Selected Questions from

ABACUS

MATH CHALLENGES

HEXAGON

for your preparation of APMOPS
AMC, IMC, Tournaments of the Towns

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ABACUS Math Challenge # 10 HEXAGON

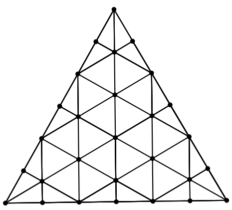
1. You may use 5 colors to color all the vertices of an equilateral triangle. How many different ways can you do this? Two colorings are different if the final results cannot be matched by rotations and/or reflections?

2. The heights of the starting 5 players of the New York Knicks basketball team this year are all different. How many different ways can they march onto the court in a line, so that none of them is in between two taller players?

3. Cut up a 2×5 rectangle into four similar pieces.

4. I gave a value to every vertex of a cube. The value of an edge is the sum if the values of the vertices at its ends. The value of a side is the sum of the values of the edges surrounding it. The value of a cube is the sum of the values of its sides. What is the value of the cube if the sum of the values of its vertices is 128?

5. How many triangles are there on the following picture?



6. How many such 8-digit number-series are there containing only the digits zero and 1, with no two 1's next to each other?

7. Find the smallest 3-digit number from which you cannot create a prime number by changing one of its digits.

8. The minute hand of a clock is exactly above the hour hand for example at 12 noon. When will they be in the same straight line again next time?

9. Can you cut up a square into two congruent polygons where the number of sides the polygons have is

a) 7

b) 8?

10. In the following addition different letters mean different numbers and the same letters mean the same numbers. What number is $ABCDEFD$?

$$\begin{array}{cccccccc} & A & B & C & D & E & F & D \\ & & B & C & D & E & F & D \\ & & & C & D & E & F & D \\ & & & & D & E & F & D \\ & & & & & E & F & D \\ & & & & & & F & D \\ + & & & & & & & D \\ \hline & A & A & A & A & A & A & A \end{array}$$

11. Rabbit made presents for all of her costumers and Eeyore for Easter. But Rabbit's costumers made presents for Eeyore, Rabbit and for each other, also. Then they all gathered at Winnie-the-Pooh's house and put their presents under the tree. Winnie and Tigger counted the presents, then Tigger told Winnie: "Hmm, the number of presents is such a 3-digit number that is greater than 200, and every digit of it is a square number. Tell me, my dearest bear friend, how many costumers does Rabbit have?"

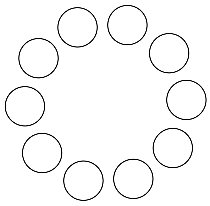
12. A soccer ball is a polyhedron that has 32 faces which are either regular pentagons or regular hexagons. How many edges does a soccer ball have?

13. A bird trader sold 10 bird cages with a bird in each, but the buyers usually wanted to buy a different cage for the bird of their choice than the one the bird was actually in. The trader, for safety reasons, switched the cages in such a way that he used an extra, empty cage and he always moved only one bird into an empty cage. At most, how many such moves do you need to satisfy all 10 costumers, even in the worst case scenario?

14. Take all those 5-digit numbers in which the sum of the digits is 37. Out of these numbers, how many are even and how many are odd?

15. Can you divide 10000 pebbles into 100 groups so that every group has a different number of pebbles, but if you make two groups out of any one of these 100 groups, the same thing is not true for the 101 groups?

1. Can you write the numbers 0, 1, 2, 3, ..., 8, 9 on the circumference of a circle so that the sum of any three consecutive numbers is less than 16 but more than 11?



2. Find all those 3-digit numbers in which every digit is a prime number and the number itself is divisible by these primes.

3. How many such number-pairs are there for which the greatest common factor is 7 and the least common multiple is 16940?

4. If you multiply the sum of the first two digits and the sum of the last two digits of a 4-digit positive whole number, you can get 187. How many such numbers are there?

5. Take 10 number cards with the following numbers on them (one number on each card): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Make a pile of them by putting one on top of another, and hold the pile in your hands. Now put the top card on the table, put the next top card on the bottom of the pile in your hands, put the next top card on the table, the next top card on the bottom of the pile, and so on, until you run out of cards. In what order do you have to stack the cards at the beginning if you want the cards to be on the table at the end in the following order : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10?

6. What is the sum of all those positive whole numbers that are smaller than 2000, and the sums of their digits are even?

7. Two kinds of people live on an island: honest and liar. Honest people always say the truth, liars always lie. One day we asked everybody in a group of 5 people from this island who know each other: "How many of you are honest?" We received the following answers: 0, 1, 2, 3, 4. How many of them are honest?

8. Alex and Burt took their rabbits to the market to trade them. Each of them got as many dollars for each of their rabbits as many rabbits they each took to the market. But, because their rabbits were so beautiful, they each got as many extra dollars for their rabbits as many rabbits they each took to the market. This way Alex received $202 more than Burt. How many rabbits did they each take to the market?

9. How many triangles are there with whole-number-long sides and a perimeter of 9?

10. In a league the teams received a total of 420 points. You got 2 points for a victory, 1 points for a tie, and 0 points for a loss. How many teams were there in the league if every team played every team twice?

11. A positive whole number is "beautiful" if it is equal to the product of its true divisors (divisors that are different from 1 and the number itself). What is the 10th smallest beautiful number?

