area of one circle is 4π cm^2. Hence, the area of the figure is $64+4\pi$ cm^2.



□

22 In a sequence of positive integers, each term is larger than the previous term. Also, after the first two terms, each term is the sum of the previous two terms. The eighth term (số hạng) of the sequence is 390. What is the ninth term?

Solution. Let a, b be the first two terms. The the first nine terms of the sequence read

$$a, b, a+b, a+2b, 2a+3b, 3a+5b, 5a+8b, 8a+13b, 13a+21b,$$

which is an increasing sequence.

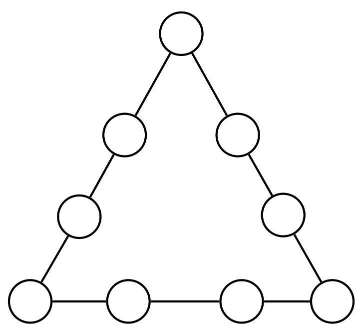
The eighth term is equal to 390, so we have the equation $8a + 13b = 390$. Since 390 is a multiple of 13, we have $8a$ is divisible by 13. Hence, $a = \{13, 26, 39, ...\}$.

If $a \geq 26$, then $13b \leq 390 - 8 \times 26$, or $b \leq 14$, which is impossible for an increasing sequence.

Thus, we conclude that $a = 13$, and then $b = 22$. Therefore, the ninth term is 631.

□

23 Each of the numbers from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is to be placed in the circles so that the sum of each line of four numbers is 17.



Solution. The sum of all the nine numbers to be filled in is

$$1 + 2 + \cdots + 9 = \frac{9 \times 10}{2} = 45.$$

Each number at the vertex of the triangle is counted twice. We have

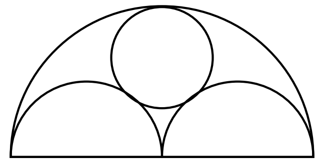
$$17 \times 3 - 45 = 6.$$

It follows that the sum of three numbers at the vertices of the triangle is 6. Notice that $6 = 1 + 2 + 3$. We have the following arrangements.

□

24 The numbers $1, 2, 3, ..., 20$ are written on a blackboard. It is allowed to erase any two numbers a, b and write the new number $a + b - 1$. What number will be on the blackboard after 19 such operations?

25 Two semicircles (nửa đường tròn) of radius 3 are inscribed in a semicircle of radius 6. A circle of radius R is tangent to all three semicircles, as shown. Find R.



Solution. Join the centres of the two smaller semicircles and the centre of the circle. This forms an isosceles triangle with equal sides $3+R$ and base 6 units. Call the altitude of this triangle h. The altitude extends to a radius of the large semicircle, so $h+R=6$. By Pythagoras, $h^2+3^2=(R+3)^2$, so

$$(6-R)^2+3^2=(R+3)^2.$$

Solving this gives $R=2$. □

26 You are given a set of 10 positive integers. Summing nine of them in ten possible ways we get only nine different sums: $86, 87, 88, 89, 90, 91, 93, 94, 95$. Find those numbers

Solution. Let S be the sum of all ten positive integers and assume that x is the sum that is repeated. Call the elements of the set $a_1, a_2, ..., a_{10}$. We have

$$S-a_1=86,\ S-a_2=87,\ \ldots, S-a_9=95,\ S-a_{10}=x.$$

Adding these equations gives

$$10S-S=813+x.$$

The only value of x from $86, 87, ...95$ which makes $813+x$ divisible by 9 is $x=87$ and then $S=100$. Hence, the ten positive integers are

$$14, 13, 12, 11, 10, 9, 7, 6, 5, 13.$$

□

27 Let natural numbers be assigned to the letters of the alphabet as follows: $A=1, B=2, C=3, \ldots, Z=26$. The value of a word is defined to be the product of the numbers assigned the the letters in the word. For example, the value of MATH is $13\times 1\times 20\times 8=2080$. Find a word whose value is 285.

Solution. By factorisation, we have

$$285=1\times 3\times 5\times 19=1\times 15\times 19=3\times 5\times 19=15\times 19.$$

Now 15 corresponds to O and 19 to S and the value of SO is 285. The other possible choices of letters $\{A, O, S\}$, $\{C, E, S\}$ and $\{A, C, E, S\}$ do not seem to give English words other than $ACES$ and $CASE$. □

28 An 80 m rope is suspended at its two ends from the tops of two 50 m flagpoles. If the lowest point to which the mid-point of the rope can be pulled is 36 m from the ground, find the distance, in metres, between the flagpoles.

Hint. Use the Pythagoras theorem. □

29 Suppose that A, B, C are positive integers such that

$$\frac{24}{5} = A + \frac{1}{B + \frac{1}{C+1}}.$$

Find the value of $A + 2B + 3C$.

Solution. Since $B + \frac{1}{C+1} > 1$, then A must represent the integer part of $\frac{24}{5}$, that is, $A = 4$. Then we have

$$\frac{4}{5} = \frac{1}{B + \frac{1}{1+C}}, \Rightarrow \frac{5}{4} = B + \frac{1}{C+1}.$$

For exactly the same reason, we have $B = 1$, and then $C = 3$. Hence,

$$A + 2B + 3C = 15.$$

□

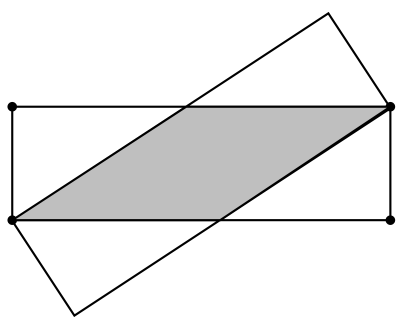
30 If $x^2 + xy + x = 14$ and $y^2 + xy + y = 28$, find the possible values of $x + y$.

Solution. Adding the two equations gives

$$(x + y)^2 + (x + y) - 42 = 0.$$

Thus, $x + y = 6$ or $x + y = -7$. □

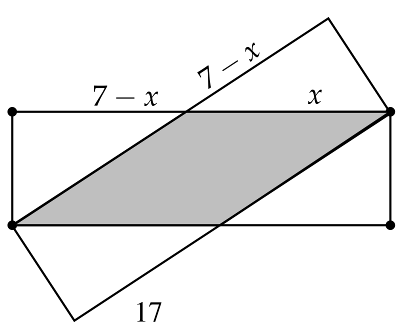
31 Two congruent rectangles each measuring 3 cm × 7 cm are placed as in the figure. Find the area of the overlap.



Solution. In the diagram, all the unshaded triangles are congruent right. By the Pythagoras theorem, we have

$$x^2 = (7 - x)^2 + 3^2.$$

Solving this equation gives $x = \frac{29}{7}$.



□

32 Find the least positive integer k such that

$$(k+1)+(k+2)+\cdots+(k+19)$$

is a perfect square.

Solution. Notice that the sum is equal to

$$19k+190.$$

Since 19 is a prime number, in order for $19k+190$ to be a perfect square, $k+10$ must contain 19 as a factor. The least value as such occurs when $k+10=19$, that is, $k=9$. □

33 A six-digit number begins with 1. If this digit is moved from the extreme left to the extreme right without changing the order of the other digits, the new number is three times the original. Find the sum of the digits in either number.