12. Out of two candles with different length and thickness, the 10 cm long one burns away in 5 hours, and the other one in 6 hours. If you start burning them at the same time, in 2 hours they have the same length. How long was the other candle originally?

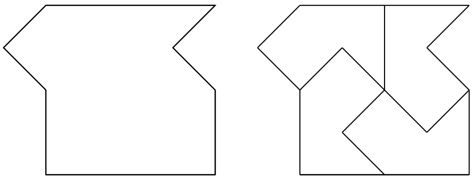
13. The sum of 10 positive whole numbers is 1001. What could the highest possible greatest common factor of these numbers be?

14. Find all those 3-digit numbers that are divisible by 7, and they give the same remainder when divided by either 4, 6, 8, or 9.

1. The sum of 49 positive whole numbers is 999. How high could the greatest common factor of these numbers be?

2. Are there three such prime numbers that have a sum of 1234 and a product of 87654321?

3. After thinking for a long time, Julie divided the first figure into 4 parts of the same size and same shape. Now you have to divide the first figure into 5 parts of the same size and same shape. How can it be done?



4. Two bicycle clubs organize a tour together. At the meeting in the morning members great each other with a handshake. Everybody shakes hands with everybody once. There were a total of 231 handshakes but 119 of them happened between members of the same club. How many members came from each club?

5. Seven dwarves are sitting around a round table with a mug in front of each with some milk in it. (Some mugs might be empty.) There is a total of a half a liter of milk in the mugs. One dwarf stood up and distributed his milk evenly among the other dwarves. Then, one by one, everybody towards his right did the same thing. After the seventh dwarf distributed his milk, everybody ended up having the same amount of milk than what they started with originally. How much milk was in each mug?

6. You had to answer 20 questions on a test. For every correct answer you get 5 points, but for every incorrect answers you lose 2 points. If you do not answer a question, you get 0 points for it. One of your classmates received 48 points on this test. How many correct answers did she give?

7. Find the greatest whole number with all different digits in which the sum of any three digits is not divisible by 19.

8. Find the smallest such number created by the digits 3 and 7 only, that is divisible by both 3 and 7.

9. What is the sum of all those 6-digit numbers that you can create by a different order of the digits 1, 2, 3, 4, 5, 6?

10. What is the smallest positive whole number that ends with 1997 and divisible by 1999?

11. Find a positive whole number which is the product of three consecutive whole numbers, and it is the product of six consecutive whole numbers, also.

12. What is the sum of all the digits of the following numbers: 1, 2, 3, ..., 1000?

13. Find the smallest positive whole number that does not contain the digit 9, but it is divisible by 999.

14. Find such a 5-digit number that is equal to 45 times the product of its digits.

15. How many such 15-digit numbers are there that are divisible by 11, and contain only the digits 3 and 8?

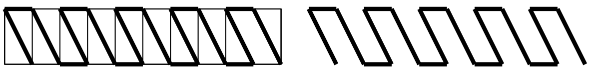
16. Find all those 4-digit numbers that end with the digit 9, and divisible by every one of their digits.

17. Find the smallest positive whole number that is equal to the product of the sum of its digits and 1998.

1. How many positive 4-digit whole numbers have all different digits and are divisible by 9 and 25?

2. One side of a parallelogram is twice as long as its other side. Its perimeter is 24 cm, and its area is 16 square cm. Find the heights and the measures of the angles of this parallelogram.

3. The diagonals of the 10 rectangles on the diagram below have a 60 degrees angle to the horizontal. How long is the shaded line if the total width of the 10 rectangles is 50 cm? (Below, we show the shaded line again without the rectangles.)



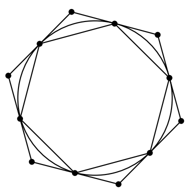
4. Place a circle, a square, and an equilateral triangle on top of each other so that their lines would have the most intersection points. What is this number of intersection points? (You may choose the size of each figure.)

5. What is the greatest two-digit divisor of 22227777?

6. We wrote down all the 3-digit numbers in an increasing order, so that we used a red pen for the even numbers' digits and a blue pen for the odd numbers' digits. How many red 8's are there on the paper?

7. Ben is 100 meters, Colby is 300 meters ahead of Andrew. The three boys run by a uniform speed. Andrew catches up with Ben in 6 minutes, and another 6 minutes he catches up with Colby, too. How long does it take Ben to catch up with Colby?

8. A circle is drawn on a piece of paper. A regular hexagon is inscribed in the circle and another regular hexagon is described around the circle. We know that the area of the smaller hexagon is 3 units. What is the area of the larger hexagon?



9. Find all those 2-digit numbers that are divisible by both the sum and the product of their digits.

10. Using all of the digits 1, 2, 3, 4, and 5, make all the 5-digit numbers. What is their sum?

11. A father distributed a basket of plumbs between his sons in the following way: he gave one plumb and 1/9 of the rest of the plumbs to the first son, he gave 2 plumbs and 1/9 of the rest of the plumbs to the second son, 3 plumbs and 1/9 of the rest of the plumbs to the third son, and so on. How many sons does he have and how many plumbs did they each get if everybody got the same amount?

12. How many such 3-digit numbers are there where the sum of the digits is 15, and the number is divisible by 15?

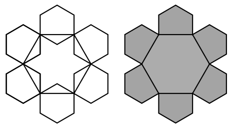
13. What is the smallest multiple of 36 that contains only the digits 5 and 0 in its form in base 10?

14. Timea wrote a few numbers on a piece of paper. She realized that the product of each number with itself is written on the paper, also. Find the sum of all these numbers she wrote on the paper.

15. How many such 4-digit numbers are there in which the sum of the first two digits is the same as the sum of the last two digits?

16. What is the maximum number of months with 5 Sundays that could occur in a year?

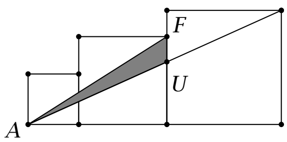
1. A company asked a designer to come up with a trade-sign. The designer used 2 different regular hexagons, and this is how



he came up with his final proposal. (See diagram below.) What is the ration of the areas of the gray and the white regions in the diagram on the right?

2. In a weekend soccer league every team plays every (other) team exactly once. The head of the league just finished the schedule of the games when a few more teams joined the league. Now he had to schedule 37 more games. How many teams were there originally, and how many new teams joined the league?

3. The sides of the squares on the diagram below are 2 cm, 3 cm, and 5 cm.

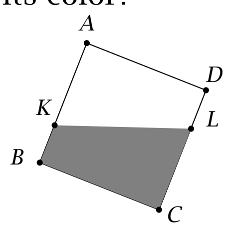


How many square centimeters is the area of the shaded triangle FAU?

4. We glue together 27 regular dice into a $3\times3\times3$ cube. What is the least amount of dots you can see on this cube? (On the regular dice then number of dots are 1 to 6, and the dots on the facing sides add up to 7.)

5. We wrote down the 3-digit numbers one after another in a row, so that the digits of the even numbers are written in red, and the digits of the odd numbers are written in blue. What is the 2005th digit, and what is its color?

6. Fill up a cube with 12-cm edges 5/8 of the way with water, then tilt it around one edge. The diagram below shows a cross section of the cube with the

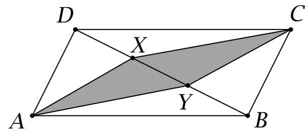


horizontal line representing the water level in it. You know that LC is twice as long as KB. How long is LC?

7. I have a red and a white candle. They have different heights and different thicknesses. I light them at the same time and notice that after 45 minutes they are the same height. The red candle burns down in 90 minutes, the white one in 2 hours. How many times taller was the red candle originally than the white candle?

8. There is a 6m by 8m sized rectangular shaped barn standing in the middle of a meadow. They tied a dog to one of the corners of the barn on a leash that is as log as the diagonal of the barn. What is the area of the territory of the dog?

9. Let points X and Y divide the diagonal BD of paralellogram



$ABCD$ into three congruent (equally long) sections. What is the ratio of the areas of $ABCD$ and $AXCY$?

10. The sum of the three different edges of a rectangular column is 35 cm. If we reduce the height of the column by 3 cm, increase its width by 3 cm, and take only a third of its length, we get a cube. How did the volume of this column change?

11. A mysterious number has 246912 digits. Each of the first 123455 digits is 3. The 123456th digit is unknown. Each of the last 123456 digits is 6. The number is divisible by 7. What is the mysterious number?

12. What is the sum of the digits of all the numbers from 1 to 2006?

13. There were 60 dancers at a party. Mary danced with the least number of boys, with 7 of them, Lucy danced with 8, Sara with 9, and so on, while the last girl danced with all the boys. How many boys and how many girls were at the party?

14. Timea, who is always in a hurry, went up the escalator by making one step every second. This way it took her 20 steps to get upstairs. Next days she made 2 steps every second on her way up, and this time it took her 32 steps to get upstairs. How many steps would it take her to go upstairs if the escalator did not work?

1. The number 3 can be written in 4 different ways as the sum of positive whole numbers: $3, 2 + 1, 1 + 2$, and $1 + 1 + 1$. (The order of the addends is important!) How many different ways can you write 20 as a sum of positive whole numbers?

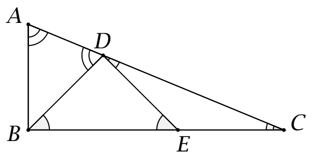
2. On the diagram below you have to get from the marked field containing 2 to the marked field containing 8.

2	3	2	4	2	5	2	6
1	2	3	1	2	3	1	2
4	5	6	7	7	6	5	4
9	8	7	6	5	4	3	2
8	3	2	2	1	1	3	4

You can step on each square field no more than once. From each field you can step only horizontally or vertically. (No diagonal steps are allowed.) Add up the numbers on the fields you step on. What is the greatest sum you can get?

3. A fully charged cellular phone can work in a standby position (which means that we do not use it for making phone calls) for 72 hours, or we can talk continually for 3 hours on it, and then we have to recharge it again. This cell phone, after it was fully charged, was in standby position for 27 hours, and the owner conducted a 45-minute conversation, also. How many more minutes could we talk on this phone?