Solution. Let n be the number in question. Then n can be written as 10^5+a, where a is a number with at most 5 digits. Moving the left-most digit (the digit 1) to the extreme right produces a number $10a+1$. The information in the problem now tells us that $10a+1 = 3(10^5+a) = 300000+3a$, or $7a=299999$. This yields $a=42857$. Thus, $n=142857$ (and the other number we created is 428571), the sum of whose digits is $1+4+2+8+5+7=27$. □

34 Find the number of positive integers between 200 and 2000 that are multiples (bội số) of 6 or 7 but not both.

Solution. The number of positive integers less than or equal to n which are multiples of k is the integer part of $n=k$ (that is, perform the division and discard the decimal fraction, if any). This integer is commonly denoted $\lfloor n=k \rfloor$. Thus, the number of positive integers between 200 and 2000 which are multiples of 6 is

$$\lfloor\frac{2000}{6}\rfloor-\lfloor\frac{200}{6}\rfloor=333-33=300.$$

Similarly, the number of positive integers between 200 and 2000 which are multiples of 7 is

$$\lfloor\frac{2000}{7}\rfloor-\lfloor\frac{200}{7}\rfloor=285-28=257.$$

In order to count the number of positive integers between 200 and 2000 which are multiples of 6 or 7 we could add the above numbers. This, however, would count the multiples of both 6 and 7 twice; that is, the multiples of 42 would be counted twice. Thus, we need to subtract from this sum the number of positive integers between 200 and 2000 which are multiples of 42. That number is

$$\lfloor\frac{2000}{42}\rfloor-\lfloor\frac{200}{42}\rfloor=47-4=43.$$

Therefore, the number of positive integers between 200 and 2000 which are multiples of 6 or 7 is $300+257-43=514$. But we are asked for the number of positive integers which are multiples of 6 or 7, but NOT BOTH. Thus, we need to again subtract the number of multiples of 42 in this range, namely 43. The final answer is $514-43=471$. □

35 Find the sum

$$\frac{25}{72}+\frac{25}{90}+\frac{25}{110}+\frac{25}{132}+\cdots+\frac{25}{9900}.$$

Solution. Notice that

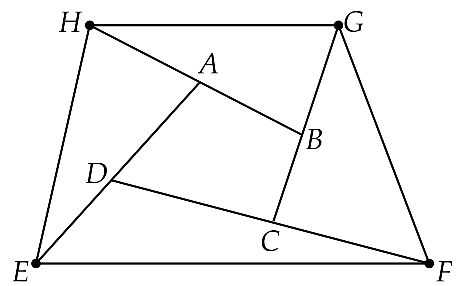
$$\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}.$$

Hence,

$$25\left(\frac{1}{8}-\frac{1}{9}+\frac{1}{9}-\frac{1}{10}+\cdots+\frac{1}{99}-\frac{1}{100}\right)=\frac{25\times 92}{8\times 100}=2.875.$$

□

36 The convex quadrilateral $ABCD$ has area 1, and AB is produced to E, BC to F, CD to G and DA to H, such that $AB = BE, BC = CF, CD = DG$ and $DA = AH$. Find the area of the quadrilateral $EFGH$.



Solution. Denote by $(.)$ the area of the polygon. Since B and A are the midpoints of AE and DH respectively, we have $(HEB) = (HAB) = (ABD)$, so that

$$(AEH) = 2(ABD).$$

Similarly, we have $(CFG) = 2(CBD)$. Thus,

$$\begin{aligned}(AEH)+(CFG) &= 2(ABD)+2(CBD)\\ &= 2(ABCD)\\ &= 2.\end{aligned}$$

Similarly, we get $(BEF) + (DGH) = 2$. Thus,

$$(EFGH) = (ABCD) + (AEH) + (CFG) + (BEF) + (DGH) = 1 + 2 + 2 = 5.$$

□

37 The corners of a square of side 100 cm are cut off so that a regular octagon (hình bát giác đều) remains. Find the length of each side of the resulting octagon.

Solution. Let a be the side length of removed triangles. Using the Pythagoras theorem, we have

$$(100-2a)^2 = a^2 + a^2 = 2a^2.$$

Solving this gives $a = \frac{100}{2+\sqrt{2}}$. The side of the octagon is

$$100 - 2a = 100(\sqrt{2}-1).$$

□

38 When 5 new classrooms were built for Wingerribee School, the average class size was reduced by 6. When another 5 classrooms were built, the average class size reduced by another 4. If the number of students remained the same throughout the changes, how many students were there at the school?

Solution. Let x be the number of students and y be the original number of classrooms. The addition of the first five classrooms gives

$$\frac{x}{y} = 6 + \frac{x}{y+5},$$

while the second addition of another five students gives

$$\frac{x}{y+5} = 4 + \frac{x}{y+10}.$$

Solving the system gives

$$5x = 6y(y+5),\ 5x = 4(y+5)(y+10),$$

we have $y = 20$. Hence, $x = 600$.

□

39 The infinite sequence

$$12345678910111213141516171819202122232425\ldots$$

is obtained by writing the positive integers in order. What is the 210^{th} digit in this sequence?

Solution. The digits $1, 2, \ldots, 9$ occupy 9 positions, and the digits in the numbers $10, 11, \ldots 99$ occupy $2 \times 90 = 180$ positions. Further, the digits in the numbers $100, 101, \ldots, 199$ occupy $3 \times 100 = 300$ posititions. Similarly, 300 positions are required for the numbers 200 to 299, etc ... Hence, the digits in the numbers up to and including 699 occupy the first

$$9 + 180 + 6 \times 300 = 1989 \text{ positions.}$$

A further 21 positions are required to write $700, 701, \ldots, 707$. so that the 2010^{th} digit is the 7 in 707. □

40 Alice and Bob play the following game with a pile of 2009 beans. A move consists of removing one, two or three beans from the pile. The players move alternately, beginning with Alice. The person who takes the last bean in the pile is the winner. Which player has a winning strategy for this game and what is the strategy?

Solution. Alice has the winning strategy. On her first move, she takes one bean. On subsequent moves, Alice removes $4 - t$ beans, where t is the number of beans that Bob removed on the preceding turn.

Right after Alice's first move, the pile has 2008 beans. Moreover, after every pair of moves, a move by Boby followed by a move by Alice, the pile decreases by exactly 4 beans. Eventually, after a move by Alice, there will be 4 beans left in the pile. Then, after Bob takes one, two, or three, Alice takes the remainder and wins the game.

□

41 For how many integers n between 1 and 2010 is the improper fraction $\frac{n^2+4}{n+5}$ NOT in lowest terms?

Solution. For some integer between 1 and 2010, that $\frac{n^2+4}{n+5}$ is not in lowest terms. That is, there is some integer d greater than 1 such that d is a common factor of n^2+4 and $n+5$. Now, d divides $n+5$ implies that d divides $(n+5)^2$. Hence, d divides $n^2+10n+25-(n^2+4)=10n+21$.

Since d is a factor of $n+5$, we have d is also a factor of $10n+50$, then d is a factor of

$$10n+50-(10n+21)=29.$$

Since $d>1$ and 29 is a prime, we have $d=29$.

Thus, $\frac{n^2+4}{n+5}$ is not in lowest terms only if 29 is a factor of $n+5$.

Assume that 29 divides $n+5$. Then $n+5=29k$ for some positive integer k and we get $n=29k-5$ so that $n^2=841k^2-145k+25$, and $n^2+4=841k^2-145k+29=29(29k^2-5k+1)$, which means that n^2+4 is divisible by 29.

Hence, $\frac{n^2+4}{n+5}$ is not in lowest terms if and only if n is divisible by 29. There are 69 multiples of 29 between 1 and 2010. □

42 Find the number of digits 1s of number n,

$$n=9+99+999+\cdots+\underbrace{9999\cdots99999}_{2010\text{ digits}}.$$

Solution. We have

$$n=\sum_{k=1}^{2010}(10^k-1)=\sum_{k=1}^{2010}10^k-\sum_{k=1}^{2010}1=\underbrace{111111...111}_{2010\text{ digits}}0-2010.$$

Hence,

$$n=\underbrace{1111\ldots1111}_{2009\text{ digits}}09101.$$

There are 2011 digits 1 in n.