4. Find the angles of the right triangle if we could cut it up to three isosceles triangles the following way:



5. There are 8 identical boxes in which there are 1, 2, 4, 8, ..., 128 pearls, but we do not know which box has how many pearls in it. Cecilia picks a few of the boxes, and gives the rest of the boxes to Mary. When they both opened their boxes, it turned out that Cecilia received 31 more pearls than Mary. How many boxes did Cecilia choose, and how many pearls were in them each?

6. A math teacher was hit by a car, which drove away right after the accident. The victim could remember only that the sum of the digits of the 4-digit number on the plate of the car was 6. He noticed also that the letters at the beginning of plate number were his own initials (TD), and that the two middle digits were identical. In the middle of the night after the accident he also realized that the sum of the different prime factors of the number on the plate is 172. Next morning he called the police to let them know the plate number of the car. What was it?

7. At least how many consecutive integers do you have to multiply in order for the product to be divisible by 2004 for sure, no matter how you pick that many consecutive numbers?

8. Joe can build a brick wall alone in 9 hours. Pete can build the same wall alone in 10 hours. If the two of them work together they lay a total of 10 bricks less every hour than each working alone, but they get the job done in 5 hours. How many bricks are there in the wall?

9. Peter has 100 books. One day he rearranged them on his bookshelf. He put half of the books on the middle self to the bottom shelf. Then he took a third of the books, which were originally on the bottom shelf, and put one third of these on the middle shelf, and the rest on the top shelf. Finally he took 10 books from the top shelf and put half of them on the middle shelf, and the other half on the bottom shelf. Now every shelf had the same numbers of books on them as originally. How many books are on each shelf?

10. We painted a few sides of a cube, and then we cut it up to smaller but equally sized cubes. We got 45 smaller cubes with no paint on them. How many sides of the original cube did we paint?

11. Grandma bought 2 candles. The red is 1 cm longer than the blue candle. In the afternoon of the Day of Christmas at 17:30 she lit the red candle, at 19:00 she lit the blue candle, also, and let them burn until they were finished. The two candles had the same length at 21:30. The red was finished at 23:30, and the blue was finished at 23:00. How long was the red candle originally?

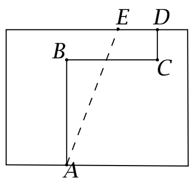
1. The digits of a 4-digit number from left to right are odd, even, odd, and even. When you double this number, you get a number which contains only even digits, but only the last digit of half of the original number is even. Find all of these 4-digit numbers.

2. In this addition same letters mean same digits, different letters mean different digits. What number is $FIVE$?

$$\begin{array}{r} O\ N\ E \\ +\ F\ O\ U\ R \\ \hline F\ I\ V\ E \end{array}$$

3. There were 60 dancers at a party. Mary danced with the least number of boys, with 7 of them, Lucy danced with 8, Sara with 9, and so on, while the last girl danced with all the boys. How many boys and how many girls were at the party?

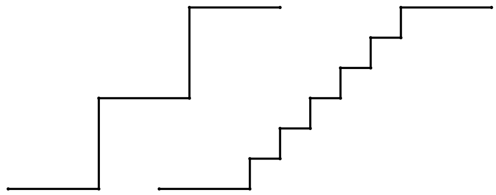
4. Two pieces of land are separated by the line $ABCD$, as shown on the diagram below. $AB = 30$ m, $BC = 24$ m, and $CD = 10$ m. AB, CD, and BC are parallel to the sides of the rectangle. We want to straighten the border line by the AE straight line, so that the areas of the two lands does not change. How far is E from D?



5. Timea, who is always in a hurry, went up the escalator by making one step every second. This way it took her 20 steps to get upstairs. Next days she made 2 steps every second on her way up, and this time it took her 32 steps to get upstairs. How many steps would it take her to go upstairs if the escalator did not work?

6. Anne, Bea, Cecilia and Dori found a secret staircase leading to the attic. Nobody used it for a long time, so it was covered with dust. The girls ran down on it side-by-side as fast as they could: Anne stepped on every other step, Bea used every 3rd step, Cecilia used every 4th step, and Dori used only every 5th step. Everybody started from the step on top. Once they all got downstairs, they realized that their every step is shown in the dust, and that only the very top and the very bottom steps have all of their shoe prints on them. How many steps have only one shoe print on them?

7. A 1-meter wide concrete stairway at the terrace has two 60 cm high and 60 cm deep steps. We would like to have 6 smaller steps instead, so that the steps are still equally high and equally deep. We do not want anything demolished, we just want to add stairs made of concrete only. How many cubic meters of concrete do we need the least?



8. How many positive 3-digit numbers are there in which the sum of the digits is odd?

9. There were 5 chess players on a tournament. Everybody had one game with everybody. You get 1 point for winning, half a point for a draw, and zero point for losing a game. We know that:
The winner of the tournament had no draw.
The second place winner did not lose any game.
Everybody finished the tournament with a different number of points.
How many points did each player finished with?

10. The remainders when the 5-digit number abcde is divided by 2, 3, 4, 5, and 6 are a, b, c, d, and e, in that order. Find this 5-digit number.

11. We arranged the consecutive positive whole numbers in the chart below. (The first number in each row indicates how many numbers we wrote in that row.) Which row contains the number 2015?

1							
2	3						
4	5	6	7				
8	9	10	11	12	13	14	15
...	...	...	...	...	...	...	...

1. A 5 cm long snail wanted to climb out of a dry well using the vertical walls of the well. The snail was rested and climbed up 10 body lengths in the first minute, 9 body lengths in the second minute, and so on. After the 10th minute the snail stops to rest for awhile. After resting a little, it continues to climb the same way as before. The snail started at the bottom of the well, but half way up it slipped and slid back down to one quarter of the whole depth. Here it rested again and then after 10 minutes of climbing the same way, it was still only at 2/3 of the way up. How deep is this well?

2. Several consecutive sheets fell out of a thick book. On the sheets that fell out, the lowest page number was 387, and the highest page number on these pages had the same digits of 387 but in a different order, of course. How many sheets did fall out of the book?

3. In a heard of sheep there are two sheep that limp on their front right legs, and there are three sheep that limps on their front left legs. Exactly 4 sheep do not limp on their front right legs, and exactly 5 sheep do not limp on their front left legs. At least how many sheep are there in the heard?

4. How many 3-digit numbers have exactly one digit 5 in them?

5. Start with the following series of letters: *AAAABBBB*. In every step change the order of two neighboring letters. At least how many steps do you need to create the *ABABABAB* series?

6. Seven cups are standing in a row. One of them is made of Gold, the others are made of a mixture of Gold and Copper, but they all look alike. Mark the cups in order by A, B, C, D, E, F, and G. If you count them back and forth in the order of *ABCDEFGFEDCBABCD*... starting with 1, then at 1000 you will be at the golden cup. What letter marks the golden cup?

7. Delete a few digits from the 17-digit number

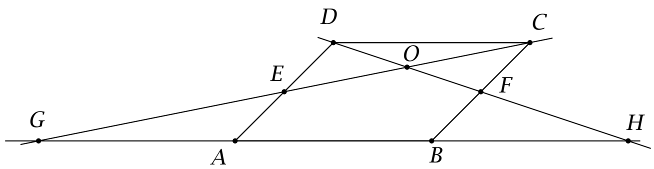
82077875072562386

so that you get the greatest possible number that is divisible by 36.

8. You have three number cards. Each card has a digit on it that is different from the other two digits. Using these cards, create all possible one-digit, two-digit, and three-digit numbers. The sum of these numbers is 5635. What are the digits on the cards?

9. Sam transferred an ever-growing bean plant into his garden from the greenhouse. On the first day the plant grew a half of its original height. On the second day it grew $\frac{1}{3}$ of its height at the beginning of that day. On the third day it grew $\frac{1}{4}$ of its height at the beginning of that day, and so on. How many days did it take the plant to grow until it became as tall as 100 times its original height?

10. Quadrilateral $ABCD$ is a parallelogram. Its area is 24 area units. Points E and F are the midpoints of sides AD and BC. What is the area of triangle OGH?



11. John, Frank and Peter are competing in a league of the three of them. Everybody plays the same number of games with everybody. There is no tie, somebody always wins a game. Frank won twice against Peter and four times against John. John won four times against Peter and three times against Frank. Peter won 5 times against Frank and three times against John. How many games were there all together, and how many games did each player play?

1. Teri thought of a positive whole number. She told me that it has 12 positive divisors, including 6 and 25. What is Teri's number?

2. Six people are estimating how many balls there are in a box. The guesses are: 52, 59, 62, 65, 49, 42. Nobody guessed it right, and the differences between the guesses and the actual number of balls are: 1, 4, 6, 9, 11, 12. How many balls are in the box?

3. One day 12 children from a class went to see a movie. On another day 9 children from the same class went to see a puppet show. 5 children attended both programs, and 10 children from the class did not go to either program. How many children are there in the class?

4. The sum of the digits in the number 2345678923456789 is 88. Take out a few digits so that the remaining number is the greatest number you can get in which the sum of the digits is 44.

5. One third of the farmers goats gave birth to a baby goat each. A quarter of the babies were eaten by wolves, half of the rest of the babies got lost in the woods, and two thirds of the rest died of illness. In his sorrow, the poor farmer sold his remaining two baby goats. How many goats does he have?

6. Imagine a city with a central metro station from which the metro lines are running in straight lines in different directions. Every line has 12 stations on it. The final stops (different from the central station) of these metro lines are connected with a circular metro line with no additional station on it. This metro line has 11 stations. How many stations are there all together in this metro system?

7. Find the smallest positive whole number that is divisible by 5 and the sum of its digits is 99.

8. In how many 4-digit numbers is the sum of the digits 5?

9. Write down the positive whole numbers from 1 to 100 directly one after the other. From the number 123456789101112...99100 you got this way, pull out ten digits so that the remaining number is the greatest possible. What digits did you pull out?

10. Find a 6-digit number that gets 6 times greater when you move its last 3 digits to the front of the number without changing the order of these three digits.

11. Four people, A, B, C and D, stand at the end of a bridge. It is very dark and they have only one torch. The bridge can support the weight of no more than two people at a time, so when a group of two people would reach the other end of the bridge one person would have to carry the torch back. A, B, C and D take 5, 10, 20 and 25 minutes respectively to cross the bridge. Assuming that when two people cross the bridge together they take the time of the slower person, find the minimum time the four of them can cross the bridge.