□

43 In the figure, the seven rectangles are congruent and form a larger rectangle whose area is 336 cm^2. What is the perimeter of the large rectangle?

Hint. Let x, y be the dimension of the small rectangles. □

44 Determine the number of integers between 100 and 999, inclusive, that contains exactly two digits that are the same.

Solution. There are three cases to consider. Case 1 includes numbers like 100, 122, 133, etc ... There are 9 such numbers in each hundred group, for a total of $9\times9=81$ numbers. Case 2 includes numbers like 121, 131, etc, ... of which there are 9 in each hundred group, for a total of $9\times9=81$ numbers. Case 3 includes numbers like $112, 113$, ... of which there are 9 numbers in each hundred group, for a total of $9\times9=81$ numbers. All three cases give us a total of 243 numbers. □

45 Two buildings A and B are twenty feet apart. A ladder thirty feet long has its lower end at the base of building A and its upper end against building B. Another ladder forty feet long has its lower end at the base of building B and its upper end against building A. How high above the ground is the point where the two ladders intersect?

Hint. The Pythagoras theorem works!

□

46 A regular pentagon is a five-sided figure that has all of its angles equal and all of its side lengths equal. In the diagram, $TREND$ is a regular pentagon, PEA is an equilateral traingle, and $OPEN$ is a square. Determine the size of $\angle EAR$.

47 Let p, q be positive integers such that $\frac{72}{487} < \frac{p}{q} < \frac{18}{121}$. Find the smallest possible value of q.

48 Someone forms an integer by writing the integers from 1to 82 in ascending order, i.e.,

$$12345678910111213...808182.$$

Find the sum of the digits of this integer.

Solution. We don't have to care about zeros as far as sum of digits is concerned, so we simply count the number of occurrences of the non-zero digits. The digit '3' occurs 18 times (10 as tens digits in 30, 31, ..., 39 and 8 as unit digits in 3, 13, ..., 73). The same is true for the digits '4', '5'. '6' and '7'. A little modification shows that the digits '1' and '2' each occurs 19 times. Finally the digit '8' occurs 11 times $(8, 18, 28, \ldots, 78, 80, 81, 82)$ and the digit '9' occurs 8 times $(9, 19, 29, \ldots, 79)$. Hence the answer is

$$(3+4+5+6+7) \times 18 + (1+2) \times 19 + 8 \times 11 + 9 \times 8 = 667.$$

□

49 How many digits are there before the hundredth 9 in the following number?

$$97977977797777977777977777797777777 \cdots ?$$

Solution. After the first digit 9, there is 1 digit 7. After the second digit 9, there are two 7s. After the third digit 9, there are 3 digits 7s. Hence, after the 99^{th} digit 9, there are 99 digits 7s. Therefore, the totol number of digits 7 before the 100^{th} digit 9 is

$$1 + 2 + 3 + \cdots + 99 = \frac{99 \times 100}{2}.$$

And before the hundredth 9, there are 99 digits 9. Thus, the number of digits (7s and 9s) before the hundredth 9 is

$$99 + 50 \times 99 = 5049.$$

□

50 Find the value of

$$\frac{1}{4 \times 9} + \frac{1}{9 \times 14} + \frac{1}{14 \times 19} + \cdots + \frac{1}{2005 \times 2010}.$$

51 What is the missing number in the following number sequence?

$$2,\ 2,\ 3,\ 5,\ 14,\ \ldots,\ 965.$$

Solution. The rule in the sequence is that after the second term, the third is obtained by multiplying the two previous terms and then subtract 1 from the product. That is,

$$3 = 2 \times 2 - 1,\ 5 = 2 \times 3 - 1,\ 14 = 3 \times 5 - 1.$$

Hence, the missing number is $5 \times 14 - 1 = 69$.

□

52 A confectionery shop sells three types of cakes. Each piece of chocolate and cheese cake costs \$ 5 and \$ 3 respectively. The mini-durian cakes are sold at 3 pieces a dollar. Mr Ngu bought 100 pieces of cakes for \$ 100. How many chocolate, cheese and durian cakes did he buy? Write down all the possible answers.

Solution. Let x, y, z represent the number of chocolate, cheese, and durian cakes respectively. Then we have two simultaneous equations

$$x + y + z = 100,\ 5x + 3y + \frac{z}{3} = 100.$$

Subtracting the two equations gives

$$4x + 2y - \frac{2}{3}z = 0,$$

or $6x + 3y = z$, which is equivalent to

$$7x + 4y = 100.$$

Since 4 is a factor of 100, and $4y$ also divides 100, we need to have $7x$ divides 100, and thus 4 divides x.

If $x = 4$, then $y = 18$, $z = 78$.

If $x = 8$, then $y = 11$, and $z = 81$.

If $x = 12$, then $y = 4$, and $z = 84$.

If $x \geq 16$, then $7x > 100$.

In conclusion,

$$(x, y, z) = \{(4, 18, 78),\ (8, 11, 81),\ (12, 4, 84)\}.$$

□

53 The Sentosa High School telephone number is an eight digit number. The sum of the two numbers formed from the first three digits and the last five digits respectively is 66558. The sum of the two numbers formed from the first five digits and the last three digits is 65577. Find the telephone number of the Sentosa High School.

Answer. 64665912. □

54 There are 50 sticks of lengths 1 cm, 2 cm, 3 cm, 4 cm, ..., 50 cm. Is it possible to arrange the sticks to make a square, a rectangle?

Solution. The sum of lengths of the sticks is

$$1 + 2 + 3 + \cdots + 50 = 1275 \text{ cm}.$$

Notice that the length of the square to be formed is a natural number. In order make a square using all the sticks, the length must be a multiple of 4. But 1275 is not divisible by 4. Hence, it is impossible to make such a square.

The total sum of length of the sticks is the perimeter of the rectangle. But 1275 is not divisible by 2. So impossible to make a rectangle either using the sticks. □

55 Find the least natural three-digit number whose sum of digits is 20.

Answer. 299. □

56 Given four digits $0, 1, 2, 3$, how many four digit numbers can be formed using the four numbers?

Hint. Use tree diagramm. There are 18 numbers. □

57 One person forms an integer by writing the integers from 1 to 2010 in ascending order, i.e.

$$12345678910111213141516171819202 1\ldots 2010.$$

How many digits are there in the integer.

Solution. Divide the digits of the integer into the following categories

$$\underbrace{123456789}_{\text{group 1}}\underbrace{101112\ldots 99}_{\text{group 2}}\underbrace{100101\ldots 999}_{\text{group 3}}\underbrace{10001001\ldots 2010}_{\text{group 4}}.$$

The number of digits in group 1 is

$$9 - 1 + 1 = 9.$$

The number of digits in group 2 is

$$(99 - 10 + 1) \times 2 = 180.$$

The number of digits in group 3 is

$$(999 - 100 + 1) \times 3 = 2700.$$

The number of digits in group 4 is

$$(2010 - 1000 + 1) \times 4 = 4044.$$

Hence, the total number of digits in the integers is

$$9 + 180 + 2700 + 4044 = 6933.$$

□

58 A bag contains identical sized balls of different colours: 10 red, 9 white, 7 yellow, 2 blue and 1 black. Without looking into the bag, Peter takes out the balls one by one from it. What is the least number of balls Peter must take out to ensure that at least 3 balls have the same colour?

Answer. 10 balls. □

59 Three identical cylinders weigh as much as five spheres. Three spheres weigh as much as twelve cubes. How many cylinders weigh as much as 60 cubes?

Solution. Let c, s, b be the weights of a cylinder, a sphere, and a cube respectively. We have $3c = 5s$, and $3s = 12b$. From these ratios, we have $60b = 15s = 9c$. □

60 Two complete cycles of a pattern look like this

$$AABBBCCCCCAABBBCCCCC\ldots$$

Given that the pattern continues, what is the 103^{rd} letter?

Solution. One cycle consists of $2+3+5$ letters. Notice that 103 gives remainder 3 when divided by 10. Hence, the 103^{rd} letter is B. □

61 Set A has five consecutive positive odd integers. The sum of the greatest integer and twice the least integer is 47. Find the least integer.

Solution. Let x be the least integer in the set. Then the other four integers read

$$x+2, x+4, x+6, x+8,$$

and we have $x+8+2x = 47$. Solving this gives $x = 13$. □

62 Which fraction is exactly half-way between $\frac{2}{5}$ and $\frac{4}{5}$?

63 Let n be the number of sides in a regular polygon where $3 \leq n \leq 10$. What is the value of n that result in a regular polygon where the common degree measure of the interior angles is non-integral?

Hint. The sum of the interior angles of a polygon is $180 \times (n-2)$. Then check for $n = \{3, 4, 10\}$. □

64 What is the 200^{th} term of the increasing sequence of positive integers formed by omitting only the perfect squares?

Solution. First we enumerate the number of perfect square less than 200.

$$1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 169, 196.$$

So if we remove these 14 perfect squares, we have $200 - 14 = 186$ integers that are left. Hence, the 200^{th} term of the sequence is 214. □

65 If w, x, y, z are consecutive positive integers such that $w^3 + x^3 + y^3 = z^3$, find the least value of z.

Answer. $z = 6$. □

66 The mean of three numbers is $\frac{5}{9}$. The difference between the largest and smallest number is $\frac{1}{2}$. Given that $\frac{1}{2}$ is one of the three numbers, find the smallest number.

Answer. $\frac{1}{3}$. □

67 Five couples were at a party. Each person shakes hands exactly one with everyone else except his/her spouse. So how many handshakes were exchanged?

Solution. If spouses shook hands too, then there would be $10 \times 9 = 90$ handshakes. But, remember that person X shaking hands with person Y is the same as person Y shaking hands with person X. So there are only half the number of handshakes or 45. Now how many handshakes do we remove for the spouses not shaking hands. Since there are 5 couples there are 5 handshakes to be removed, $45 - 5 = 40$. □

68 What is the positive difference between the sum of the first 20 positive multiples of 5 and the sum of the first 20 positive, even integers?

Solution. The sum of the 20 first positive multiples of 5 is

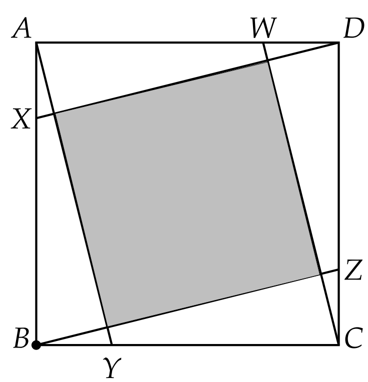
$$5 + 10 + 15 + \cdots + 100 = 5(1 + 2 + 3 + \cdots + 20) = \frac{5 \times 20 \times 21}{2} = 1050.$$

The sum of the first 20 positive even integers is

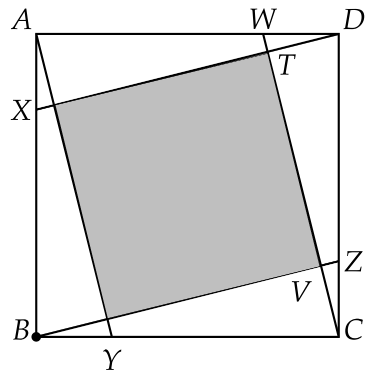
$$2 + 4 + 6 + \cdots + 40 = 420.$$

Then difference is $1050 - 420 = 630$. □

69 The sides of unit square $ABCD$ have trisection points X, Y, Z and W as shown. If $AX : XB = BY : YC = CZ : ZD = DW : WA = 3 : 1$, what is the area of the shaded region?



Solution. The square is a unit one, hence $AB = BC = CD = DA1$. Thus, $AX = BY = CZ = DW = \frac{1}{4}$ and $AW = DZ = YC = BX = \frac{3}{4}$. Hence, the area of triangle ABY is $\frac{1}{8}$. We can prove that the shaded figure is also a square.



Now we shall find the lengths of DT and WT.

By the Pythagoras theorem,

$$WC = \sqrt{DW^2 + CD^2} = \sqrt{\frac{1}{4^2} + 1^2} = \frac{\sqrt{17}}{4}.$$

The area of triangle WDC can be computed in two ways. That is,

$$\frac{1}{2}DW \times CD = \frac{1}{2}TD \times WC = \frac{1}{8}.$$

Hence, $TD = 2 \times \frac{1}{8} \times \frac{4}{\sqrt{17}} = \frac{1}{\sqrt{17}}$. Thus,

$$TW = \sqrt{WD^2 - TD^2} = \sqrt{\frac{1}{16} - \frac{1}{17}} = \frac{1}{4\sqrt{17}}.$$

The area of right triangle WTD is $\frac{1}{2} \times \frac{1}{4\sqrt{17}} \times \frac{1}{\sqrt{17}} = \frac{1}{8\times 17}$. The area of the shaded region is

$$1 - \frac{4}{8} - \frac{1}{34} = \frac{1}{2} - \frac{1}{34} = \frac{16}{34} = \frac{8}{17}.$$

□

70 We have a box with red, blue and green marbles. At least 17 marbles must be selected to make sure at least one of them is green. At least 18 marbles must be selected without replacement to be sure that at least 1 of them is red. And at least 20 marbles must be selected without replacement to be sure all three colors appear among the marbles selected. So how many marbles are there in the box?

Solution. Let r the number of red marbles. Let b the number of blue marbles. Let g the number of green marbles. $r + b = 16$ (the 17th would be green) $g + b = 17$ (the 18th would be red) $r + g = 19$, $b = 16 - r$, $g + 16 - r = 17$, $g - r = 1$, $g + r = 19$, $2g = 20$, $g = 10$, $g - r = 1$, $10 - r = 1$, $r = 9$, $b = 16 - r$, $b = 16 - 9 = 7$, $r + b + g = 9 + 7 + 10 = 26$. □

71 The three-digit integer N yields a perfect square when divided by 5. When divided by 4, the result is a perfect cube. What is the value of N?

Solution. We have $N = 4x^2$ and $N = 5y^3$ for some integers x, y. Hence,

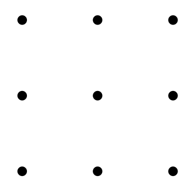
$$x^2 = \frac{4}{5}y^3.$$

The list of perfect cubes between 1 and 1000 is

$$1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.$$

Notice that y^3 is divisible by 5. Among the perfect cubes listed, only 125 and 1000 qualify as multiples of 5. But 1000 is rejected after some inspection. The answer is 125. □

72 How many different sets of three points in this 3 by 3 grid of equally spaced points can be connected to form an isosceles triangle (having two sides of the same length)?



73 Given the list of integer

$$12345678987654321234 56\cdots,$$

find the 1000st integer in the list.