12. Find the 4-digit number $\overline{abcd}$ so that the sum of the four numbers $\overline{abcd} + \overline{abc} + \overline{ab} + a = 2006$.

13. Sometimes you can see interesting things looking at the date of a day. How many days are there in a year when the number of the day is divisible by the number of the month?

14. Positive real numbers are arranged in the form:

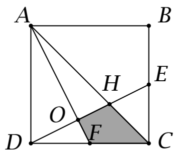
1	3	6	10	15	...
2	5	9	14	...	
4	8	13	...		
7	12	...			
11	...				

Find the number of the line and column where the number 2002 stays.

1. The ages of a father and his two kids are all different exponents of the same prime number. A year ago the ages of all three of them were primes. How old are they now?

2. Find prime numbers x, y, and z, for which $2x + 3y + 6z = 78$.

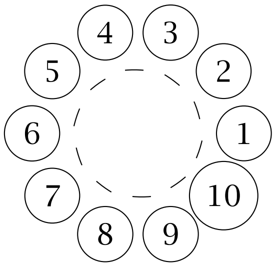
3. E and F are the midpoints of the sides of square $ABCD$. What fraction of the area of the square is the shaded area?



4. What is the sum of all the positive 3-digit whole numbers with even digits only?

5. There are 100 groups of pebbles on a table containing 1, 2, 3, ..., 99, 100 pebbles, respectively. In one step you may reduce the number of pebbles of any number of groups as long as you take the same number of pebbles from each group you selected. What is the least number of steps you can take all the pebbles off of the table?

6. Ten people are sitting around a round table. Everybody thinks of a number and whispers it to his two neighbors. Then, everybody announces the average of the two numbers he heard. These results are shown on the diagram. What number did the person who said 6 think of?



7. Mr. and Mrs. Brown gave a party for their friends they have not seen for a long time. Three couples came. During the party, some of the people were so happy to see each other again, that they even shook hands. (None of the men shook hands with their own wives.) Later on, Mr. Brown asked everybody how many people he/she shook hands with. He received seven different answers. How many guests did Mrs. Brown shake hands with?

8. Where is the ratio of the numbers containing the digit 7 to those that do not greater: among the 2-digit numbers or among the 4-digit numbers?

9. A and B are positive whole numbers. None of them is divisible by 10, and their product is 10,000. What is their sum?

10. Find the last two digits of

$$7 + 7^2 + 7^3 + \cdots + 7^{2008}.$$

11. There are many different roads between Town A, Town K, and Town F. We know that between any two of these towns the number of direct roads is at least 3 but no more than 10. You can get from Town A to Town F directly or through Town K in a total of 33 different ways. Similarly, you can get from Town K to Town F directly or through Town A in a total of 23 different ways. In how many different ways can you get from Town K to Town A?

12. Angle A of the convex quadrilateral $ABCD$ is 100 degrees. We know that diagonal AC breaks up the quadrilateral into an equilateral and an isosceles triangle. How big are the inner angles in $ABCD$?

13. We glue together 27 regular dice into a $3\times3\times3$ cube. What is the least amount of dots you can see on this cube? (On the regular dice then number of dots are 1 to 6, and the dots on the facing sides add up to 7.)

14. The sides of a rectangle are 14 cm and 24 cm. We draw the diagonal from one vertex, and we draw straight segments from this vertex to the mid-, third-, and quarter-points of the longer side facing this vertex, a total of 6 segments including the diagonal). What are the areas of the triangles we created?

15. One year a monthly calendar looked like the diagram below. The sum of the numbers in one of the 3×3 segments of this calendar is 162. What is the smallest number in that 3×3 section?

M	Tu	W	Th	F	Sa	Su
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

1. The ages of a father and his two different-aged-sons are the powers of the same prime number. Last year everybody's age was a prime number. How old are they now?

2. Could the sum of seven consecutive whole numbers be a prime number?

3. A 5-digit number is divisible by 7, 8, and 9. The number created from the first two digits is a prime number, 1 greater than a square number, and the sum of these two digits is a two-digit number. Find this 5-digit number.

4. Every digit of a 5-digit number is either 1 or a prime number. Not only that, but any number created by any 2, 3, or 4 consecutive digits of this number are prime numbers also. Find this number, and check if it is a prime number or not.

5. Can you put the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 into two groups so that the sums of the numbers in each group is the same? Can you do this to get the same product in each group?

6. Are there any five consecutive whole numbers which can be put into two groups so that the product of the numbers in each group is the same?

7. When 22022 and 20222 are divided by the same 3-digit number, they give the same remainder. Which one of these divisors can be determined by the remainder?

8. Find a, b, c, and d, such that $a^{bb} = bccbdc$.

9. An extra terrestrial from Mars came to visit the Earth. Martians eat only once a day the most, either in the morning, or at noon, or in the evening, but only if they feel like eating. They can go without eating for any number of days. While the Martian was here, it ate 7 times. We also know that it spent 7 mornings, 6 noons, and 7 evenings without eating. How many days did the Martian spend here on Earth?

10. There are 3 diagonals running into the same vertex of a rectangular based column. Prove that you can always construct a triangle from these three segments.

11. Is it true that if the product of 6 positive whole numbers ends in exactly two zeros, then you can find 4 out of these 6 numbers for which the same is true?