Solution. Notice that 1 appears in the first postion and in the 17th position and the next 1 appears in the 33rd position. In other words, the number 1 appears every 16 entries. □

74 A person write the letters from the words $LOVEMATH$ in the following way

$$LOVEMATHLOVEMATHLOVEMATH\cdots.$$

i) Which letter is in the 2010th place?

ii) Assume that there are 50 letters M in a certain sequence. How many letters E are there in the sequence?

iii) If the letters are to be coloured blue, red, purple, yellow, blue, red, purple, yellow, ... What colour is the letter in the 2010th place?

4 Problems from APMOPS 2010, First Round

1 Find the value of

$$\left(1-\frac{1}{2}\right)\times\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{4}\right)\times\cdots\times\left(1-\frac{1}{99}\right)\times\left(1-\frac{1}{100}\right).$$

2 Find the value of

$$\frac{1}{2}+\frac{1}{2\times 2}+\frac{1}{2\times 2\times 2}+\cdots+\frac{1}{\underbrace{2\times 2\times 2\times\cdots\times 2\times 2\times 2}_{\text{10 of 2s}}}.$$

3 Find the total number of ways to arrange 3 indentical white balls and 3 indentical black balls in a circle on a plane. The two layouts below are considered as one way of arrangement.

4 Given that one and only one of the following statements (mệnh đề) is correct, which one is correct?

i) All of the statements below are correct.

ii) None of the statements below is correct.

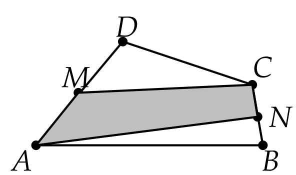
iii) One of the statements above is correct.

iv) All of the statements above are correct.

v) None of the statements above is correct.

5 Let n be a whole number greater than 1. It leaves a remainder of 1 when divided by any single digit whole number greater than 1. Find the smallest possible value of n.

6 M and N are the mid-points of the lines AD and BC respectively. Given that the area of $ABCD$ is 2000 cm^2 and the area of the shaded region $ANCM = x$ cm^2, find the value of x.



7 Your pocket money had previously been decreased by x %. To get back to the same amount of pocket money before the decrease, you need to have an increase of 25%. Find the value of x.

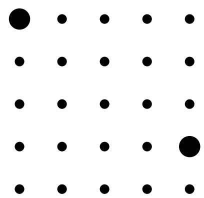
8 A circle of diameter 2 cm rolls along the circumference of a circle of diameter 12 cm, without slipping, until it returns to its starting position. Given that the smaller circle has turned x° about its centre, find the value of x.

9 Find the last digit of the number

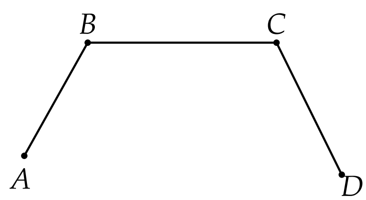
$$\underbrace{2\times 2\times 2\cdots\times 2}_{\text{85935 of 2s}}.$$

10 Three bus services operate from the same bus interchange. The first service leaves at 24 minute intervals, the second at 30 minute intervals and the third at 36 minute intervals. All three services leave the bus interchange together at 0900. Find the number of minutes that has passed when they next leave the interchange together.

11 Twenty five boys position themselves in a 5 by 5 formation such that the distances between two adjacent boys in the same row or the same column are equal to 1 m. The two dark circles indicate a pair of boys whose distance apart is exactly 5 m. Given that there are n pairs whose distance apart are exactly 5 m, find the value of n.



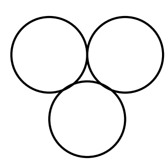
12 When Albert begins walking up slope AB (1 km distance), across level ground BC (12 km distance), and down slope CD (3 km distance), Daniel begins his journey in the opposite direction from D at the same time. Given that the speeds of both traveling up slope, on level ground and down slope are 2 km/h, 4 km/h and 5 km/h respectively, find the number of hours that has passed when they meet.



13 Find the value of

$$1^3 + 2^3 + 3^3 + \cdots + 20^3 + 21^3.$$

14 Three identical circles have at most three points of contact as shown below. Find the least number of identical circles required to have nine points of contact.



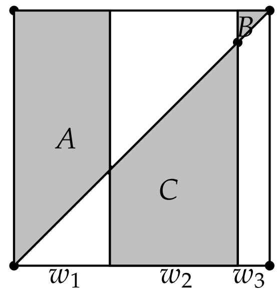
15 A goat in a horizontal ground is tied to one end of 14 m long rope. The other end of the rope is attached to a ring which is free to slide along a fixed 20 m long horizontal rail. If the maximum possible area that the goat can graze is x m^2, find the value of x.

Ignore the dimension of the ring and take π to be $\frac{22}{7}$.

16 The 13 squares are to be filled with whole numbers. If the sum of any three adjacent numbers is 21, find the value of x.

				4				9				x

17 A square is divided into three rectangles of widths w_1, w_2 and w_3 as shown. If $w_1 + w_3 = w_2$ and the areas of the shaded regions A, B and C are 8 cm^2, x cm^2 and 10 cm^2 respectively, find the value of x.

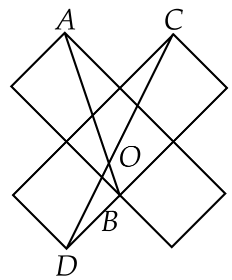


18 Given that

$$S = \frac{1}{\frac{1}{2001} + \frac{1}{2002} + \frac{1}{2003} + \cdots + \frac{1}{2009} + \frac{1}{2010}},$$

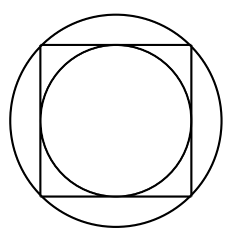
find the largest whole number smaller than S.

19 The figure shown comprises five identical squares. A, B, C and D are vertices of the squares. AB cuts CD at O and angle $AOC = x^\circ$, find the value of x.

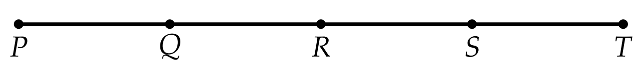


20 Find the smallest whole number that is not a factor of $1 \times 2 \times 3 \times \cdots \times 21 \times 22 \times 23$.

21 A square has its four vertices touching a circle and its four sides touching another smaller circles as shown below. If the area of the larger circle is x times that of the smaller one, find the value of x.



22 P, Q, R, S and T are equally spaced on a straight rod. If the rod is first rotated 180° about T, then 180° about S and finally 180° about P, which point's position remains unchanged?



23 Given that the product of four different whole number is $10,000$, find the greatest possible value of the sum of the four numbers.

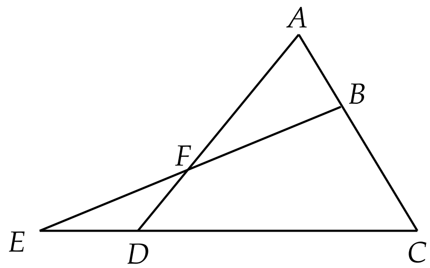
24 An equilateral triangle PQR of side 32cm has three equilateral triangles cut off from its corners to give rite to a hexagon $ABCDEF$. Another equilateral triangle LMN of side x cm gives rise to the same hexagon when subjected to the same treatment. If $AB = 8$cm, $BC = 15$cm, $CD = 9$cm, $DE = 10$ cm, $EF = 13$ cm and $FA = 11$ cm, find the value of x.