12. We write all the integers from 1 to 2001 on a circular line in a random order. Then we switch some of the neighbors using the following procedure. We start at one of the numbers. We compare this number to its neighbor in the clockwise direction. If the neighbor is smaller then we switch the two numbers, otherwise we leave them as they were. In the next step we compare the bigger of these two numbers to its clockwise neighbor and do a switch if the neighbor is smaller. We keep repeating these steps. We say that we completed a cycle if we compared the number in the last position to the number in the position we started at. How many cycles do we have to complete before the positions of the numbers are the same as their original positions were?

13. Using the grid lines, Tom draws a rectangle on a paper with unit-square grid lines on it. Staying on the grid lines, he wants to draw a closed figure inside of this rectangle, so that he would not go outside of this rectangle, but he would go through every grid point on the border and the inside of this rectangle exactly once. How long is the line he has to draw if the dimensions of the rectangle are 2000×2001?

14. Find such digits x and y for which the 6-digit number $xyxyxy$ (in base 10) has a 3-digit prime divisor.

15. With its 2 diagonals, brake a convex quadrilateral into 4 triangles. Prove that the products of the areas of the 2-2 triangles facing each other are the same.

16. From the number

$$1234567890123456789 0 \ldots 1234567890,$$

which has 5000 digits, take out the digits located on an odd number location. We do the same with the remaining 2500-digit number, and continue until we have a one-digit number. What number is this last digit?

1. What is the greatest possible common divisor of 6 different 2-digit positive whole numbers?

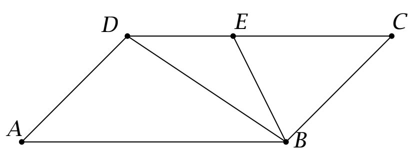
2. We built a big cube from identical size smaller cubes.1/8 of the small cubes used was red, 1/4 was white and the rest were green. We used more than 300 green cubes. How many small cubes did we use if we tried to use as few as possible?

3. In a store you ask for 5 pieces of candy put in a paper bag. Then you ask for 10 pieces of candy put in another similar paper bag. Each piece of candy has the same mass. The first bag measures 85 grams, the second bag measures 165 grams. How much does each paper bag cost if the price of a bag of candy weighing 1 kg is $12?

4. Draw an octagon into a circle. Find the sum of four of its inner angles if no two of these angles are next to each other.

5. Cut the perimeter of a convex quadrilateral $ABCD$ at its vertices. Place the four segments parallel to their original positions starting from one common point O. Connecting the outer ends of these segments you get a new quadrilateral $XYZV$. How many times greater is the area of $XYZV$ than the area of $ABCD$?

6. The parallelogram $ABCD$ on the diagram below is cut into three isosceles triangles such that $DA = DB$, $EB = ED$, $CE = CB$. If the measure of the angle $\angle DAB = x^\circ$, find the value of x.



7. We want to cut up a big cube into smaller pieces. First we cut it a few times with vertical planes parallel to two of it facing sides, then we cut it a few times with vertical planes again but in a perpendicular direction to the previous planes. Finally we cut it a few times horizontally, also. All this time the cube was resting on one of its faces on the floor. With every cut we cut through the cube completely. We cut the cube a total of 175 times, at least once in all three of the previously mentioned directions. Could the number of smaller pieces created be 2145?

8. How many times does the digit 1 appear in base 10 in the number $N = 9 + 99 + 999 + \cdots + \underbrace{999\ldots99}_{1998 \text{ nines}}$?

9. Find the smallest positive whole number that defeats the following statement: "If the sum of the digits of a whole number n is divisible by 6, then n is divisible by 6."

10. Write a digit in front and at the end of the number 1998, so that the new 6-digit number is divisible by 99.

11. Write a digit in front and at the end of the number 1998, so that the new 6-digit number is divisible by 88.

12. Find the smallest positive whole number in which the number created from the first two digits is divisible by 2, the number created from the first three digits is divisible by 3, ..., the number created from the first eight digits is divisible by 8, and the number itself is divisible by 9.

13. Using the digits 1, 2, 3, 4, 5, and 6 once and only once, find 6-digit numbers in which the number created from the first two digits is divisible by 2, the number created from the first three digits is divisible by 3, the number created from the first four digits is divisible by 4, the number created from the first five digits is divisible by 5, and the number itself is divisible by 6.

1. Are there three such prime numbers that have a sum of 1234 and a product of 87654321?

2. Using the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 once and only once, find 9-digit numbers in which the number created from the first two digits is divisible by 2, the number created from the first three digits is divisible by 3, ..., the number created from the first eight digits is divisible by 8, and the number itself is divisible by 9.

3. By adding an extra digit anywhere in the number 975 312 468, create a 10-digit number that is divisible by 33.

4. The sum of 49 positive whole numbers is 999. How high could the greatest common factor of these numbers be?

5. Laurie picked three consecutive numbers. Then she took two of them in every possible combination and multiplied them. Could the sum of these products be 3 000 000?

6. Two bicycle clubs organize a tour together. At the meeting in the morning members great each other with a handshake. Everybody shakes hands with everybody once. There were a total of 231 handshakes but 119 of them happened between members of the same club. How many members came from each club?

7. Find all those 3-digit numbers that are divisible by 7, and they give the same remainder when divided by either 4, 6, 8, or 9.

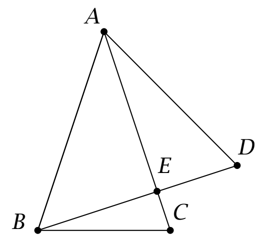
8. Susie was wondering: "Isn't it interesting that my mother's age is half of the sum of my father's and my age; my father and my mother together are 100 years old; both my father's and my mother's age is prime?" How old is Susie?

9. In the following multiplication ⊕ same letters mean the same digits, different letters mean different digits. What could the value of the product be?

$$\text{BIG}\oplus\text{BIG} = \text{LOTBIG}.$$

10. Seven dwarves are sitting around a round table with a mug in front of each with some milk in it. (Some mugs might be empty.) There is a total of a half a liter of milk in the mugs. One dwarf stood up and distributed his milk evenly among the other dwarves. Then, one by one, everybody towards his right did the same thing. After the seventh dwarf distributed his milk, everybody ended up having the same amount of milk than what they started with originally. How much milk was in each mug?

11. Triangles ABC and ABD are isosceles so that $AB = AC = BD$. Segments AC and BD intercept each other in point E. What is the sum of angles ACB and ADB if BD and AC are perpendicular to each other?



12. Using 6 given colors, you color each side of a cube to a different color, then you write the six numbers on it so that the numbers 6 and 1; 2 and 5; 3 and 4 are facing each other. How many different cubes can you make? (Two cubes are considered to be the same if you can rotate one cube into the position of the other.)

13. How many different ways can you cover a 2×10 rectangle by using 2×1 dominoes?

14. In an isosceles triangle T, the length of one of the segments connecting a vertex and the midpoint of the opposite side is the same as the length of one of the segments connecting the midpoints of two sides. How big could the greatest angle of T be?

15. Steve forgot the combination of his lock. He remembers only that the first digit is 7, and the fifth digit is 2. He knows that it is a 6-digit odd number, and that it gives the same remainder when divided by 3, 4, 7, 9, 11, and 13. What is the number?

16. There are two American, one English, one French, one Russian, and and German swimmers in the final of a swimming competition. How many different possible final results have at least one American swimmer on the top three places if every swimmer finished on a different place?

1. N is a five-digt number that has the following property: if you write down from left to right the remainders N gives when dividedy by 2, 3,4,5 and 6, you get the original number N. What is the sum of all possible values of N?

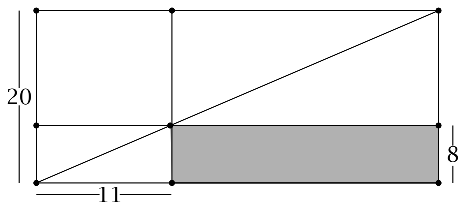
2. Divide the number $3\underbrace{00000\cdots00000}_{99\text{ zeros}}7$ by 37. What is the remainder? What are the first nine and last nine digits of the quotient?

3. A tile worker ordered tiles to cover the floor of a squared shape hall. However, he was so absent minded that, instead of the number of tiles needed along one side of the hall, he put down his own age. This way he received 1111 more tiles than necessary. How old is the tile worker?

4. We added three consecutive numbers then we added the next three consecutive numbers. Could the product of these two numbers be 111111111?

5. The sides of a right triangle are 5, 12 and 13 units. What is the radius of the inscribed circle of this triangle?

6. We divided a rectangle by two straight lines parallel to its sides, so that the lines intersect on the diagonal of the rectangle, as shown on the diagram. With the distances given, determine the area of the section shaded.



7. A flee is jumping randomly to the left and to the right on a long, thin stick. Every jump is 10 cm long. How many different ways can it get 60 cm to the right of its starting point using 10 jumps?

8. Find such a 6-digit number (in base 10) that if you multiply the number by either 2, 3, 4, 5, or 6, you get a number that you could have gotten from the original number, too, just by sliding its digits by a few places. Sliding here means that you take a few digits from the end of the number and write them in the front of the number in the same order. (For example: from $abcdef$, you can get $fabcde$, or $efabcd$, or $defabc$, or the like)

9. Find the fraction p/q with the smallest denominator, so that

$$\frac{99}{100} < \frac{p}{q} < \frac{100}{101},$$

where p and q are positive whole numbers.

10. Steve forgot the combination of his lock. He remembers only that the first digit is 7, and the fifth digit is 2. He knows that it is a 6-digit odd number, and that it gives the same remainder when divided by 3, 4, 7, 9, 11, and 13. What is the number?

11. There are two American, one English, one French, one Russian, and and German swimmers in the final of a swimming competition. How many different possible final results have at least one American swimmer on the top three places if every swimmer finished on a different place?

12. Teri thought of a positive whole number. She told me that it has 12 positive divisors, including 6 and 25. What is Teri's number?

13. Six people are estimating how many balls there are in a box. The guesses are: 52, 59, 62, 65, 49, 42. Nobody guessed it right, and the differences between the guesses and the actual number of balls are: 1, 4, 6, 9, 11, 12. How many balls are in the box?

14. One day 12 children from a class went to see a movie. On another day 9 children from the same class went to see a puppet show. 5 children attended both programs, and 10 children from the class did not go to either program. How many children are there in the class?