25 Given the following three numbers A, B, C

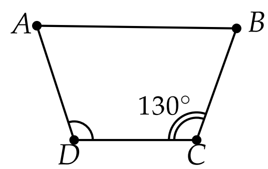
$$A = \underbrace{3 \times 3 \times 3 \times \cdots \times 3}_{\text{40 of 3's}},\ B = \underbrace{5 \times 5 \times 5 \times \cdots \times 5}_{\text{30 of 5's}},\ C = \underbrace{7 \times 7 \times 7 \times \cdots \times 7}_{\text{20 of 7's}},$$

arrange the numbers from largest to smallest.

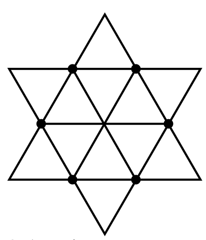
26 B and D lie on AC and CE respectively and AD cuts BE at F. If $BC = 2AB$, $AF = 2FD$, area of $EFD = x$ cm^2 and area of $BCDF = 1750$ cm^2, find the value of x.



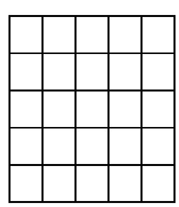
27 In the diagram, $AD = DC = CB$, angle $ADC = 110°$, angle $DCB = 130°$ and angle $ABC = x°$, find the value of x.



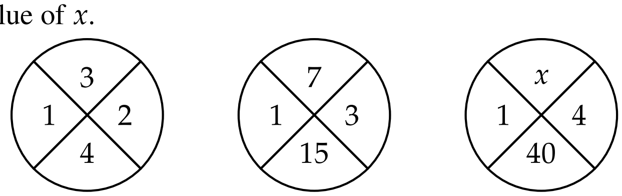
28 The figure comprises twelve equilateral triangles. Find the total number of trapeziums in the figure. Here we define a trapezium to be a 4-sided figure with exactly one pair of parallel sides.



29 The following 5 by 5 grid consists of 25 unit squares. Find the largest number of unit squares to be shaded so that each row, each column and each of the two main diagonal lines has at most 2 unit squares that are shaded.



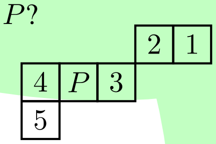
30 Find the value of x.



Asia Pacific Mathematical Olympiad for Primary Schools 2011

First Round: 2 hours

1 If the following figure is folded into the shape of a cube, what is the number opposite to the face marked P?



2 Find the value of

$$\left(1-\frac{1}{2}\right)\times\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{4}\right)\times\cdots\left(1-\frac{1}{2010}\right)\left(1-\frac{1}{2011}\right).$$

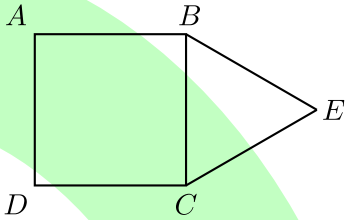
3 A circle of radius 1 m has some points lying on its circumference. Find the minimum number of points such that at least two points are less than 1 m apart.

4 Three sides of a four-sided figure are of lengths 4 cm, 9 cm and 14 cm respectively. If the largest possible length of the fourth side is x cm where x is a whole number, find the value of x.

5 Find the largest prime number that divides the number

$$(1\times 2\times 3\times\cdots\times 97\times 98)+(1\times 2\times 3\times\cdots\times 98\times 99\times 100).$$

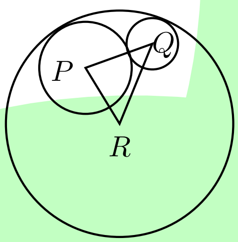
6 $ABCD$ is a square and BCE is an equilateral triangle. If BC is 8 cm, find the radius of the circle passing through A, E and D in cm.



7 Find the value of

$$\frac{19}{20}+\frac{1919}{2020}+\frac{191919}{202020}+\cdots+\frac{\underbrace{1919\ldots19}_{\text{2011 of 19's}}}{\underbrace{2020\ldots20}_{\text{2011 of 20's}}}.$$

8 The following diagram shows a circle of radius 8 cm with the centre R. Two smaller circles with centres P and Q touch the circle with centre R and each other as shown in the diagram. Find the perimeter of the triangle PQR in cm.

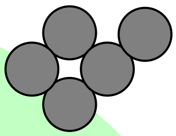


9 Peter and Jane competed in a 5000 m race. Peter's speed was 4 times that of Jane's. Jane ran from the beginning to the end, whereas Peter stopped running every now and then. When Jane crossed the finish line, Peter was 100 m behind. Jane ran a total of x m during the time Peter was not running. Find the value of x.

10 If numbers are arranged in three rows A, B, and C in the following manner, which row will contain the number 1000?

A	1	6	7	12	13	18	19	...
B	2	5	8	11	14	17	20	...
C	3	4	9	10	15	16	21	...

11 The following diagram shows 5 identical circles. How many different straight cuts are there so that the five shaded circles can be divided into two parts of equal areas?



12 A test with a maximum mark of 10 was administered to a class. Some of the results are shown in the table below. It is know that the average mark of those scoring more than 3 is 7 while the average mark of those getting below 8 is 4. Given that none scored zero, find the number of pupils in the class.

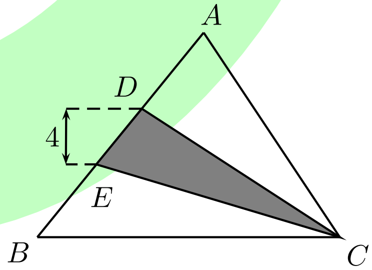
Score	1	2	3	...	8	9	10
Number of pupils	1	3	6	...	4	6	3

13 A test comprises 10 true or false questions. Find the least number of answer scripts required to ensure that there are at least 2 scripts with identical answers to all the 10 questions.

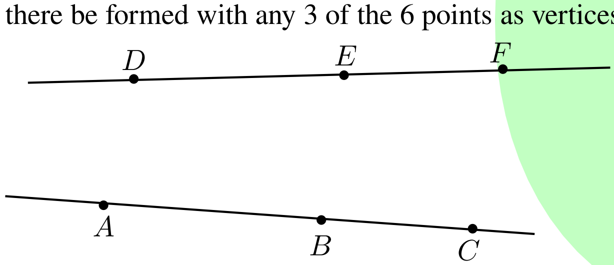
14 P_n is defined as the product of the digits in the whole number n. For example, $P_{19} = 1 \times 9 = 9$, $P_{32} = 3 \times 2 = 6$. Find the value of

$$P_{10} + P_{11} + P_{12} + \cdots + P_{98} + P_{99}.$$

15 ABC is a triangle with $BC = 8$ cm. D and E line on AB such that the vertical distance between D and E is 4 cm. Find the area of the shaded region CDE in cm^2.

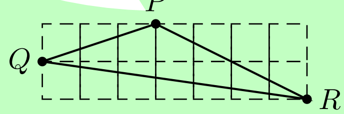


16 The points A, B, C, D, E and F are on the two straight lines as shown. How many triangles can there be formed with any 3 of the 6 points as vertices?

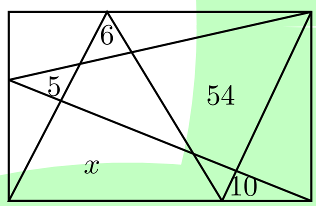


17 A group of 50 girls were interviewed to find out how many books they had borrowed from the school library in April. The total number of books borrowed by the girls in April was 88, and 18 girls had borrowed only 1 book each. If each girl had borrowed either 1, 2, or 3 books, find the number of girls that had borrowed 2 books each.

18 The following diagram shows a triangle PQR on a 2 by 7 rectangular grid. Find the sum of the angle of PQR and angle PRQ in degrees.

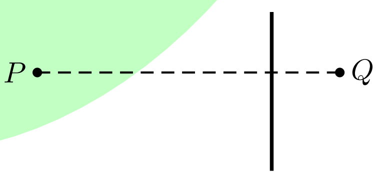


19 The following diagram shows a rectangle where the areas of the shaded regions are 5 cm^2, 6 cm^2, 10 cm^2, 54 cm^2 and x cm^2 respectively. Find the value of x.

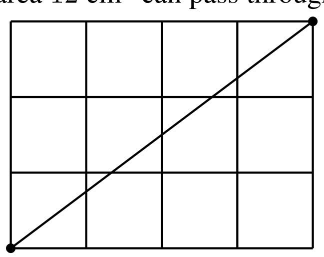


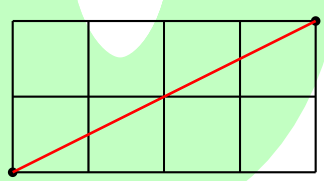
20 The following diagram shows two squares $ABCD$ and $DGFE$. The side CD touches the side DG. If the area of $DEFG$ is 80 cm^2, find the area of the triangle BGE in cm^2.

21 Two points P and Q are 11 cm apart. A line perpendicular to the line PQ is 7 cm from P and 4 cm from Q. How many more lines, on the same plane, are 7 cm from P and 4 cm from Q?



22 The greatest number of points of intersection of the grid lines that the diagonal of a rectangle with area 12 cm^2 can pass through is 3 as shown.

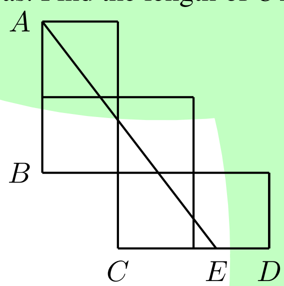




Find the greatest possible number of points of intersection that the diagonal of a rectangle with area 432 cm^2 can pass through.

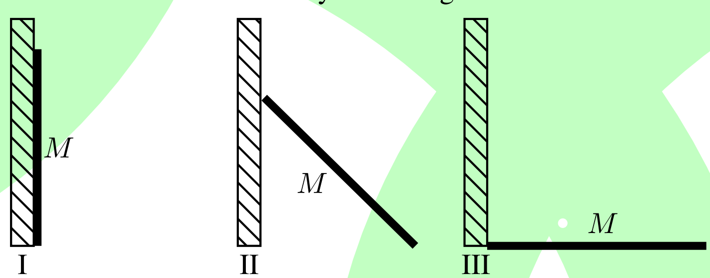
23 Given that $(100 \times a + 10 \times b + c) \times (a + b + c) = 1926$ where a, b and c are whole numbers, find the value of $a + b + c$.

24 The following figure comprises 5 identical squares each of area 16 cm^2. A, B, C and D are vertices of the squares. E lies on CD such that AE divides the 5 squares into two parts of equal areas. Find the length of CE in cm.



25 Whole numbers from 1 to 10 are separated into two groups, each comprising 5 numbers such that the product of all the numbers in one group is divisible by the product of all the numbers in the other. If n is the quotient of such a division, find the least possible value of n.

26 Diagram I shows a ladder of length 4 m leaning vertically against a wall. It slides down without slipping to II, and then finally to a horizontal position as shown in III. If M is at the mid-point of the ladder, find the distance travelled by M during the slide in m.



27 A theme part issues entrance tickets bearing 5-digit serial numbers from 00000 to 99999. If any adjacent numbers in the serial numbers differ by 5 (for example 12493), customers holding such a ticket could use the ticket to redeem a free drink. Find the number of tickets that have serial numbers with this property.

28 S_n is defined as the sum of the digits in the whole number n. For example, $S_3 = 3$ and $S_{29} = 2 + 9 = 11$. Find the value of

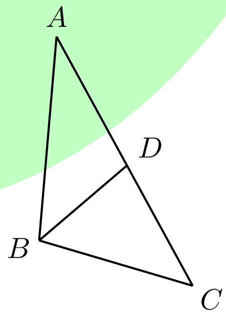
$$S_1 + S_2 + S_3 + \cdots + S_{2010} + S_{2011}.$$

29 Anthony, Benjamin and Cain were interviewed to find out how many hours they spend on the computer in a day. They gave the following replies.

- **Anthony:**
  I spend 4 hours on the computer.
  I spend 3 hours on the computer less than Benjamin.
  I spend 2 hours on the computer less than Cain.
- **Benjamin:**
  Can spends 5 hours on the computer.
  The time I spend on the computer differs from Cain's time by 2 hours.
  The time I spend on the computer is not the least among the three of us.
- **Cain:**
  I spend more time on the computer than Anthony.
  I spend 4 hours on the computer.
  Benjamin spends 3 hours on the computer more than Anthony.

If only two of the three statements made by each boy are true, find the number of hours that Anthony spends on the computer in a day.

30 ABC is a triangle and D lies on AC such that $AD = BD = BC$. If all the three interior angles of triangle ABC, measured in degrees, are whole numbers, find the greatest possible value of angle ABC in degrees.



Lời giải

Lời giải APMOPS 2011 sẽ được nhóm giáo viên cập nhật và trình bài trong lớp học APMOPS 2012 trên website hexagon.

Lớp ôn thi APMOPS 2012

Chuẩn bị cho các em (sinh năm 1998, 1999) thi APMOPS năm 2012, chúng tôi đã chuẩn bị các bài giảng, bài tập chuyên đề về số học, hình học, suy luận logic, đại số bám theo chương trình, đề thi của APMOPS trong những năm qua.

Lớp học được giảng dạy bởi các thầy giáo có kinh nghiệm, công tác tại các trường đại học, tạp chí toán học

- Nguyễn Tiến Lâm
- Hoàng Trọng Hảo
- Phạm Văn Thuận
- Nguyễn Việt Hùng
- Trần Quang Hùng
- Trần Tuấn Anh

Mỗi thầy giáo sẽ phát triển một hướng chuyên đề và có thảo luận nhóm, giúp cho các bài giảng vừa đảm bảo được tính sâu sắc cũng như bao quát khá đủ các nội dung thi APMPOS.

- **Lịch học:** 8h ngày chủ nhật, tại Nguyễn Trãi, Thanh Xuân, Hanoi (chi tiết báo học sinh sau) học trong 15 buổi học.
- **Đăng ký học:** Đăng ký qua mục Liên hệ của website **www.hexagon.edu.vn** từ ngày 30 tháng 4 đến ngày 1 tháng 6.
- **Khai giảng:** Tuần đầu tháng 6 năm 2011.

HEXAGON®
inspiring minds always

Singapore-Asia Pacific Mathematical Olympiad for Primary Schools (Mock Test for APMOPS 2012)

Practice Problems for APMOPS 2012, First Round

1 Suppose that today is Tuesday. What day of the week will it bee 100 days from now?

2 Ariel purchased a certain amount of apricots. 90% of the apricot weight was water. She dried the apricots until just 60% of the apricot weight was water. 15 kg of water was lost in the process. What was the original weight of the apricots (in kg)?

3 Notice that $1 - 2 = -1$, $1 - (2 - 3) = 2$, and $1 - (2 - (3 - 4)) = -2$. What is the value of

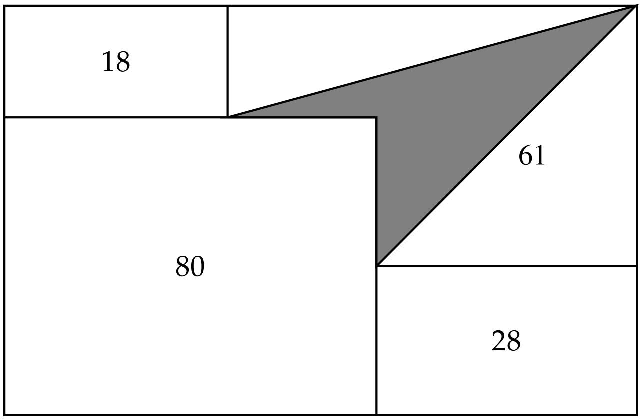
$$1 - (2 - (3 - (4 - \ldots - 100)))\ldots)?$$

4 You are preparing skewers of meatballs, where each skewer has either 4 or 6 meatballs on it. Altogether you use 32 skewers and 150 meatballs. How many skewers have only 4 meatballs on them?

5 The numbers $1, 2, 3, \ldots, 100$ are written in a row. We first remove the first number and every second number after that. With the remaining numbers, we again remove the first number and every second number after that. We repeat this process until one number remains. What is this number?

6 P and Q are whole numbers so that the ratio $P : Q$ is equal to $2 : 3$. If you add 100, 200 to each of P and Q, the new ratio becomes equal to $3 : 4$. What is P?

7 The following figure consists of 3 smaller rectangles and a hexagon whose areas are 18 cm^2, 80 cm^2, 28 cm^2, and 61 cm^2. If all the side-lengths in centimetre of the rectangles and the hexagon are integers, find the area of the shaded region.



8 Suppose that a, and b are positive integers, and the four numbers

$$a+b,\ a-b,\ a\times b,\ a\div b$$

are different and are all positive integers. What is the smallest possible value of $a+b$?

9 You are given a two-digit positive integer. If you reverse the digits of your number, the result is a number which is 20% larger than your number. What is your number?

10 The number $N = 111\ldots1$ consists of 2006 ones. It is exactly divisible by 11. How many zeros are there in the quotient $\frac{N}{11}$?

11 This mock test, prepared by Thuận from **HEXAGON** centre in Hanoi, consists of 30 problems. Pupil A gets a score that is an odd multiple of 5 and pupil B gets a score that is a even multiple of 7. The mark of each problem is an integer and each of the two pupils' score is an integer the difference of which is 3 and sum is less than 100. Find the higher score of the pupils.

12 In a large hospital with several operating rooms, ten people are each waiting for a 45 minute operation. The first operation starts at 8:00 a.m., the second at 8:15 a.m., and each of the other operations starts at 15 minute intervals thereafter. When does the last operation end?

13 Each of the integers 226 and 318 have digits whose product is 24. How many three-digit positive integers have digits whose product is 24?

14 A work crew of 3 people requires 20 days to do a certain job. How long would it take a work crew of 4 people to do the same job if each of both crews works at the same rate as each of the others?

15 Each of the nine numbers $1, 2, \ldots, 9$ is to be placed inside the cell of the following 3×3 grid once. The product of three numbers in each row and in each column is given: the product of numbers in the first column is 35, the second column is 96, the product of three numbers in the first row is 54, etc. Find the value of $p + q$.

35	96	108	
			54
	p		42
		q	160

16 N is a whole number greater than 2. The six faces of a $5 \times 5 \times N$ block of wood are painted red and then the block cut into 25 $N 1 \times 1 \times 1$unit cubes. If exactly 92 unit cubes have exactly two faces painted red, what is N?

17 Eleven people are in a room for a meeting. When the meeting ends, each person shakes hands with each of the other people in the room exactly once. The total number of handshakes that occurs is x. Find the value of x

18 Mark has a bag that contains 3 black marbles, 6 gold marbles, 2 purple marbles, and 6 red marbles. Mark adds a number of white marbles to the bag and tells Susan if she now draws a marble at random from the bag, the probability of it being black or gold is $\frac{3}{7}$. The number of white marbles that Mark adds to the bag is n. Find the value of n.

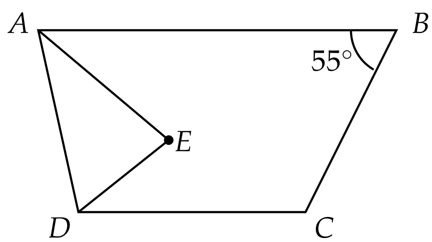
19 A number line has 40 consecutive integers marked on it. If the smallest of these integers is ˘11, what is the largest?

20 If I add 5 to $\frac{1}{3}$ of the number, the result is $\frac{1}{2}$ of the number. What is the number?

21 If n is a positive inter such that all the following numbers are prime, find the value of n.

$$5n - 7,\ 3n - 4,\ 7n + 3,\ 6n + 1,\ 9n + 5.$$

22 The diagram below shows a trapezium with base AB and CD, $\angle ABC = 55°$. E is inside the trapezium such that AE bisects angle BAD and ED bisects angle ADC. If the measure of $\angle DAE$ is $x°$, find the value of x.



23 Let $\lfloor a \rfloor$ denote the integer not exceeding a. If n is a whole number, $n \geq 2$, find $\lfloor p \rfloor$

$$p = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{3^3} + \cdots + \frac{n}{2^n}.$$

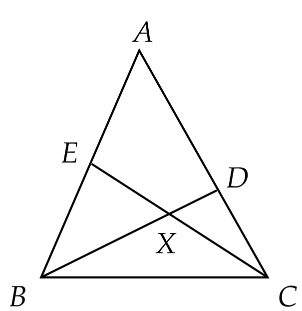
24 How many numbers are there that appear both in the arithmetic sequence $10, 16, 22, 28, \ldots, 1000$ and the arithmetic sequence $10, 21, 32, 43, \ldots, 1000$?

25 The whole numbers from the set $\{1, 2, 3, \ldots, 2020\}$ are arranged in the following manner.

1					
2	3				
4	5	6			
7	8	9	10		
11	12	13	14		
15	16	17	18	19	
20	21	22	23	24	25
$\vdots$					

What is the number that will appear directly below the number 2012?

26 In triangle ABC, produce a line from B to AC, meeting at D, and from C to AB, meeting at E. Let BD and CE meet at X. Let triangle BXE have area 4 cm^2, triangle BXC have area 8, and triangle CXD have area 10. Find the area of quadrilateral $AEXD$.



27 The length of a rectangle is 1 cm more than twice its width. If the perimeter of the rectangle is 74 cm, what is the area of the rectangle?

28 At Maths Project Olympiad, prize money is awarded for 1st, 2nd and 3rd places in the ratio 3 : 2 : 1. Last year John and Ashley shared third prize equally. What fraction of the total prize money did John receive?

29 A goat is tied to one of the corners of a rectangular barn on a rope that is 50 feet long. The dimensions of the barn are 40 feet by 30 feet. Assuming that the goat can graze wherever its rope allows it to reach, what is the square footage of the grazing area for the goat?

30 Find a nine-digit positive integer, $d_1d_2d_3d_4d_5d_6d_7d_8d_9$ with distinct digits, so that 1 divides d_1, 2 divides d_1d_2 , 3 divides $d_1d_2d_3$, ... , 8 divides $d_1d_2d_3$ $d_4d_5d_6d_7d_8$, and 9 divides $d_1d_2d_3d_4d_5d_6d_7$ d_8d_9. Only digits $1, 2, \ldots, 9$ are to be used.

Answer Key

Question	Answer	Question	Answer	Question	Answer
1	Thursday	11	28	21	30
2	20	12	11AM	22	90°
3	-50	13	21	23	1
4	21	14	15	24	16
5	64	15	6	25	2075
6	200	16	19	26	50
7	20	17	55	27	200 m^2
8	8	18	4	28	$\frac{1}{10}$
9	45	19	28	29	2000π
10	1002	20	30	30	29381654729